R. GLADKOVA and N. KUTYLOVSKAYA

Selected Questions and Problems in Physics

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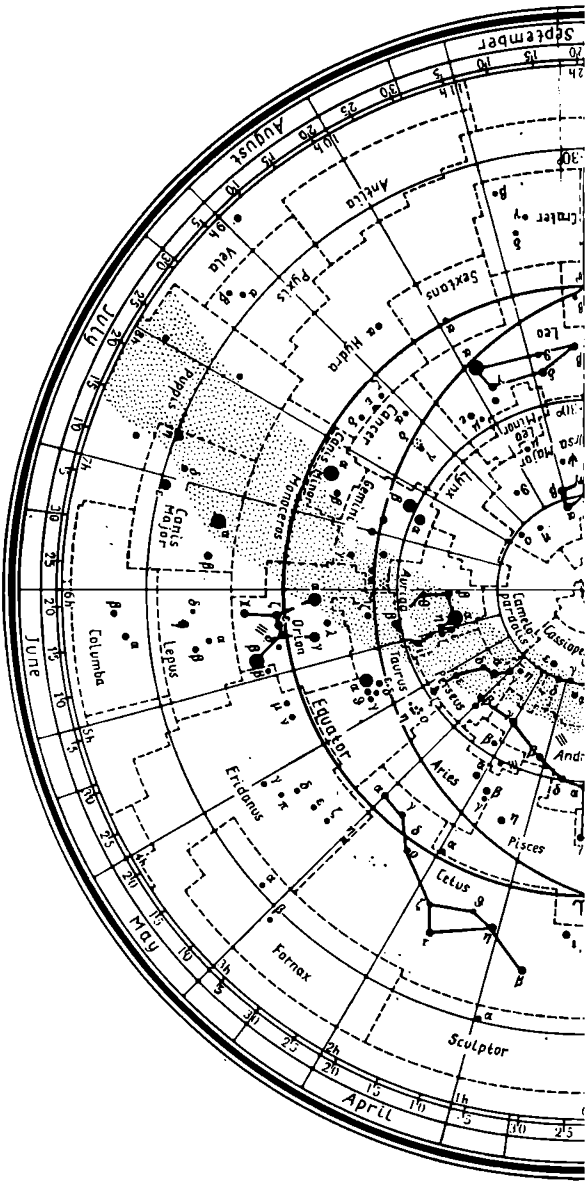

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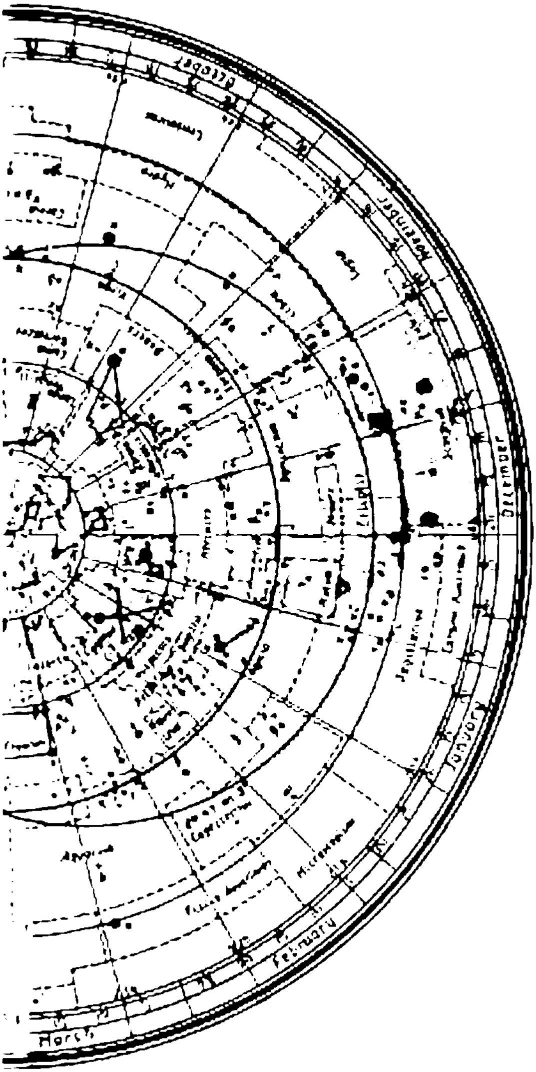

SELECTED QUESTIONS AND PROBLEMS IN PHYSICS

Р. А. Гладкова, Н. Н. Кутыловская

Сборник задач и вопросов по физике

«Высшая школа» Москва

R. GLADKOVA and N. KUTYLOVSKAYA

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Contents

Preface

This collection of questions and problems in physics is intended for the students of correspondence courses and evening classes in intermediate colleges and is in accord with the existing curriculum.

The purpose of this book is to teach the students how to solve problems in physics. This should stimulate correspondence course students to work independently, encourage them to accumulate an adequate theoretical background, and develop in them the requisite aptitude for practical activity in various branches of the economy.

Each section begins with a brief description of the basic theoretical concepts, laws, and formulas. This provides the maximum possible help to correspondence course students in solving problems. A large number of problems are supplied with detailed solutions and an analysis of the results, while in some cases different approaches are used to solve the same problem so that the student can discover the most rational form of independent study.

The theoretical material is presented in a lucid form, and most problems are of medium complexity. However, each section contains tougher problems as well. Their solution requires a broader range of theoretical data, and will facilitate a deeper understanding of the physics course.

In keeping with the existing curriculum, problems in astronomy have also been included in the collection. Their solution requires the use of a star chart, which is printed on the fly-leaf.

The authors are grateful to A. L. Kosorukov, a researcher at the Institute of Applied Mathematics of the USSR Academy of Sciences, for the help in compiling the problems.

R. Gladkova
N. Kutylovskaya

General Methodical Instructions

Before starting a new chapter in this course, go through the contents of the textbook for this chapter, and write down the topics in a separate notebook. Having carefully read the list of topics, write down the numbers of the relevant sections in the margin.

While studying the material in a section, it is necessary to read through the entire section without stopping at the difficult parts. During the second reading, think about the meaning of each sentence, and write down the derivation of the formulas, the definitions of the physical quantities and their units, as well as the formulation of the relevant laws of physics. Use the textbook whenever necessary. Your study of the section should be completed by revising the material, repeating the examples and explanations, and making notes of the derivations and diagrams. If you can reproduce the entire material without looking at either the textbook or your notes, it is safe to assume that you have grasped the subject.

If difficulties are encountered while reading the theoretical material, and if it is not possible to overcome these difficulties independently, you must ask your teacher and get oral or written consultation before revising the main and supplementary material.

It is not possible to get a firm grip over theoretical material without solving problems. Moreover, the solution of problems helps when memorizing laws and formulas. Hence attention must be paid to the solution of problems.

Before solving a problem, read the conditions and write them in your notebook without abbreviations after understanding the essence of the problem. Using the standard

notation for physical quantities, write down the given quantities in SI units in the notebook. After this, write down the quantities to be determined. Use the tables in the Appendices for the extra quantities required to solve the problem.

Having understood the physical phenomenon and using the physical laws applicable to a current problem, write down the necessary formulas to express the required quantity. In other words, solve the problem in the general form analytically. Numerical values should be substituted in the final expression together with the units of the quantities. The units of quantities should be simplified first, followed by the numerical values.

Obey the rules for operations with numbers. Whenever possible, use mathematical tables. All calculations should be made with a pocket calculator or a slide rule. The correctness of the solution can be verified by comparing the result with the actual values of the quantities. Explanatory notes and figures should accompany each solution. The answers must be based on the laws studied before solving the problem, and should reveal the essence of the phenomenon involved. Solutions of typical problems and detailed explanations are included to help students tackle the problems. While studying the course on physics, students must pass tests to determine the level of their understanding of a topic, and also do practical work to strengthen their theoretical background as well as to acquire a practical knowledge of the way equipment is handled.

At the end of the course, correspondence course students have to pass examinations, where they must demonstrate a sound knowledge of theory, the formulation of laws, formulas, and units of physical quantities. They must also learn how to identify problems in everyday practice on the basis of their knowledge of physical laws, and also how to solve physical problems.

Chapter I

Fundamentals of Molecular Physics and Thermodynamics

§ 1. MOLECULAR KINETIC THEORY. MOTION OF MOLECULES, THEIR SIZE AND MASS

Basic Concepts and Formulas

The molecular kinetic theory explains the structure and properties of bodies through the motion and interaction of the small particles constituting them. The theory is based on three postulates:

1. All substances consist of very small particles, viz. molecules.[1] Molecules, in turn, consist of still smaller particles, viz. atoms, which themselves are made up of protons, electrons, and neutrons.

2. Molecules are in a state of perpetual random motion known as thermal motion. As a result, every molecule possesses a kinetic energy.

3. Molecules of a substance interact with one another. The forces of interaction (attractive and repulsive) are electrical in origin. They depend both on the nature of the molecules and on their separation. Consequently, molecules possess a potential energy.

The internal energy of a body is defined in molecular physics as the sum of the kinetic energies of all the molecules and the potential energies of their interaction.

The number of molecules in any body is extremely large, while their size and mass are very small.

The SI unit of the amount of substance, viz. the mole, is a base unit. The mole is the amount of substance containing the same number of structural elements as the number of ^{12}C atoms in a mass of 0.012 kg.

A mole of any substance contains the same number of molecules. This number is known as the Avogadro constant

[1] A molecule is the smallest particle of a substance that retains its chemical properties.

N_A. The mass of a molecule is

$$m_0 = M/N_A,$$

where M is the molar mass.

The number of molecules in 1 m^3 of a substance is determined from the formula

$$n = N_A\rho/M,$$

where ρ is the density of the substance.

The number of molecules in a given mass of a substance is

$$N = mN_A/M,$$

where m is the mass of the substance and $m/M = \nu$ is the number of moles.

A gas whose molecules do not attract one another is called an ideal gas.

Under normal conditions ($T_0 = 273$ K and $p_0 = 101325$ Pa), the molar volumes of all ideal gases are the same: $V_m = 22.4 \times 10^{-3}$ m^3/mol.

The Loschmidt number N_L is the number of gas molecules in 1 m^3 of a substance under normal conditions:

$$N_L = N_A/V_m,$$

where V_m is the molar volume defined as

$$V_m = M/\rho_0.$$

The mean distance $\bar{l}$ between molecules is defined as

$$\bar{l} = \sqrt[3]{V/N},$$

where V/N is the elementary volume per molecule.

The mean free path $\bar{\lambda}$ is the mean distance covered by a molecule between two successive collisions:

$$\bar{\lambda} = \bar{v}/\bar{z},$$

where $\bar{v}$ is the arithmetic mean velocity of the molecules and $\bar{z}$ is the average number of collisions per second:

$$\bar{z} = \sqrt{2}\,\pi d_{eff}^2 \bar{v} n_0.$$

Here d_{eff} is the effective diameter of a molecule and n_0 is the number of molecules in 1 m^3 of a substance.

Worked Problems

Problem 1. Determine the amount of substance (in moles) contained in (a) 1 kg of mercury, and (b) 5.6 dm^3 of oxygen under normal conditions.

Given: $m_1 = 1$ kg is the mass of mercury, $V_0 = 5.6$ dm^3 = 5.6×10^{-3} m^3 is the volume of oxygen under normal conditions. From tables, we find the molar mass of mercury, $M_1 = 200.6 \times 10^{-3}$ kg/mol, the molar mass of oxygen, $M_2 = 32 \times 10^{-3}$ kg/mol, and the density of oxygen under normal conditions, $\rho_0 = 1.43$ kg/m^3.

Find: the amount of substance ν_1 in 1 kg of mercury and the amount of substance, ν_2, in 5.6 dm^3 of oxygen under normal conditions.

Solution. The amount of substance in 1 kg of mercury can be determined as follows:

$$\nu_1 = m_1/M_1, \quad \nu_1 = \frac{1 \text{ kg}}{200.6 \times 10^{-3} \text{ kg/mol}} = 4.98 \text{ mol}.$$

In order to determine the amount of substance in 5.6 dm^3 of oxygen, we calculate the mass m_2 of oxygen: $m_2 = \rho_0 V_0$. This gives

$$\nu_2 = \frac{\rho_0 V_0}{M_2}, \quad \nu_2 = \frac{1.43 \text{ kg/m}^3 \times 5.6 \times 10^{-3} \text{ m}^3}{32 \times 10^{-3} \text{ kg/mol}} = 0.25 \text{ mol}.$$

Answer. One kg of the mercury contains about 5 mol, and 5.6 dm^3 of the oxygen contain about 0.25 mol.

Problem 2. Calculate the number of molecules contained in 0.5 kg of oxygen and in 5.0 cm^3 of carbon dioxide under normal conditions.

Given: $m_1 = 0.5$ kg is the mass of oxygen, $V_0 = 5.0$ cm^3 = 5.0×10^{-6} m^3 is the volume of carbon dioxide under normal conditions. From tables, we find the molar mass of oxygen, $M_1 = 32 \times 10^{-3}$ kg/mol, and the molar mass of carbon dioxide, $M_2 = 44 \times 10^{-3}$ kg/mol (it is equal to the sum of the molar masses of carbon and oxygen), the Avogadro constant $N_A = 6.022 \times 10^{23}$ mol^{-1}, the molar volume V_m = 22.4×10^{-3} m^3/mol, and the Loschmidt number N_L = 2.68×10^{25} m^{-3}.

Find: the number N_1 of molecules in 0.5 kg of oxygen and the number N_2 of molecules in 5.0 cm^3 of carbon dioxide under normal conditions.

Solution. The amount of substance in moles is $\nu = m_1/M_1$. Knowing that one mole contains N_A molecules, we determine the number of molecules in 0.5 kg of oxygen:

$$N_1 = \nu N_A = m_1 N_A/M_1;$$

$$N_1 = \frac{0.5\ \text{kg}}{32 \times 10^{-3}\ \text{kg/mol}} \times 6.022 \times 10^{23}\ \text{mol}^{-1} \simeq 9.4 \times 10^{24}.$$

In order to find the number of carbon dioxide molecules, N_2, contained in 5.0 cm^3 under normal conditions, we use the relation

$$N_2 = N_A V_0/V_m,$$

where N_A/V_m is the number of molecules in 1 m^3. This gives

$$N_2 = \frac{6.022 \times 10^{23}\ \text{mol}^{-1}}{22.4 \times 10^{-3}\ \text{m}^3 \cdot \text{mol}^{-1}} \times 5 \times 10^{-6}\ \text{m}^3 = 1.34 \times 10^{20}.$$

Remark. The second part of the problem can be solved quite easily if we consider that 1 m^3 of a gas contains N_L molecules under normal conditions. Then $N_2 = N_L V_0 = 2.68 \times 10^{25}\ \text{m}^{-3} \times 5.0 \times 10^{-6}\ \text{m}^3 = 1.34 \times 10^{20}$.

Answer. The number of molecules in 0.5 kg of oxygen is approximately 9.4×10^{24}, while 5.0 cm^3 of carbon dioxide contain 1.34×10^{20} molecules.

Problem 3. The molar mass of oxygen is 32×10^{-3} kg/mol. Determine the molar mass of air if the densities of oxygen and air under normal conditions are 1.43 and 1.29 kg/m^3 respectively.

Given: the molar mass of oxygen, $M_1 = 32 \times 10^{-3}$ kg/mol, the density of oxygen, $\rho_{01} = 1.43$ kg/m^3, and the density of air, $\rho_{02} = 1.29$ kg/m^3.

Find: the molar mass M_2 of air.

Solution. The molar mass M can be expressed in terms of the mass m_0 of a molecule and the Avogadro constant N_A:

$$M = m_0 N_A.$$

The density ρ_0 of a gas under normal conditions is determined from the relation $\rho_0 = m_0 N_L$, where N_L is the Loschmidt number.

Let us divide termwise the expressions for molar mass and density: $M/\rho_0 = N_A/N_L$. But the ratio N_A/N_L of the two constants is a constant. Consequently, the molar masses of

two gases are directly proportional to their densities under normal conditions:

$$M_1/\rho_{01} = M_2/\rho_{02} = \ldots = N_A/N_L.$$

This gives

$$M_2 = \rho_{02}M_1/\rho_{01},$$

$$M_2 = \frac{1.29\ \text{kg/m}^3}{1.43\ \text{kg/m}^3} \times 32 \times 10^{-3}\ \text{kg/mol} \simeq 29 \times 10^{-3}\ \text{kg/mol}.$$

Answer. The molar mass of air is approximately 29×10^{-3} kg/mol.

Problem 4. Determine the mass of an acetylene molecule C_2H_2 and the density of acetylene under normal conditions.

Given: the chemical formula of acetylene is C_2H_2. From tables, we find that the molar masses of carbon and hydrogen are $M_{C_2} = 24 \times 10^{-3}$ kg/mol and $M_{H_2} = 2 \times 10^{-3}$ kg/mol respectively. The Loschmidt number is $N_L = 2.68 \times 10^{25}\ \text{m}^{-3}$ and the Avogadro constant is $N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$.

Find: the mass m_0 of an acetylene molecule and the density ρ_0 of acetylene under normal conditions.

Solution. The mass of an acetylene molecule can be determined from the relation

$$m_0 = M/N_A.$$

The acetylene molecule C_2H_2 consists of carbon and hydrogen. Therefore, the molar mass of acetylene can be determined from the molar masses of its components: $M = M_{C_2} + M_{H_2} = 26 \times 10^{-3}$ kg/mol.

Remark. The molar mass of any chemical compound, for example, ammonia NH_3, can be determined in this way:

$$M = M_N + 3M_H = 17 \times 10^{-3}\ \text{kg/mol}.$$

For an acetylene molecule, we have

$$m_0 = \frac{26 \times 10^{-3}\ \text{kg/mol}}{6.022 \times 10^{23}\ \text{mol}^{-1}} = 4.32 \times 10^{-26}\ \text{kg}.$$

Knowing the mass m_0 of a molecule and the number of molecules in 1 m^3 (equal to N_L), we can determine the density ρ_0 of acetylene under normal conditions:

$$\rho_0 = m_0 N_L, \quad \rho_0 = 4.32 \times 10^{-26}\ \text{kg} \times 2.68 \times 10^{25}\ \text{m}^{-3} \simeq 1.16\ \text{kg/m}^3.$$

Answer. The mass of an acetylene molecule is approximately 4.3×10^{-26} kg. The density of acetylene under normal conditions is 1.16 kg/m^3.

Problem 5. Calculate the approximate size of a water molecule assuming that molecules are spherical and touch each other.

Given: the chemical formula of water is H_2O. From tables, we find the density of water, $\rho = 1 \times 10^3$ kg/m^3, at room temperature and the molar mass of water, $M = 18 \times 10^{-3}$ kg/mol (see Problem 4). The Avogadro constant is $N_A = 6.022 \times 10^{23}$ mol^{-1}.

Find: the volume V and the diameter d of a water molecule.

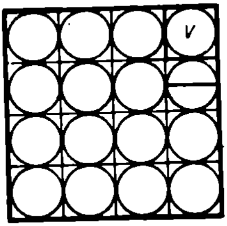

Fig. 1

Solution. Knowing the density and the molar mass of water, we can determine its molar volume V_m:

$$V_m = M/\rho, \quad V_m = \frac{18 \times 10^{-3}\ \text{kg/mol}}{1 \times 10^3\ \text{kg/m}^3} = 1.8 \times 10^{-5}\ \text{m}^3/\text{mol}.$$

One mole of any substance is known to contain N_A molecules. Consequently, the volume of a water molecule can be determined by dividing the molar volume by the Avogadro constant:

$$V = \frac{V_m}{N_A}, \quad V = \frac{1.8 \times 10^{-5}\ \text{m}^3/\text{mol}}{6.022 \times 10^{23}\ \text{mol}^{-1}} \simeq 0.3 \times 10^{-28}\ \text{m}^3.$$

It was stated in the problem that water molecules are spherical and closely packed (Fig. 1), i.e. the gaps between the molecules are negligibly small. Hence we can assume that the elementary volume V taken from the molar volume V_m will contain one molecule whose diameter is approximately equal to the edge of the cube:

$$d = \sqrt[3]{V},$$

$$d = \sqrt[3]{0.3 \times 10^{-28}\ \text{m}^3} \simeq 3 \times 10^{-10}\ \text{m}.$$

In the general form, the solution is

$$d = \sqrt[3]{V} = \sqrt[3]{V_m/N_A} = \sqrt{M/\rho N_A}.$$

Answer. The volume of a water molecule is approximately 3×10^{-29} m^3, and the diameter of a molecule is about 3×10^{-10} m.

Problem 6. Calculate the mean distance between the centres of molecules of an ideal gas under normal conditions.

Solution. The molar volume under normal conditions is $V_m = 22.4 \times 10^{-3}$ m³/mol. One mole of any substance contains $N_A = 6.022 \times 10^{23}$ molecules. If the molecules are assumed to be distributed uniformly over the entire volume, the volume per molecule is $V = V_m/N_A$.

This volume can be regarded as a cube whose edge is equal to the mean distance d between the molecules (see Fig. 1):

$$d = \sqrt[3]{V_m/N_A}, \quad d = \sqrt[3]{\frac{22.4 \times 10^{-3}\ \text{m}^3/\text{mol}}{6.022 \times 10^{23}\ \text{mol}^{-1}}} \simeq 3.3 \times 10^{-9}\ \text{m}.$$

Answer. The mean distance between gas molecules under normal conditions is approximately 3.3×10^{-9} m.

Problem 7. The arithmetic mean velocity of nitrogen molecules under normal conditions is 453 m/s. Find the mean free path, the mean free time, and the mean momentum of a molecule if it undergoes 7.55×10^9 collisions per second.

Given: the arithmetic mean velocity $\bar{v} = 453$ m/s of a nitrogen molecule under normal conditions, the average number of collisions of the molecule per second, $\bar{z} = 7.55 \times 10^9\ \text{s}^{-1}$. From tables, we find the molar mass of nitrogen, $M = 28 \times 10^{-3}$ kg/mol, and the Avogadro constant, $N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$.

Find: the mean free path $\bar{\lambda}$, the mean free time $\bar{t}$, and the mean momentum $\bar{p}$ of a molecule.

Solution. As a result of the random motion, the values of λ, t, and p for molecules do not remain constant. Therefore, we must determine the mean values of these quantities:

$$\bar{\lambda} = \frac{\bar{v}}{\bar{z}}, \quad \bar{\lambda} = \frac{453\ \text{m/s}}{7.55 \times 10^9\ \text{s}^{-1}} = 6.0 \times 10^{-8}\ \text{m},$$

$$\bar{t} = \frac{\bar{\lambda}}{\bar{v}}, \quad \bar{t} = \frac{6.0 \times 10^{-8}\ \text{m}}{453\ \text{m/s}} = 1.3 \times 10^{-10}\ \text{s}$$

In order to determine the momentum of a nitrogen molecule, we must find its mass: $m_0 = M/N_A$.

The mean momentum $\bar{p}$ of the molecule can be calculated thus

$$\bar{p} = m_0\bar{v} = (M/N_A)\,\bar{v},$$

$$\bar{p} = \frac{28\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\ \text{mol}^{-1}}\times 453\ \text{m/s} \simeq 2.11\times10^{-23}\ \text{kg}\cdot\text{m/s}.$$

Answer. The mean free path and the mean free time are 6.0×10^{-8} m and 1.3×10^{-10} s respectively. The mean momentum of a nitrogen molecule is 2.11×10^{-23} kg·m/s.

Questions and Problems

1.1. What experimental facts clearly confirm the random nature of molecule motion and the relationship between the intensity of this motion and the temperature?

1.2. Why is diffusion in liquids much slower than it is in gases?

1.3. What physical process occurs when the surface of a solid is painted?

1.4. Common salt placed in water is uniformly distributed over the entire volume a certain time after being added. Explain this phenomenon.

1.5. Why do gauge blocks (Johansson blocks) stick together when their end faces are brought in contact (Fig. 2)?

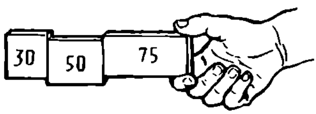

Fig. 2

1.6. What phenomenon is responsible for gluing solids?

1.7. One of the largest nuggets of gold, with a mass of 62.3 kg, was found at the mouth of the Amazon river. How much substance is contained in it?

1.8. Determine the mass of 1 kmol of carbon, nitrogen, and helium.

1.9. What is the mass of 50 mol of oxygen?

1.10. How many molecules are contained in 32 kg of oxygen and in 2 g of hydrogen?

1.11. What is the volume occupied by 7×10^{28} molecules of carbon dioxide under normal conditions?

1.12. Determine the amount of substance contained in 6 g of carbon dioxide. How many molecules constitute this mass?

1.13. What is the volume occupied by 0.6×10^{23} atoms of graphite? The density of graphite is known.

1.14. The density of brass is 8500 kg/m^3. What does this mean?

1.15. The density of aluminium is 2.7×10^3 kg/m^3. How much substance is contained in 1 m^3 of aluminium?

1.16. Determine the mass of a molecule and of an atom of oxygen, nitrogen, and helium.

1.17. Calculate the molar mass of methane and the mass of a methane molecule CH_4.

1.18. A 1-μm layer of silver is deposited on the surface of a metal mirror. How many silver atoms would be in a surface layer with area 25 cm^2?

1.19. A grain of common salt having a mass of 3×10^{-3} g is dissolved in 10 l of water and uniformly distributed over the entire volume. How many molecules of salt are contained in 5 cm^3 of the solution?

1.20. What is the ratio between the masses of a 10^{-8}-g dust particle and an air molecule? The molar mass M of air is 29×10^{-3} kg/mol.

1.21. Comparing the densities of air and hydrogen, find the ratio between the masses of their molecules.

1.22. Determine the mass of a propane molecule C_3H_8 and the density of propane under normal conditions.

1.23. Calculate the mass of a butane molecule C_4H_{10} if its density under normal conditions is 2.67 kg/m^3.

1.24. Given the density of hydrogen under normal conditions, determine its molar mass.

1.25. A drop of mineral oil with a mass of 0.023 mg is poured on the surface of water and forms a film 60 cm^2 in area. Assuming that the molecules in the film are arranged in one row, determine the size of a molecule.

1.26. Determine the approximate mass and size of a molecule of carbon disulfide CS_2 assuming that the molecules are closely packed and spherical.

1.27. What will be the length of a chain of the molecules contained in 1 mg of water and closely arranged in a row? How many times can such a chain be wound around the globe along the equator if the equator is 4×10^7-m long and the diameter of a molecule is 2.69×10^{-10} m?

1.28. The densities of hydrogen and methane under normal conditions are 0.09 and 0.72 kg/m^3 respectively. Find the molar mass of methane if the molar mass of hydrogen is 2×10^{-3} kg/mol.

1.29. Determine the fraction of volume occupied by the gas molecules in a vessel containing a gas under normal conditions. The diameter of a molecule should be assumed to be 10^{-10} m.

1.30. The mean velocity of a carbon dioxide molecule is 362 m/s, and the average number of collisions of a molecule per second is 9×10^9. Calculate the mean free path.

1.31. The mean free path of molecules in a high vacuum is about 5000 km. What is the average number of collisions of gas molecules per second if the mean molecular velocity is 560 m/s?

1.32. The maximum altitude of the orbit of the Vostok spaceship is about 327 km above the ground. The mean free path of gas molecules at such an altitude is about 5 km. What would be the velocity of molecules if the number of collisions per second were approximately 0.11 s^{-1}?

1.33. The mean velocity of an oxygen molecule under normal conditions is 425.1 m/s. Determine the mean free path of the molecule if it undergoes on the average 6.57×10^9 collisions per second.

1.34. Determine the mean free path of air molecules under normal conditions. The effective diameter of the molecules should be taken to be 3×10^{-10} m.

1.35. Determine the mean free path of helium atoms under the conditions such that the number of atoms per unit volume (number density) is 3.2×10^{24} m^{-3}, and the effective diameter of helium atoms is 1.9×10^{-10} m.

1.36. A nitrogen molecule moves under normal conditions at a mean velocity of 454 m/s. Determine the mean momentum of the molecule.

1.37. A carbon dioxide molecule having a momentum of 2.7×10^{-23} kg·m/s undergoes 9.5×10^9 collisions per second. Determine the mean distance covered by the molecule between collisions.

§ 2. VELOCITIES OF MOLECULES. BASIC EQUATION IN THE KINETIC THEORY OF GASES

Basic Concepts and Formulas

In their thermal motion, the molecules of monatomic gases undergo translation. Molecules consisting of several atoms are in translatory as well as in rotary motion.

Brownian movement and molecular collisions point to a permanent change both in magnitude and in direction of molecular velocities. For this reason, the properties of gases are studied by using the statistical approach, which makes it possible to calculate the mean values of velocities of molecules, their energy, and other parameters.

The arithmetic mean velocity of all the molecules is

$$\bar{v} = \frac{v_1 + v_2 + \ldots + v_N}{N}, \quad \bar{v} = \sqrt{\frac{8RT}{M\pi}} \simeq 1.6 \sqrt{\frac{RT}{M}},$$

where M is the molar mass of the gas, $R = 8.31$ J/(mol·K) is the molar gas constant, and N is the number of the molecules.

The root-mean-square velocity of molecules is

$$\bar{v}_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} \simeq 1.73 \sqrt{\frac{RT}{M}}.$$

The most probable velocity of molecules is

$$v_{\mathrm{p}} = \sqrt{\frac{2RT}{M}} \simeq 1.41 \sqrt{\frac{RT}{M}}.$$

The basic equation in the kinetic theory of gases establishes a relation between the pressure of gas molecules and the kinetic energy of their translatory motion:

$$p = \frac{2}{3} n_0 \bar{E}_{\mathrm{k}} = \frac{2}{3} n_0 \frac{m_0 \bar{v}_{\mathrm{rms}}^2}{2},$$

where n_0 is the number of molecules per m^3 (number density) and $\bar{E}_{\mathrm{k}}$ is the mean kinetic energy of a molecule.

In an isochoric process, the pressure of a gas is proportional to its thermodynamic temperature: $p_1/p_2 = T_1/T_2$, and

$$E_{\mathrm{k}} = \frac{3}{2} kT.$$

This gives

$$p = n_0 kT,$$

where $k = R/N_{\mathrm{A}} = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant.

The mean values of the kinetic energy of translatory motion of molecules of different gases at the same temperature

are the same:

$$\frac{m_{01}\bar{v}^2_{rms1}}{2} = \frac{m_{02}\bar{v}^2_{rms2}}{2}.$$

Consequently,

$$\frac{\bar{v}_{rms1}}{\bar{v}_{rms2}} = \sqrt{\frac{m_{02}}{m_{01}}}.$$

Worked Problems

Problem 8. Determine the mean value of the kinetic energy and the root-mean-square velocity of helium molecules under normal conditions.

Given: the pressure and temperature of helium under normal conditions are $p_0 = 1.013 \times 10^5$ Pa and $T_0 = 273$ K respectively. From tables, we find the molar mass of helium, $M = 4 \times 10^{-3}$ kg/mol, the Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K, the Avogadro constant $N_A = 6.02 \times 10^{23}$ mol^{-1}, and the density of helium under normal conditions, $\rho_0 = 0.18$ kg/m^3.

Find: the mean kinetic energy of a helium molecule, $\bar{E}_k$, and the root-mean-square velocity of helium molecules, $\bar{v}_{rms}$.

Solution. We express the mean kinetic energy of a helium molecule in terms of the temperature: $\bar{E}_k = (3/2)\, kT_0$. For monatomic gases like helium, this will be the total kinetic energy of molecules:

$$\bar{E} = \frac{3}{2} \times 1.38 \times 10^{-23}\ \text{J/K} \times 273\ \text{K} \simeq 5.65 \times 10^{-21}\ \text{J}.$$

Since $\bar{E} = m_0\bar{v}^2_{rms}/2$, we have $\bar{v}_{rms} = \sqrt{2\bar{E}/m_0}$, where m_0 is the mass of a helium molecule, which can be expressed as the ratio of the molar mass and the Avogadro constant: $m_0 = M/N_A$.

Finally, we obtain

$$\bar{v}_{rms} = \sqrt{\frac{2\bar{E}N_A}{M}},$$

$$\bar{v}_{rms} = \sqrt{\frac{2 \times 5.65 \times 10^{-21}\ \text{J} \times 6.02 \times 10^{23}\ \text{mol}^{-1}}{4 \times 10^{-3}\ \text{kg/mol}}} \simeq 1300\ \text{m/s}.$$

Answer. The mean kinetic energy of a molecule is 5.65×10^{-21} J, the root-mean-square velocity is about 1300 m/s.

Remark. The root-mean-square velocity can also be found from the formula $\bar{v}_{rms} = \sqrt{3p_0/\rho_0}$.

Problem 9. Calculate the number of air molecules in a room of size $6 \times 4 \times 2.5$ m^3 at a temperature of 27°C and a pressure of 99.8 kPa.

Given: $V = 60$ m^3 is the volume of air in the room, $T = 300$ K is the air temperature, $p = 99.8 \times 10^3$ Pa is the air pressure. We know the Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K from tables.

Find: the number N of air molecules.

Solution. The number N of air molecules in the room can be determined from their number density (their number in 1 m^3) and the volume of the air:

$$N = n_0 V.$$

In order to find n_0, we shall use the basic equation $p = n_0 kT$ in the kinetic theory of gases, whence $n_0 = p/kT$. This gives

$$N = \frac{pV}{kT}.$$

Substituting the numerical values, we get

$$N = \frac{99.8 \times 10^3 \text{ Pa} \times 60 \text{ m}^3}{1.38 \times 10^{-23} \text{ J/K} \times 300 \text{ K}} = 1.45 \times 10^{27}.$$

Answer. The room contains 1.45×10^{27} air molecules.

Questions and Problems

2.1. Determine the arithmetic mean and the root-mean-square velocity of air and oxygen molecules at a temperature of 300 K.

2.2. Helium and neon have the same temperature. Molecules of which of the gases have a higher mean kinetic energy?

2.3. At what temperature is the root-mean-square velocity of oxygen molecules equal to 500 m/s?

2.4. What is the ratio between the root-mean-square velocities of helium and neon molecules at the same temperature?

2.5. The critical temperature of hydrogen is 32 K, while the hydrogen in the Sun's atmosphere is at about 6000 K.

Determine the root-mean-square velocities of hydrogen molecules at these two temperatures.

2.6. Calculate the mean kinetic energy of hydrogen molecules at the temperatures mentioned in Problem 2.5.

2.7. What is the temperature of a monatomic gas if the mean kinetic energy of its molecules is 0.8×10^{-19} J?

2.8. Determine the mean kinetic energy of translatory motion of all the neon molecules in 1 mol and in 1 kg at 1000 K.

2.9. What must be the temperature of hydrogen for its molecules to have the same root-mean-square velocities as that of helium at 580 K?

2.10. One cubic metre of a gas contains 2.4×10^{10} molecules at 27°C. Determine the gas pressure. What is the term applied to such a degree of rarefaction?

2.11. How many molecules are there in 0.5 m^3 of a gas at 300 K and a pressure of 120 kPa?

2.12. Determine the mean kinetic energy of monatomic gas molecules at 310 K. How many molecules are there in 1 m^3 of such a gas at 0.4 MPa?

2.13. Determine the pressure of nitrogen in an ampoule if the number density of the molecules in it at 0°C is 3.5×10^{14}.

2.14. Find the root-mean-square velocity and the mean kinetic energy of helium molecules at 20°C.

2.15. The pressure of the rarefied air between the walls of a Dewar flask is 1.33×10^{-2} Pa at 0°C. How many molecules does a cubic centimetre contain?

2.16. A vessel whose volume is 2 m^3 contains 2.4 kg of a gas. What is the gas pressure if the root-mean-square velocity of its molecules is 500 m/s?

2.17. How many air molecules are contained in a $4 \times 5 \times 3$ m^3 room at 20°C under a pressure of 90 kPa?

2.18. What is the increment in the mean kinetic energy of translation of molecules in a monatomic gas heated from 0°C to 373 K?

2.19. A gas has leaked from a 5-l cylinder because of a faulty valve. As a result, the gas pressure dropped by 2.9 kPa. The temperature in the cylinder remained unchanged at 17°C. How many molecules escaped from the cylinder?

2.20. After an electric heater had been switched on, the air temperature in the room was raised from 17 to 22°C at a

constant pressure. By what amount (in percent) was the number of air molecules in the room reduced?

2.21. Hydrogen leaked from a 1-m^3 cylinder because of a damaged valve. Initially, when the hydrogen was under a pressure of 5 MPa, its temperature was 280 K. After some time, the temperature rose to 290 K at the same pressure. How many hydrogen molecules escaped from the cylinder? What was the decrease in hydrogen mass?

2.22. The root-mean-square velocity of thermal motion of gas molecules was increased by a factor of two. Determine the change in the thermodynamic temperature of the gas and in the kinetic energy of thermal motion of molecules if the gas was monatomic.

§ 3. EQUATION OF STATE FOR AN IDEAL GAS. ISOTHERMAL, ISOCHORIC, AND ISOBARIC PROCESSES

Basic Concepts and Formulas

The physical quantities characterizing the state of a body are known as parameters. In order to characterize the state of a gas, three parameters are needed: pressure p, volume V, and temperature T. The equation relating the three parameters is known as the equation of state for an ideal gas.

For a fixed mass of a gas, the equation of state has the form

$$\frac{pV}{T} = \text{const}, \quad \text{or} \quad \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}.$$

If the gas was first under normal conditions (p_0, V_0, T_0) and then changed to a state for which the parameters became p, V, and T, the equation of state has the form

$$p_0V_0/T_0 = pV/T.$$

It should be noted that p_0 and T_0 are known and equal to 1.013×10^5 Pa and 273 K. Therefore, in order to determine V_0 it is sufficient to know the values of p, V, and T.

The equation of state of a gas is applicable to isothermal, isobaric, and isochoric processes. Indeed, by cancelling out the parameter which is constant for a given process, we obtain:

for an isothermal process

$$p_1V_1 = p_2V_2 \quad \text{for } T = \text{const},\ m = \text{const},$$

for an isobaric process

$$V_1/T_1 = V_2/T_2 \text{ or } V_1/V_2 = T_1/T_2 \text{ for } p = \text{const},\ m = \text{const},$$

and for an isochoric process

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}, \quad \text{or} \quad \frac{p_1}{p_2} = \frac{T_1}{T_2} \quad \text{for } V = \text{const},\ m = \text{const}.$$

In the general case when the mass m is known or is to be determined, the equation of state of an ideal gas is

$$pV = \frac{m}{M} RT,$$

where M is the molar mass of the gas and $R = 8.314$ J/(mol·K) is the molar gas constant.

Using this equation, we can determine the density of a gas as a function of its temperature and pressure. Dividing both sides of the equation by volume V and substituting density ρ for m/V, we obtain

$$\rho = \frac{M}{R} \frac{p}{T}.$$

For a fixed mass of a gas, M/R is constant. Consequently, the gas density is directly proportional to pressure and inversely proportional to thermodynamic temperature.

Dalton's law. If a vessel contains a mixture of several gases which do not react chemically, the pressure of the gas mixture is equal to the sum of the partial pressures of each gas taken separately, i.e.

$$p = p_1 + p_2 + p_3 + \ldots,$$

where p_1, p_2, p_3, ... are the partial pressures, i.e. the pressures exerted by each gas separately as if it alone were occupying the entire vessel.

Worked Problems

Problem 10. A cylinder contains a gas at a temperature of 17°C and a pressure of 1.0 MPa. What will be the change in the pressure if the temperature is lowered to —23°C?

Given: $t_1 = 17°C$ and $p_1 = 1.0$ MPa are respectively the temperature and pressure of the gas in the first state, and $t_2 = -23°C$ is the gas temperature in the second state.

Find: the change in pressure, Δp, in the cylinder as a result of the transition of the gas from the first to the second state.

Solution. We use the equation of state for the gas, $p_1V_1/T_1 = p_2V_2/T_2$. Since the change in gas pressure occurs with decreasing temperature but at a constant volume (an isochoric process), we have $V_1 = V_2$ and $p_1/p_2 = T_1/T_2$. Hence $p_2 = p_1T_2/T_1$ and

$$\Delta p = p_1 - p_2.$$

We shall write the parameters for the first and second states of the gas separately. The parameters of the first state are $p_1 = 1.0 \times 10^6$ Pa, $T_1 = 290$ K, and the parameters of the second state are $T_2 = 250$ K, $p_2 = ?$

Let us determine the gas pressure p_2 after the temperature has been lowered and calculate the pressure difference Δp:

$$p_2 = \frac{250\ \text{K}}{290\ \text{K}} \times 1.0 \times 10^6\ \text{Pa} = 0.86 \times 10^6\ \text{Pa},$$

$$\Delta p = (1.0 - 0.86) \times 10^6\ \text{Pa} = 0.14 \times 10^6\ \text{Pa}.$$

Answer. The gas pressure in the cylinder dropped by 0.14 MPa.

Problem 11. A 20-l vessel is filled with air at a pressure of 0.4 MPa and connected to another vessel from which all the air has been pumped out. The pressure in the two vessels equalizes at 1.0×10^5 Pa. Determine the volume of the second vessel, assuming that the process is isothermal.

Given: $p_1 = 0.4$ MPa $= 0.4 \times 10^6$ Pa and $V_1 = 20$ l $= 0.02$ m^3 are respectively the pressure and the volume of the gas in the first state and $p_2 = 1.0 \times 10^5$ Pa is the gas pressure after the second vessel has been connected to the first one.

Find: volume V of the second vessel.

Solution. An isothermal process occurs at a constant tempetature ($T_1 = T_2$) and obeys Boyle's law, which is formalized by the equation $pV =$ const, or $p_1V_1 = p_2V_2$. After the second vessel is connected to the first, the gas occupies the volume $V_2 = V_1 + V$. Consequently, $p_1V_1 = p_2 (V_1 +$

V), or

$$\frac{p_1V_1}{p_2}=V+V_1,$$

whence

$$V=\frac{p_1V_1}{p_2}-V_1, \quad V=\frac{0.4\times10^6\ \text{Pa}\times0.02\ \text{m}^3}{1.0\times10^5\ \text{Pa}}-0.02\ \text{m}^3$$
$$=0.06\ \text{m}^3.$$

Answer. The volume of the second vessel is 60 l.

Problem 12. What was the temperature of a gas if as a result of isobaric heating through 1 K its volume increased by 0.0035 of the initial value?

Given: $\Delta T = 1$ K is the change in the gas temperature and $\Delta V = 0.0035\,V_1$ is the increment in the gas volume.

Find: T_1, the initial temperature of the gas.

Solution. As a result of heating, the gas temperature has increased by 1 K, hence $T_2 = T_1 + \Delta T$. The gas volume has increased thereby from V_1 to V_2, i.e. $V_2 = V_1 + \Delta V$.

Since the process occurs at a constant pressure ($p_1 = p_2$), the equation of state can be transformed as follows:

$$\frac{V_1}{T_1}=\frac{V_2}{T_2}, \quad \text{or} \quad \frac{V_1}{T_1}=\frac{V_1+\Delta V}{T_1+\Delta T} \ \text{(Charles' law)}.$$

Let us solve this equation for T_1: $V_1T_1 + V_1\,\Delta T = V_1T_1 + \Delta VT_1$, whence

$$T_1=\frac{V_1\,\Delta T}{\Delta V}, \quad T_1=\frac{V_1\times1\ \text{K}}{0.0035V_1}=286\ \text{K}.$$

Answer. The gas temperature before heating was 286 K.

Problem 13. Two cylinders having volumes 3 l and 7 l are filled respectively with oxygen at a pressure of 200 kPa and nitrogen at a pressure of 300 kPa at the same temperature. The cylinders are connected and after a certain time each will contain a gas mixture at the same temperature. Determine the pressure of the gas mixture in the cylinders.

Given: $V_{1\text{ox}} = 3$ l and $p_{1\text{ox}} = 200$ kPa are the volume and the pressure of the oxygen prior to the connection of the cylinders, $V_{1\text{nit}} = 7$ l and $p_{1\text{nit}} = 300$ kPa are the volume and the pressure of the nitrogen prior to the connection of the cylinders.

Find: the pressure p of the gas mixture.

Solution. According to Dalton's law, the pressure of a gas mixture is equal to the sum of the partial pressures of the gases: $p = p'_{2ox} + p'_{2nit}$, where p'_{2ox} and p'_{2nit} are the partial pressures of the oxygen and the nitrogen.

After the cylinders have been connected, each gas occupies a volume equal to the sum of the volumes of the cylinders: $V'_{2ox} = V'_{2nit} = V_{1ox} + V_{1nit}$.

Let us write the parameters of the two states for the oxygen and nitrogen in SI units:

for oxygen

state *1*	state *2*
$V_{1ox} = 3 \times 10^{-3}$ m^3,	$V'_{2ox} = 3 \times 10^{-3}$ m^3 $+ 7 \times 10^{-3}$ m^3 $= 1 \times 10^{-2}$ m^3,
$p_{1ox} = 2 \times 10^5$ Pa,	$p'_{2ox} = ?$

for nitrogen

state *1*	state *2*
$V_{1nit} = 7 \times 10^{-3}$ m^3,	$V'_{2nit} = 1 \times 10^{-2}$ m^3,
$p_{1nit} = 3 \times 10^5$ Pa,	$p'_{2nit} = ?$

The process is isothermal, hence T = const.

In order to find p'_{2ox} and p'_{2nit}, we use Boyle's law $p_{1ox}V_{1ox} = p'_{2ox}V'_{2ox}$, $p_{1nit}V_{1nit} = p'_{2nit}V'_{2nit}$ for each gas separately:

$$p'_{2ox} = \frac{p_{1ox}V_{1ox}}{V'_{2ox}} = \frac{2 \times 10^5 \text{ Pa} \times 3 \times 10^{-3} \text{ m}^3}{1 \times 10^{-2} \text{ m}^3} = 0.6 \times 10^5 \text{ Pa},$$

$$p'_{2nit} = \frac{p_{1nit}V_{1nit}}{V'_{2nit}} = \frac{3 \times 10^5 \text{ Pa} \times 7 \times 10^{-3} \text{ m}^3}{1 \times 10^{-2} \text{ m}^3} = 2.1 \times 10^5 \text{ Pa}.$$

Finally, we obtain

$$p = 0.6 \times 10^5 \text{ Pa} + 2.1 \times 10^5 \text{ Pa} = 2.7 \times 10^5 \text{ Pa}.$$

Answer. The pressure of the gas mixture in the cylinders is 270 kPa.

Problem 14. The air in a balloon at a temperature of 20°C and a pressure of 99.75 kPa has a volume of 2.5 l. When the balloon is immersed in water at a temperature of 5°C, the air pressure in it increases to 2×10^5 Pa. What is the change in the volume of the air in the balloon?

Given: $t_1 = 20°C$, $p_1 = 99.75$ kPa, and $V_1 = 2.5$ l are the temperature, pressure, and volume of the air before the balloon is immersed in water, $t_2 = 5°C$ and $p_2 = 2 \times 10^5$ Pa are the temperature and pressure of the air after the immersion of the balloon in water.

Find: the change ΔV in the volume of the air in the balloon.

Solution. Before the balloon is immersed in water, the state of the air in it is characterized by the parameters p_1, V_1, and T_1. After immersion, the parameters are p_2, V_2, and T_2.

Let us write the parameters of the gas in states *1* and *2* separately, expressing them in SI units:

state *1*	state *2*
$p_1 = 9.975 \times 10^4$ Pa,	$p_2 = 2 \times 10^5$ Pa,
$V_1 = 2.5 \times 10^{-3}$ m^3,	$V_2 = ?$
$T_1 = 293$ K,	$T_2 = 278$ K.

As the air changes from state *1* to state *2*, all three parameters change. Consequently, we must use the equation of state to determine the final volume V_2:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2},$$

whence

$$V_2 = \frac{p_1 V_1 T_2}{T_1 p_2},$$

$$V_2 = \frac{9.975 \times 10^4 \text{ Pa} \times 2.5 \times 10^{-3} \text{ m}^3 \times 278 \text{ K}}{293 \text{ K} \times 2 \times 10^5 \text{ Pa}} = 1.2 \times 10^{-3} \text{ m}^3.$$

Calculations show that the volume of the air in the balloon changes as a result of its immersion in water by

$$\Delta V = V_1 - V_2,$$

$$\Delta V = 2.5 \times 10^{-3} \text{ m}^3 - 1.2 \times 10^{-3} \text{ m}^3 = 1.3 \times 10^{-3} \text{ m}^3.$$

Answer. The change in the volume of the air is 1.3×10^{-3} m^3 = 1.3 l.

Problem 15. A cylinder having a volume of 0.6 m^3 contains oxygen at a temperature of 27°C. A pressure gauge on

the cylinder indicates 11.7 MPa of excess pressure.[2] Reduce the volume of the oxygen to normal conditions and determine its mass.

Given: $V_1 = 0.6$ m^3 is the volume and $T_1 = 27°C$ is the temperature of the oxygen in the cylinder, $p_g = 11.7$ MPa is the reading of the pressure gauge, $T_0 = 273$ K, $p_0 = 1.013 \times 10^5$ Pa are the temperature and pressure under normal conditions. From tables, we take the density $\rho_0 = 1.43$ kg/m^3 of oxygen under normal conditions, the molar mass of oxygen, $M = 32 \times 10^{-3}$ kg/mol, and the molar gas constant $R = 8.31$ J/(mol·K).

Find: the volume V_0 of the oxygen under normal conditions and the mass m of the oxygen in the cylinder.

Solution. From the reading of the pressure gauge, we determine the gas pressure in the cylinder: $p_1 = p_g + p_0 = 11.7 \times 10^6$ Pa $+ 0.1013 \times 10^6$ Pa $= 11.8 \times 10^6$ Pa. Reducing the volume to normal conditions means determining the volume the gas would occupy at a temperature of 273 K and a pressure of 1.013×10^5 Pa.

We shall write the parameters of the oxygen in SI units for the two states:

state *1*	state *2*
$p_1 = 11.8 \times 10^6$ Pa,	$p_0 = 1.013 \times 10^5$ Pa,
$V_1 = 0.6$ m^3,	$T_0 = 273$ K,
$T_1 = 300$ K,	$V_0 = ?$

We solve the problem by using the equation of state for an ideal gas:

$$\frac{p_1 V_1}{T_1} = \frac{p_0 V_0}{T_0},$$

whence

$$V_0 = \frac{p_1 V_1 T_0}{T_1 p_0},$$

$$V_0 = \frac{11.8 \times 10^6 \text{ Pa} \times 0.6 \text{ m}^3 \times 273 \text{ K}}{300 \text{ K} \times 1.013 \times 10^5 \text{ Pa}} = 63.6 \text{ m}^3.$$

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[2] The excess pressure indicated on a pressure gauge is the difference between the pressure of the gas in the cylinder and atmospheric pressure.

From V_0 and ρ_0, we can determine the mass of the oxygen:

$$m = \rho_0 V_0, \ m = 1.43 \text{ kg/m}^3 \times 63.6 \text{ m}^3 = 91 \text{ kg}.$$

Answer. The volume of the oxygen under normal conditions is 63.6 m^3, the mass is approximately 91 kg.

Remark. The problem can be solved by using the equation $p_1 V_1 = \frac{m}{M} RT_1$, from which we first determine the mass $m = \frac{M}{R}\frac{p_1 V_1}{T_1}$. We can find the gas volume from the density of the oxygen under normal conditions:

$$V_0 = m/\rho_0.$$

Problem 16. A cylinder contains acetylene at 27°C under a pressure of 4.05 MPa. What will the pressure in the cylinder be after half the mass of the gas has been used up if the temperature has fallen thereby to 12°C?

Given: $T_1 = 300$ K and $p_1 = 4.05 \times 10^6$ Pa are the initial temperature and pressure of the gas in the cylinder, $T_2 = 285$ K is the temperature of the remaining gas, $m_2 = 0.5\, m_1$ is the mass of the consumed gas.

Find: the pressure p_2 of the gas remaining in the cylinder.

Solution. In this problem, the temperature, pressure, and mass of the gas change. Therefore, it is best to use the equations of state for the two cases in the form

$$p_1 V_1 = \frac{m_1}{M} RT_1 \quad \text{and} \quad p_2 V_1 = \frac{0.5 m_1}{M} RT_2.$$

Dividing the first equation by the second termwise and cancelling out m_1, M, R, and the unknown volume V_1, we obtain

$$\frac{p_1}{p_2} = \frac{T_1}{0.5 T_2},$$

whence

$$p_2 = \frac{0.5 p_1 T_2}{T_1},$$

$$p_2 = \frac{0.5 \times 4.05 \times 10^6 \text{ Pa} \times 285 \text{ K}}{300 \text{ K}} = 192.4 \times 10^4 \text{ Pa}$$

$$\simeq 1.92 \times 10^6 \text{ Pa}.$$

Answer. The pressure of the acetylene remaining in the cylinder is approximately 1.92 MPa.

Problem 17. Determine the density of hydrogen at a temperature of 17°C and a pressure of 204 kPa.

Given: $T = 290$ K and $p = 2.04 \times 10^5$ Pa are the temperature and pressure for which the density of hydrogen has to be determined. From tables, we obtain the molar mass of hydrogen, $M = 2 \times 10^{-3}$ kg/mol, and the molar gas constant $R = 8.31$ J/(mol·K).

Find: the hydrogen density ρ.

Solution. We write the equation of state $pV = \frac{m}{M} RT$. Dividing both sides by V, we obtain $p = \frac{m}{V} \frac{RT}{M}$, where m/V is the density. Then $p = \rho \frac{RT}{M}$, whence

$$\rho = \frac{pM}{RT}, \qquad \rho = \frac{2.04 \times 10^5 \text{ Pa} \times 2 \times 10^{-3} \text{ kg/mol}}{8.31 \text{ J/(mol·K)} \times 290 \text{ K}} = 0.17 \text{ kg/m}^3.$$

Answer. The density of hydrogen is 0.17 kg/m³.

Problem 18. Plot the graphs for an isothermal, isobaric, and isochoric process in p-V coordinates.

Solution. An isothermal process occurs at a constant temperature T = const and obeys Boyle's law. According to

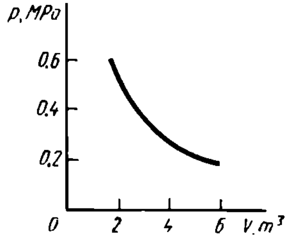

Fig. 3

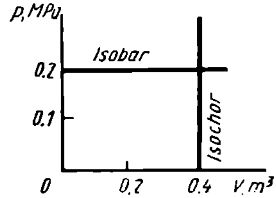

Fig. 4

this law, the pressure of a given mass of a gas varies in inverse proportion to its volume. Such a dependence is represented by a hyperbola, which is known in physics as an isotherm. In order to plot the graph, it is sufficient to keep the product pV constant, say, $pV = 1.2$. Ascribing arbitrary values 2, 4, 6, . . . to the volume, we calculate the corresponding pressures: 0.6, 0.3, 0.2, ... Having chosen a scale, we plot the graph (Fig. 3)

The volume of a gas in an isobaric process varies with temperature at constant pressure p = const. The process is graphically represented by a straight line parallel to the V-axis. For an isochoric process, the pressure varies at a constant volume V = const. This dependence is represented by a straight line parallel to the p-axis (Fig. 4).

Problem 19. Which of the two isotherms (Fig. 5) plotted for the same mass of a gas corresponds to a higher temperature?

Solution. We construct the isochor for a certain volume V_1. The isochor intersects the isotherm corresponding to the temperature T_1 at a pressure p_1, and the isochor intersects the isotherm for T_2 at a pressure p_2. Since the same volume corresponds to states *1* and *2*, we can write $p_1/T_1 = p_2/T_2$ from Gay-Lussac's law. Since $p_2 > p_1$, $T_2 > T_1$. Consequently, the upper isotherm corresponds to the higher temperature.

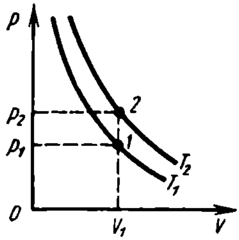

Fig. 5

Questions and Problems

3.1. A vessel contains 10.2 l of a gas under normal conditions. What volume will the gas occupy at a temperature of 40°C and a pressure of 1 MPa?

3.2. A gas occupies a volume of 4 l at a temperature of 50°C and a pressure of 196 kPa. At what pressure will this gas occupy a volume of 16 l if heated to 20°C?

3.3. At what temperature do 4.0 m^3 of a gas produce a pressure of 150 kPa if the same mass of the gas under normal conditions has a volume of 5 m^3?

3.4. A 45-l cylinder contains oxygen at a temperature of 27°C and a pressure of 1.52 MPa. What volume would the gas occupy under normal conditions?

3.5. Compressed oxygen for welding is stored in 20-l cylinders at a pressure of 9.8 MPa and a temperature of 290 K. Reduce the volume of oxygen to normal conditions.

3.6. A 6-l cylinder contains 0.1 kg of a gas at a temperature of 300 K and a pressure of 9.44×10^6 Pa. Determine the molar mass and identify the gas.

3.7. What amount of gas (in moles) is contained in a 10-l cylinder at a pressure of 0.29 MPa and a temperature of 17°C?

3.8. Determine the mass of carbon dioxide stored in a cylinder having a volume of 40 l at a temperature of 13°C and a pressure of 2.7 MPa.

3.9. Determine the amount of gas (in moles) occupying a volume of 25 l at a pressure of 1.4×10^5 Pa and a temperature of 300 K.

3.10. A mountain-climber takes in 5 g of air with each breath under normal conditions. What volume of air must he inhale in the mountains where the atmospheric pressure is lower than that at sealevel and is 79.8 kPa at a temperature of —13°C?

3.11. Under normal conditions neon occupies a volume of 12.4 l. How many times will the pressure of the gas increase if it is placed in a vessel having a volume of 5.6 l at a temperature of 318 K?

3.12. An ideal gas occupies a volume of 2 l at a pressure of 1.33 kPa and a temperature of 15°C. What will be the pressure if the temperature is doubled, while the volume decreases by 0.25 of the initial value?

3.13. A 40-l cylinder contains 64 g of oxygen under a pressure of 213 kPa. Determine the temperature of the gas.

3.14. 42 g of acetylene are kept in a 20-l cylinder at a temperature of 17°C. Determine the amount of substance in the gas and its pressure.

3.15. A 40-l cylinder contains 1.98 kg of carbon dioxide at 0°C. When the temperature is increased through 48 K, the cylinder explodes. At what pressure does the explosion occur?

3.16. Calculate the molar mass of butane if 2 l of the gas have a mass of 4.2 g at a temperature of 15°C and a pressure of 87 kPa. Calculate the number of gas molecules in 1 m^3.

3.17. At the beginning of the compression stroke in a diesel engine, the temperature of the air was 40°C and the pressure was 78.4 kPa. As a result of compression, the volume was reduced by a factor of 15, and the pressure increased to 3.5 MPa. Determine the temperature of the air at the end of the compression stroke.

3.18. The volume of 265 g of a gas at a temperature of 273 K and a pressure of 5 MPa is 60 l. What gas is it?

3.19. Determine the mass of air in a room of $6 \times 5 \times 3\ m^3$ at a temperature of 293 K and a pressure of 1.04×10^5 Pa. The molar mass M of air is 29×10^{-3} kg/mol.

3.20. What will the final temperature of a gas mixture in an internal combustion engine be if it occupies a volume of 40 dm^3 in the cylinder at a temperature of 50°C under normal atmospheric pressure and is compressed by the piston to a volume of 5 dm^3 and a pressure of 15.2×10^5 Pa?

3.21. A cylinder contains a gas at a temperature of 7°C and a pressure of 91.2 MPa. What will be the pressure after 0.25 of the mass of the gas has flown out of the cylinder and the temperature has risen to 27°C?

3.22. A closed vessel of 2-m^3 volume contains 1 kg of nitrogen and 1.5 kg of oxygen. Determine the pressure of the mixture in the vessel if the temperature of the mixture is 17°C.

3.23. Determine the density of oxygen at a temperature of 47°C and a pressure of 10^5 Pa.

3.24. The pressure of air 10 km above the surface of the Earth is about 30.6 kPa and the temperature is 230 K. Determine the density of the air, the number density of the molecules, and their root-mean-square velocity at this altitude.

3.25. 7 g of a gas contained in a cylinder at 27°C produce a pressure of 4.9×10^4 Pa. 4 g of hydrogen at 60°C produce a pressure of 43.5×10^4 Pa in the same volume. Determine the molar mass of the unknown gas and identify it.

3.26. What will be the increase in the mass of air in a room if the atmospheric pressure changes from 9.84×10^4 to 10.1×10^4 Pa and the air temperature remains unchanged and equal to 273 K? The size of the room is $4 \times 5 \times 2.5\ m^3$.

3.27. A vessel containing 10 l of air under a pressure of 1 MPa is connected to a 4-l empty vessel. Determine the final air pressure in the vessels, assuming that the process is isothermal.

3.28. A vessel contains a gas under a pressure of 5×10^5 Pa. What will the gas pressure be if 3/5 of the mass of the gas has flown out, the temperature being maintained constant?

3.29. Plot the graphs of an isothermal process in V-T and p-T coordinates.

3.30. The bladder of a football having a volume of 2.5 l

must be inflated to a pressure of 300 kPa. The pump takes in 0.14 l of air under normal atmospheric pressure during one stroke. How many strokes are required if the bladder was initially empty?

3.31. Given two equations of isothermal processes for the same mass of a gas: $p_1V_1 = 6$ and $p_2V_2 = 9$. Plot the graphs in p-V coordinates and answer the following questions: (1) in which units is the product pV expressed? (2) which of the isotherms will be further from the coordinate axes and why?

3.32. Dry atmospheric air consists of oxygen, nitrogen, and argon. Disregarding other components whose concentrations are very small, determine the masses of these gases

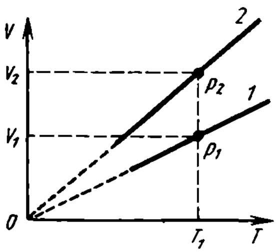

Fig. 6

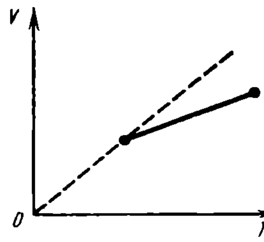

Fig. 7

in 1 m³ of atmospheric air under normal conditions if their partial pressures are 2.1×10^4 Pa for oxygen, 7.8×10^4 Pa for nitrogen, and 10^3 Pa for argon.

3.33. Represent an isobaric process graphically in V-T, p-V, and p-T coordinates.

3.34. Figure 6 shows two isobars plotted for the same mass of a gas. Which of the two isobaric processes occurs at a higher pressure and why?

3.35. A gas is heated isobarically from T_1 to T_2 (Fig. 7). What changes occur in the gas?

3.36. What volume will a gas occupy at 348 K if its volume at 35°C is 7.5 l? The process is isobaric.

3.37. A gas occupies a volume of 10 l at 27°C. To what temperature should it be cooled isobarically to reduce its volume by 0.25 of the initial value?

3.38. Through how many degrees Kelvin should the temperature of a gas be increased in an isobaric process to in-

crease its volume by a factor of 1.3 in comparison with the volume occupied by it at 0°C?

3.39. A certain mass of a gas is taken through a closed cycle *1-2-3-1* in which the state of the gas changes isobarically-isochorically-isothermally. Represent these processes in V-T and p-V coordinates.

3.40. A balloon of volume 4500 m^3 is filled with helium at 290 K. The mass of the envelope is 677 kg. Determine the lifting force of the balloon at a temperature of 27°C, assuming that the atmospheric pressure remains unchanged and is 102 kPa.

§ 4. THE CHANGE IN THE INTERNAL ENERGY DURING HEAT TRANSFER AND DUE TO MECHANICAL WORK

Basic Concepts and Formulas

When several bodies having different temperatures come into contact, heat transfer takes place, as a result of which the temperature of the bodies equalizes, and thermal equilibrium sets in. The internal energy of the bodies being heated increases at the expense of the energy given away by the bodies with higher temperatures. It has been established that the change in the internal energy of a body is proportional to its mass and to the change in its temperature: $\Delta U = Q = cm\,\Delta T$, where Q is the amount of heat (a measure of the change in the internal energy) expressed in joules (J), and c is the proportionality factor known as the specific heat of a substance. For example, the specific heat of aluminium is 920 J/(kg·K), which means that the internal energy of 1 kg of aluminium increases by 920 J when its temperature is raised through 1 K.

In order to determine a specific heat, the heat balance equation is employed. This equation is constructed by establishing the processes in which the energy is released (during cooling or as a result of fuel combustion) and the processes in which energy is absorbed (as a result of heating).

For example, a body of mass m_1 and specific heat c_1 has a temperature T_1 and is exchanging heat with a body of mass m_2, which has a specific heat c_2 and a temperature T_2. It is known that $T_1 > T_2$. Then the first body gives away

the following amount of heat:

$$Q_{\mathrm{giv}} = c_1 m_1 (T_1 - \Theta).$$

The second body receives as a result of the heat transfer the following amount of heat:

$$Q_{\mathrm{rec}} = c_2 m_2 (\Theta - T_2),$$

where Θ is the final temperature of the two bodies. Since energy is conserved, we can write $Q_{\mathrm{giv}} + Q_{\mathrm{rec}} = 0$. This gives

$$c_1 m_1 (T_1 - \Theta) = c_2 m_2 (\Theta - T_2).$$

This is the heat balance equation for the system of two bodies.

A body can be heated at the expense of the energy released during fuel combustion. The amount of heat liberated is proportional to the mass of the burnt fuel and depends on which fuel it is:

$$Q = q m_f,$$

where q is the specific heat of combustion of the fuel in joules per kilogram (J/kg) and m_f is the mass of the burnt fuel.

Not all the heat liberated during the combustion of a fuel is usefully spent on heating. A fraction of the heat dissipates. This is taken into account by introducing the efficiency of a heater:

$$\eta = \frac{Q_{\mathrm{useful}}}{Q_{\mathrm{spent}}}$$

If $Q_{\mathrm{useful}} = cm\,\Delta T$, and $Q_{\mathrm{spent}} = q m_f$, we have

$$\eta = \frac{cm\,\Delta T}{q m_f}$$

The internal energy of a body or a system of bodies may change when a mechanical work is done. The mechanical energy of a body or a system of bodies can be transformed completely into internal energy, i.e. be spent on heating:

$$Q = \Delta E_{\mathrm{k}} + \Delta E_{\mathrm{p}},$$

where ΔE_{k} and ΔE_{p} are the changes in the kinetic and potential energies of the body.

The first law of thermodynamics (the law of energy conservation). The amount of heat transferred to a closed system

is spent on increasing its internal energy and on the mechanical work done against external forces:

$$Q = \Delta U + A,$$

where Q is the amount of heat transferred to the system, ΔU is the change in its internal energy, and A is the work done by the system.

For isochoric processes, $V = \text{const}$, and hence

$$A = p\,\Delta V = 0,\ Q = \Delta U,$$

i.e. the heat transferred to a gas is completely used to increase its internal energy.

In isobaric processes, $p = \text{const}$, and hence

$$Q = \Delta U + A,$$

i.e. the heat transferred to a gas is spent on increasing its internal energy and on the work done to expand the gas.

For isothermal processes, $T = \text{const}$, and hence the internal energy does not change. Therefore, $Q = A$. The heat supplied to a gas during an isothermal process is completely spent on the mechanical work done by the gas.

Adiabatic processes occur in the absence of any heat exchange with the surroundings, i.e. $Q = 0$. Therefore, the first law of thermodynamics in this case will have the form

$$Q = \Delta U + A, \text{ or } A = -\Delta U,$$

i.e. an adiabatically expanding gas does work at the expense of the change in its internal energy, being cooled thereby.

Worked Problems

Problem 20. In order to determine the specific heat of copper, a copper cylinder of mass 0.5 kg is heated to 100°C and then placed in an aluminium calorimeter of mass 40 g, containing 300 g of water at a temperature of 15°C. As a result of heat exchange, the calorimeter stabilizes at temperature 26°C. What is the specific heat of copper in this experiment? Compare the obtained result with the tabulated value and determine the absolute and the relative error.

Given: $m_c = 0.5$ kg is the mass of the copper cylinder, $T = 373$ K is the initial temperature of the cylinder, $m_a = 0.04$ kg is the mass of the calorimeter, $m_w = 0.3$ kg is the

mass of the water, $T_1 = 288$ K is the initial temperature of the water and calorimeter, and $\Theta = 299$ K is the final temperature of the water, calorimeter, and cylinder. From tables, we take the specific heat of aluminium, $c_a = 880$ J/(kg·K), and the specific heat of water, $c_w = 4187$ J/(kg·K). The tabulated specific heat of copper is $c_{tab} = 380$ J/(kg·K).

Find: the specific heat of copper c_c, the absolute error Δ_c and the relative error Δ_c/c_{tab} of the measurement.

Solution. As a result of heat exchange, the temperature of all the bodies in the calorimeter equalizes. The heated cylinder gives off heat Q_{giv} and is cooled from T to Θ:

$$Q_{giv} = c_c m_c (T - \Theta).$$

The calorimeter and water receive heat, and their temperature rises from T_1 to Θ:

$$Q_{rec} = c_a m_a (\Theta - T_1) + c_w m_w (\Theta - T_1),$$

or

$$Q_{rec} = (c_a m_a + c_w m_w) (\Theta - T_1).$$

Proceeding from the energy conservation law, we equate the heat given off by the copper cylinder and the heat received by the calorimeter and water: $Q_{giv} = Q_{rec}$. Consequently, $c_c m_c (T - \Theta) = (c_a m_a + c_w m_w) (\Theta - T_1)$. We obtained the heat balance equation from which we determine c_c:

$$c_c = \frac{(c_a m_a + c_w m_w) (\Theta - T_1)}{m_c (T - \Theta)}.$$

Substituting the numerical values of the known quantities, we calculate the specific heat for copper:

$$c_c = \frac{(880\ \text{J/(kg·K)} \times 0.04\ \text{kg} + 4187\ \text{J/(kg·K)} \times 0.3\ \text{kg}) \times (299\ \text{K} - 288\ \text{K})}{0.5\ \text{kg}\ (373\ \text{K} - 299\ \text{K})}$$

$$\simeq 384\ \text{J/(kg·K)}.$$

The absolute error is

$$\Delta_c = c_c - c_{tab},$$

$$\Delta_c = 384\ \text{J/(kg·K)} - 380\ \text{J/(kg·K)} \simeq 4\ \text{J/(kg·K)}.$$

The relative error is

$$\frac{\Delta_c}{c_{tab}} \times 100\% = \frac{4\ \text{J/(kg·K)}}{380\ \text{J/(kg·K)}} \times 100\% \simeq 1.1\%.$$

Answer. The specific heat of copper is 384 J/(kg·K), the absolute error is approximately 4 J/(kg·K), and the relative error is about 1.1%.

Problem 21. 150 g of water is contained in a brass calorimeter of mass 0.2 kg at a temperature of 12°C. Find the final temperature in the calorimeter after an iron weight having a mass of 0.5 kg and heated to 100°C is immersed in water. Plot the temperature versus the amount of heat graph for this heat transfer process.

Given: $m_w = 150$ g $= 0.15$ kg is the mass of water in the calorimeter, $m_{cal} = 0.2$ kg is the mass of the calorimeter, $T_w = 285$ K is the initial temperature of the water and calorimeter, $m_i = 0.5$ kg is the mass of the weight, and $T_i = 373$ K is the initial temperature of the weight. From tables, we take the specific heat of iron, $c_i = 460$ J/(kg·K), the specific heat of water, 4187 J/(kg·K), and the specific heat of brass, $c_b = 380$ J/(kg·K).

Find: the final temperature Θ in the calorimeter.

Solution. As a result of heat exchange in the calorimeter, the internal energy of the weight decreases, while the internal energy of the calorimeter and water increases. The amount of heat is a measure of the change in the internal energy. The amount of heat $Q_i = c_i m_i (T_i - \Theta)$ liberated during the cooling of the weight is absorbed by heating the water ($Q_w = c_w m_w (\Theta - T_w)$) and the calorimeter ($Q_{cal} = c_{cal} m_{cal} (\Theta - T_w)$).

The heat balance equation is

$$c_i m_i (T_i - \Theta) = c_w m_w (\Theta - T_w) + c_{cal} m_{cal} (\Theta - T_w).$$

We shall solve this equation for the unknown temperature Θ in the calorimeter at thermal equilibrium. For this purpose, we open the parentheses:

$$c_i m_i T_i - c_i m_i \Theta = c_w m_w \Theta - c_w m_w T_w + c_{cal} m_{cal} \Theta - c_{cal} m_{cal} T_w.$$

All the terms containing Θ can be transferred to the right-hand side of the equation:

$$c_i m_i T_i + c_w m_w T_w + c_{cal} m_{cal} T_w = c_w m_w \Theta + c_i m_i \Theta + c_{cal} m_{cal} \Theta.$$

We can now write the expression for Θ in the general form:

$$\Theta = \frac{c_1 m_1 T_1 + (c_w m_w + c_{cal} m_{cal}) T_w}{c_1 m_1 + c_w m_w + c_{cal} m_{cal}}.$$

Substituting the numerical values, we obtain

$$\Theta = \frac{460 \times 0.5 \times 373 + (4190 \times 0.15 + 380 \times 0.2)\, 285}{4190 \times 0.15 + 460 \times 0.5 + 380 \times 0.2}$$

$$\times \frac{J \cdot kg^{-1} \cdot K^{-1} \cdot kg \cdot K}{J \cdot kg^{-1} \cdot K^{-1} \cdot kg} \simeq 307 \text{ K}, \quad \Theta = 34°C.$$

The graph of $t = f(Q)$ is represented in Fig. 8. The straight line AB shows the change in the temperature of the weight (the temperature decreases from 100 to 34°C since the process is accompanied by the liberation of heat). Straight line BC shows the change in the temperature of the calorimeter and water (the temperature increases from 12 to 34°C, the process involves the absorption of heat). Straight lines CE and CD show the change in the temperature of the water and the calorimeter respectively. Even though the temperature difference is the same in both cases (22 K), the lines CD and CE have different slopes. This is because the water absorbs more heat than the calorimeter.

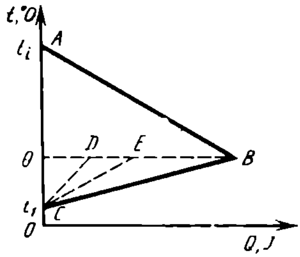

Fig. 8

Answer. The equilibrium temperature in the calorimeter is about 307 K.

Problem 22. Determine the efficiency of a melting furnace in which 70 kg of A-1 grade coal was burnt to heat 0.5 t of aluminium from 282 K to the melting point.

Given: $m_a = 0.5$ t $= 500$ kg is the mass of the aluminium, $T_1 = 282$ K is the initial temperature of the aluminium, $m_{coal} = 70$ kg is the mass of the coal. From tables, we obtain the melting point of aluminium $T_m = 932$ K, the specific heat of aluminium $c_a = 880$ J/(kg·K), and the specific heat of combustion of coal $q = 2.05 \times 10^7$ J/kg.

Find: the efficiency η of the furnace.

Solution. The amount of heat Q_{useful} required to heat the aluminium from T_1 to its melting point can be deter-

mined from the formula

$$Q_{\text{useful}} = c_a m_a (T_m - T_1).$$

The amount of heat Q_{spent} liberated as a result of the combustion of the coal is given by

$$Q_{\text{spent}} = q m_{\text{coal}}.$$

The efficiency of the furnace is defined as the ratio of the amount of heat spent in heating the aluminium to the amount of heat obtained as a result of the fuel combustion:

$$\eta = \frac{Q_{\text{useful}}}{Q_{\text{spent}}} \times 100\%, \text{ or } \eta = \frac{c_a m_a (T_m - T_1)}{q m_{\text{coal}}} \times 100\%.$$

Substituting the numerical values, we obtain

$$\eta = \frac{880 \text{ J/(kg} \cdot \text{K)} \times 500 \text{ kg} \times 650 \text{ K}}{2.05 \times 10^7 \text{ J/kg} \times 70 \text{ kg}} \times 100\% \simeq 20\%.$$

Answer. The efficiency of the furnace is about 20%.

Problem 23. The engine of a scooter develops a power of 3.31 kW at a speed of 58 km/h. How far will the scooter go by consuming 3.2 l of petrol if the efficiency of the engine is 20%?

Given: $P = 3.31$ kW $= 3310$ W is the power developed by the engine, $v = 58$ km/h $= 16.1$ m/s is the speed of the scooter, $V = 3.2$ l $= 3.2 \times 10^{-3}$ m^3 is the volume of consumed petrol, and $\eta = 20\%$ is the efficiency of the engine. From tables, we obtain the specific heat of combustion of petrol, $q = 4.6 \times 10^7$ J/kg, and the density of petrol, $\rho = 700$ kg/m^3.

Find: the distance s covered by the scooter.

Solution. The energy $Q = qm$ liberated by burning the petrol is spent on the work done to move the scooter and the driver. Only 20% of the energy received from the fuel will be spent in doing mechanical work. Using the law of energy conservation, we can write $\eta Q = A$, or $\eta q m = Pt$. We express the time required for the motion in terms of the distance covered and the velocity: $t = s/v$. Then $\eta q m = Ps/v$, whence

$$s = \frac{\eta q m v}{P}.$$

In order to determine the mass of the petrol, we express it in terms of density and volume ($m = \rho V$) and substitute

it into the expression for the distance:

$$s = \frac{\eta q \rho V v}{P}.$$

Substituting the numerical values, we obtain

$$s = \frac{0.2 \times 4.6 \times 10^7 \text{ J/kg} \times 700 \text{ kg/m}^3 \times 3.2 \times 10^{-3} \text{ m}^3 \times 16.1 \text{ m/s}}{3.31 \times 10^3 \text{ W}}$$

$$= 10^5 \text{ m} = 100 \text{ km}.$$

Answer. There is enough petrol to cover 100 km.

Problem 24. A freely falling steel ball hits the ground with a velocity of 41 m/s. After the impact it bounces to a height of 1.6 m. Determine the change in the temperature of the ball during the impact, assuming that only the internal energy of the ball changed as a result of the impact against the ground.

Given: $v = 41$ m/s is the velocity of the ball when it hits the ground, and $h = 1.6$ m is the height to which the ball rises after the impact. From tables, we obtain the specific heat of steel, $c = 460$ J/(kg·K), and the free fall acceleration $g = 9.8$ m/s^2.

Find: the change in the ball's temperature, ΔT, as a result of impact.

Solution. The kinetic energy acquired by the ball by the moment of impact is $E_k = mv^2/2$. A fraction Q of this energy is spent on increasing the internal energy of the ball, i.e. on heating it, while the remaining energy is spent on the ascent, i.e. converted into the potential energy of the ball $E_p = mgh$. According to the energy conservation law, we can write $E_k = Q + E_p$.

Since $Q = cm\,\Delta T$, we can write $\frac{mv^2}{2} = cm\,\Delta T + mgh$, or $\frac{mv^2}{2} - mgh = cm\,\Delta T$. We eliminate m from both sides: $\frac{v^2}{2} - gh = c\,\Delta T$, and solve this equation for ΔT:

$$\Delta T = \frac{v^2 - 2gh}{2c},$$

$$\Delta T = \frac{(41 \text{ m/s})^2 - 2 \times 9.8 \text{ m/s}^2 \times 1.6 \text{ m}}{2 \times 460 \text{ J/(kg·K)}} \simeq 1.8 \text{ K}.$$

Answer. The temperature of the ball has increased through about 1.8 K.

Problem 25. The volume of 80 g of oxygen at an initial temperature of 300 K increases by a factor of 1.5 as a result of isobaric expansion. Determine the amount of heat spent heating the oxygen, the work done during its expansion, and the change in its internal energy.

Given: $m = 80$ g $= 0.08$ kg is the mass of the oxygen, $T_1 = 300$ K is its initial temperature, $V_2 = 1.5V_1$ is the final volume of the gas. From tables, we find the molar mass of oxygen, $M = 32 \times 10^{-3}$ kg/mol, the molar gas constant $R = 8.31$ J/(mol·K), and the specific heat of oxygen at constant pressure $c_p = 0.92 \times 10^3$ J/(kg·K).

Find: the amount of spent heat Q, the work A done during the isobaric expansion, and the change in the internal energy, ΔU, of the gas.

Solution. The amount of heat required to heat the gas is

$$Q = c_p m \,\Delta T,$$

where $\Delta T = T_2 - T_1$.

The temperature T_2 can be found from Charles' law, which governs the isobaric process: $V_2/V_1 = T_2/T_1$. Since $V_2 = 1.5V_1$ by hypothesis, $T_2 = 1.5\, T_1 = 1.5 \times 300$ K $=$ 450 K.

After substituting the numerical values, we obtain

$$Q = 0.92 \times 10^3 \text{ J/(kg·K)} \times 0.08 \text{ kg} \times 150 \text{ K}$$
$$= 11\,040 \text{ J} = 11.04 \text{ kJ}.$$

In an isobaric process, $A = p\,\Delta V$. Applying the equation of state, we can write

$$A = p\,\Delta V = \frac{m}{M} R\,\Delta T,$$

$$A = \frac{0.08 \text{ kg}}{32 \times 10^{-3} \text{ kg/mol}} \times 8.31 \text{ J/(mol·K)} \times 150 \text{ K}$$
$$= 3116 \text{ J} = 3.12 \text{ kJ}.$$

The first law of thermodynamics for isobaric processes has the form $Q = \Delta U + A$, and hence

$$\Delta U = Q - A, \quad \Delta U = 11.04 \text{ kJ} - 3.12 \text{ kJ} = 7.92 \text{ kJ}.$$

Answer. The amount of heat spent heating oxygen is 11.04 kJ, 3.12 kJ is spent on the expansion work, and 7.92 kJ on increasing the internal energy.

Questions and Problems

4.1. How much energy is required to raise the temperature of 1 kg of tin through 1 K?

4.2. The temperature of a 1-kg copper weight has decreased from 293 K to 19°C. What is the decrease in its internal energy due to cooling?

4.3. Two metal bars, one aluminium and one nickel, with the same mass, have their temperatures lowered through 1 K. Which bar liberated a larger amount of heat? What is the ratio of the amounts of liberated heat?

4.4. 1 kg of water and 1 kg of steel are heated through 1 K. What is the change in their internal energy?

4.5. Three cylinders having the same volume are made of lead, copper, and aluminium. Which of them has the largest heat capacity?

4.6. A copper weight and an iron weight of the same mass were dropped from the same height to the ground. Which of the weights had the higher temperature after the impact? Does the answer depend on the mass of the weights?

4.7. Gases have larger specific heats at constant pressure than at constant volume. How can this be explained?

4.8. How much energy will be liberated as a result of the complete combustion of 1 kg of Donetsk coal?

4.9. How much fuel oil has to be burnt to obtain 4×10^7 J of heat?

4.10. How much heat will be liberated as a result of the complete combustion of 5 m^3 of natural gas?

4.11. How much heat must be spent in heating a copper plate of mass 180 g through 15°C?

4.12. 200 l of water are drawn at 283 K for a bath. How much boiling water has to be added to elevate the water temperature to 37°C?

4.13. To fill an aquarium, 20 l of water at 15°C are mixed with 2 l of water at 70°C. Determine the water temperature in the aquarium.

4.14. What will be the change in the internal energy of 1 l of mercury as a result of heating it from 283 K to 50°C? The change in the mercury's density with temperature should be neglected.

4.15. 8 l of water at 20°C were poured into an aluminium

kettle whose mass is 1.5 kg. How much heat is required to bring the water to boil?

4.16. 240 g of water at 288 K were poured into an aluminium calorimeter of mass 40 g. After a lead bar having a mass of 100 g and heated to 100°C had been placed into the calorimeter with water, a temperature of 289 K was established. Write the heat balance equation and determine the specific heat of lead.

4.17. In order to cool a copper component having a temperature of 373 K, it was placed into 420 g of water at a temperature of 15°C. Determine the mass of the component if the water gets heated to 18°C as a result of the heat transfer.

4.18. Calculate the specific heat of brass if 334.4 J of heat are required to heat a brass weight having a mass of 200 g from 285 K to 289.4 K.

4.19. The amount of heat required to raise the temperature of 200 g of mercury through 58.8°C is the same as that required to raise the temperature of 50 g of water through 7°C. Using these data, determine the specific heat of mercury.

4.20. Determine the temperature of a furnace in which 0.5 t of steel was hardened if 175 MJ of heat were spent to heat the steel from 20°C to the hardening temperature.

4.21. A steel component having a mass of 0.3 kg was heated to a high temperature and then hardened in 3 kg of machine oil having a temperature of 283 K. Determine the initial temperature of the component if its final temperature at thermal equilibrium was 303 K.

4.22. A metal cylinder having a mass of 146 g and heated to 100°C is immersed in a brass calorimeter having a mass of 128 g and containing 240 g of water at 8.5°C. The temperature established as a result of heat transfer was 283 K. Determine the specific heat of the metal of which the cylinder is made and identify the metal.

4.23. To heat 3 l of water from 20 to 100°C on a gas burner, 0.06 m^3 of natural gas was burnt. Determine the efficiency of the burner.

4.24. Determine the amount of coke required to heat 1.5 t of scrap iron from 20°C to its melting point, if the efficiency of the cupola is 60%.

4.25. A bullet having a mass of 9 g and shot from a gun

acquires a velocity of 800 m/s. Determine the mass of the powder charge if the efficiency of the shot is 24%.

4.26. What is the change in the temperature of 2.0 m^3 of water in a boiler if 25 kg of Ekibastuz coal have been burnt in the furnace whose efficiency is 50%?

4.27. A tumbler contains 250 g of water at 80°C. What will the decrease in the water temperature be if a silver spoon having a mass of 50 g and a temperature of 293 K is immersed in it?

4.28. A blast furnace consumes 2200 m^3 of air per minute. The air is heated in Cowper stoves by burning the blast-furnace gas. Determine the volume of the gas burnt daily to heat air from 273 K to 1200°C if the energy losses amount to 30%.

4.29. A train having a mass of 2×10^6 kg and moving at a velocity of 54 km/h comes to a halt. How much heat is liberated in the brakes?

4.30. A lead bullet flies at a velocity of 300 m/s. What will the change in its temperature be when it is stopped abruptly? Assume that 50% of its energy is spent heating the bullet.

4.31. To what height should it be possible to lift a load whose mass is 0.5 t if the entire energy given away by a brass weight of 10 kg upon cooling by 100 K were spent for lifting?

4.32. What will be the change in the temperature of water falling from 120 m if 60% of its potential energy is spent to heat it.

4.33. Determine the change in the temperature of water falling from 96 m onto the blades of the turbine in the Bratsk hydroelectric plant, assuming that 50% of the energy of the falling water is spent on increasing the water's internal energy.

4.34. The striker of a hammer whose mass is 10^4 kg freely falls from a height of 2.5 m onto a forged iron piece whose mass is 200 kg. How many blows does the hammer strike if the forged piece is heated through 20 K? Assume that 30% of the energy of the hammer is spent on heating.

4.35. A worker pins together wooden struts by butting a 500-g iron nail into them with a 3-kg hammer moving at 12 m/s before the impact. Assuming that the entire energy

of the hammer is spent on heating the nail, determine the change in its temperature after 20 blows.

4.36. Two identical copper balls have received the same amount of energy, as a result of which the first ball, remaining at rest, is heated through 40 K, and the second is set in motion without heating. Determine the velocity of the second ball.

4.37. A lead bullet passes through a wooden wall so that its velocity is 400 m/s at the moment of impact against the wall and 100 m/s when it emerges from it. Determine the change in the temperature of the bullet assuming that 40% of the mechanical energy spent by the bullet piercing the wall is converted into heat.

4.38. Determine the efficiency of a tractor engine which consumes 292 g of diesel fuel per kilowatt per hour.

4.39. The engine of a crawler tractor develops a power of 73.6 kW by consuming 285 g of diesel fuel per kilowatt per hour. Determine the efficiency of the engine.

4.40. A locomotive driven by an internal combustion engine has an efficiency of 25% at a power of 3 MW. Determine the consumption of the diesel fuel per hour during operation at maximum power.

4.41. Modern cars consume on the average 330 g of petrol per hour per kilowatt of developed power. Determine the efficiency of a car engine.

4.42. Determine the power developed by the engine of a "Zaporozhets" car if it consumes 74 g of petrol per kilometre at a velocity of 60 km/h. The efficiency of the engine is 30%.

4.43. The capacity of the fuel tank of a car is 60 l. How far can the car go on the fuel contained in a full tank at a constant speed if the mass of the loaded car is 1800 kg and the efficiency of the engine is 20%? The drag coefficient is 0.04.

4.44. A volume of a gas has increased by 0.02 m^3 as a result of heating, while its internal energy has increased by 1280 J. How much heat was supplied to the gas if the process occurred at a constant pressure of 1.5×10^5 Pa?

4.45. 3 m^3 of air is stored under a pressure of 2×10^5 Pa at 0°C. Determine the work done by the air during isobaric heating through 12 K.

4.46. How much work is done to heat 160 g of oxygen through 20 K at constant pressure?

4.47. 580 g of air is heated isobarically through 10 K. How much heat is supplied to the air? What work is done in the process?

4.48. 2 kg of air are contained in a cylinder with a piston at 289 K. What work will be done by it during isobaric heating to 373 K?

4.49. 3.47 kg of a gas does 144 kJ of work when isobarically heated through 159 K. Determine the molar mass of the gas and identify it.

4.50. 22 g of carbon dioxide are contained in a cylinder under a heavy piston. What work is done by the gas as a result of heating it from 17 to 117°C?

4.51. What work is done by an ideal gas filling a rubber balloon when heated from 10 to 70°C? The initial volume of the balloon is 5 l and the atmospheric pressure is 10^5 Pa. The elasticity of the balloon should be neglected.

4.52. A gas occupying a volume of 30 l at a pressure of 1.2×10^5 Pa is isobarically heated from 300 to 450 K. Determine the work done by the gas.

4.53. What work is done by 1 mol of an ideal gas when heated isobarically through 1 K? Does this work depend on the pressure and initial temperature?

4.54. Oxygen is contained in a cylinder under a piston. Determine the mass of the oxygen if the work done to heat it from 273 to 473 K is 16 kJ. Friction should be neglected.

4.55. Can the temperature of a gas be changed without any heat exchange with the environment?

4.56. The volume of a gas can be reduced either by isothermal or by adiabatic compression. In which case will the change in pressure be higher?

§ 5. PROPERTIES OF VAPOURS

Basic Concepts and Formulas

The conversion of a liquid to vapour is known as evaporation, and the reverse process is called condensation. Vapour is formed as the result of either the evaporation or boiling of a liquid. During evaporation, vapour is only formed at the free surface of the liquid, which is possible at any temperature. During boiling, vapour is formed in the bulk of the liquid (vaporization).

Every liquid boils at a certain temperature known as its boiling point. Evaporation is always accompanied by an absorption of energy. Vaporization during boiling is accompanied by absorption of energy from outside. Evaporation occurs due to a decrease in the internal energy of the evaporating liquid (which is cooled thereby).

The specific latent heat of vaporization r is the amount of heat required for the vaporization of 1 kg of a liquid at a constant temperature (boiling point):

$$Q = rm, \ r = Q/m.$$

The specific latent heat of vaporization is measured in joules per kilogram (J/kg).

The specific latent heat of vaporization depends on the pressure and temperature. It decreases with increasing pressure.

The problems on vaporization are solved by using the heat balance equation. Since vaporization during boiling occurs at a certain temperature, while writing the heat balance equation one should take into account not only $Q = rm$, but also $Q_1 = cm (T_b - T_1)$, where T_b is the boiling point of a liquid.

One should distinguish between the processes of evaporation and condensation: heat is absorbed in the former case and liberated in the latter case.

Special attention should be paid to the physical meaning of the specific heat of vaporization. For example, the specific latent heat of vaporization of ammonia is 1.37×10^6 J/kg. This means that the conversion of 1 kg of ammonia into vapour at the boiling point (—33.4°C) requires 1.37×10^6 J of energy (the same amount of energy is liberated during condensation). If the temperature of ammonia is below the boiling point, an additional amount of heat is required to heat the ammonia to this temperature.

Vapour is called saturated if its pressure and density are the maximum possible at a given temperature. Unsaturated vapours have properties close to those of gases and hence obey all the laws for ideal gases.

An unsaturated vapour can be converted into a saturated vapour either by reducing its volume or by decreasing the temperature.

The humidity of air is characterized by the presence of vapour in the atmosphere. The absolute humidity ρ_a is a quantity equal to the density of the water vapour in air, or to its pressure p_a. The absolute humidity is measured in kilograms per cubic metre (kg/m^3).

The relative humidity B is equal to the ratio of the absolute humidity ρ_a (or the pressure p_a of the water vapour in air) to the density ρ_s (or pressure p_s) of the saturated vapour at a given temperature:

$$B = \frac{\rho_a}{\rho_s} 100\%, \text{ or } B = \frac{p_a}{p_s} 100\%.$$

The values of density ρ_s or pressure p_s of saturated vapours are given in Table 6.

When the temperature of air drops to the dew point, the relative humidity becomes 100%.

Worked Problems

Problem 26. How much heat is required to raise the temperature of 200 g of water from 10°C to the boiling point and to vaporize 10% of it? Assume that there are no energy losses.

Given: $m_w = 0.2$ kg is the mass of the cold water, $t =$ 10°C, or $T = 283$ K, is the temperature of the cold water, $m_{steam} = 0.1\, m_w$ is the mass of steam. From tables, we find the specific heat of water, $c_w = 4187$ J/(kg·K) $\simeq$ 4190 J/(kg·K), the boiling point of water, $t_b = 100$°C, or $T_b = 373$ K, and the specific latent heat of vaporization of water, $r = 2.26 \times 10^6$ J/kg.

Find: the amount of heat Q (spent energy).

Solution. Since the water temperature is below the boiling point, it must be heated from T to T_b, for which the amount $Q_1 = c_w m_w (T_b - T)$ of heat is required. The amount of heat required to vaporize the water is $Q_2 = m_{steam} r$, or $Q_2 = 0.1\, m_w r$.

The process is known to occur without energy losses. Consequently, the total amount of heat that must be spent is

$$Q = Q_1 + Q_2,$$
$$Q_1 = 4190 \text{ J/(kg·K)} \times 0.2 \text{ kg} \times 90 \text{ K} = 75420 \text{ J}$$
$$= 75.42 \text{ kJ},$$

$Q_2 = 0.1 \times 0.2\ \text{kg} \times 2.26 \times 10^6\ \text{J/kg} = 4.52 \times 10^4\ \text{J}$
$= 45.2\ \text{kJ},$
$Q = 75.42\ \text{kJ} + 45.2\ \text{kJ} = 120.62\ \text{kJ}.$

This process can be represented graphically (Fig. 9). We shall plot the amount of spent heat along the abscissa axis. For the sake of simplicity, we round off the values of Q_1 and Q_2 to integers: 75 and 45 kJ. The temperature in degrees Celsius (this is more convenient for choosing a scale) is plotted along the ordinate axis. When analyzing the graph, we pay attention to the fact that vaporization occurs at a constant temperature (in this case, at 100°C). Consequently, the water must be heated to this temperature. The intercepts OA and AB on the abscissa correspond to the values Q_1 and Q_2.

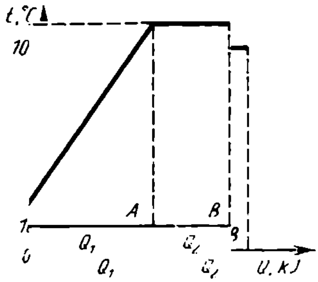

Fig. 9

Answer. About 121 kJ of energy is required to heat the water and vaporize part of it.

Problem 27. How much charcoal must be burnt in order to heat 6 t of water taken at 283 K to boiling and vaporize 1 t of it? The efficiency of the boiler is 70%.

Given: $m_w = 6 \times 10^3$ kg is the mass of water in the boiler, $T = 283$ K is the initial temperature of the water, $T_b = 373$ K is the boiling point of water, $m_{steam} = 10^3$ kg is the mass of the steam, and $\eta = 70\% = 0.7$ is the efficiency of the boiler. From tables, we take the specific heat of water, $c = 4187\ \text{J/(kg}\cdot\text{K)} \simeq 4190\ \text{J/(kg}\cdot\text{K)}$, the specific latent heat of vaporization of water, $r = 2.26 \times 10^6$ J/kg, and specific heat of combustion of charcoal, $q = 3.1 \times 10^7$ J/kg.

Find: the mass m_c of charcoal.

Solution. The energy $Q = qm_c$ liberated during the combustion of the charcoal is spent on heating the water to boiling point, $Q_1 = cm_w (T_b - T)$, and on vaporizing 1 t of the water, $Q_2 = rm_{steam}$.

From the energy conservation law and given that only 70% of energy liberated by burning the charcoal is spent on heating and vaporizing the water, we write the heat balance equation $\eta Q = Q_1 + Q_2$, or $\eta q m_c = cm_w (T_b - T) +$

rm_{steam}, whence $m_c = \frac{cm_w(T_b - T) + rm_{steam}}{\eta q}$.

Substituting the numerical values, we obtain

$$m_c = \frac{4190 \text{ J/(kg·K)} \times 6 \times 10^3 \text{ kg} \times 90 \text{ K} + 2.26 \times 10^6 \text{ J/kg} \times 10^3 \text{ kg}}{0.7 \times 3.1 \times 10^7 \text{ J/kg}} =$$
$$= 208.$$

Answer. About 208 kg of charcoal are spent.

Problem 28. 200 kg of steam at a temperature of 373 K are passed through 4 t of water at a temperature of 293 K. To what temperature will the water be heated? Energy losses should be neglected. Graph the function $t = f(Q)$.

Given: $m_w = 4 \times 10^3$ kg is the mass of the water, $T_w =$ 293 K is the temperature of the water, $m_{steam} = 200$ kg is the mass of the steam, $T_{steam} = 373$ K is the temperature of the steam. From tables, we take the specific heat of water, $c_w = 4187$ J/(kg·K) $\simeq 4190$ J/(kg·K), and the specific latent heat of vaporization of water, $r = 2.26 \times 10^6$ J/kg.

Find: the temperature Θ at the end of the process.

Solution. We have here a heat exchange: steam at its boiling point (which equals its condensation temperature) gives to water the amount of heat $Q_1 = rm_{steam}$ and is converted into water at the same temperature (during condensation, as in boiling, the temperature remains constant). The water obtained from the steam is cooled from T_b to Θ, liberating the amount of heat $Q_2 = c_w m_{steam}(T_{steam} - \Theta)$. Since the final temperature Θ is the same for all components, the internal energy of the cold water increases by $Q = c_w m_w \times (\Theta - T_w)$.

According to the energy conservation law, we can write

$$Q_1 + Q_2 = Q, \text{ or } rm_{steam} + c_w m_{steam}(T_{steam} - \Theta)$$
$$= c_w m_w(\Theta - T_w).$$

We transform the heat balance equation removing the parentheses and gathering the terms containing the unknown temperature on the right-hand side:

$$rm_{steam} + c_w m_{steam} T_{steam} + c_w m_w T_w = c_w m_w \Theta$$
$$+ c_w m_{steam} \Theta = (c_w m_w + c_w m_{steam})\Theta,$$

whence

$$\Theta = \frac{rm_{steam} + c_w(m_{steam} T_{steam} + m_w T_w)}{c_w(m_w + m_{steam})}.$$

Substituting the numerical values, we determine Θ:

$$\Theta = \frac{2.26 \times 10^6 \text{ J/kg} \times 200 \text{ kg}}{4190 \text{ J/(kg}\cdot\text{K)} (4 \times 10^3 \text{ kg} + 200 \text{ kg})}$$

$$+ \frac{4190 \text{ J/(kg}\cdot\text{K)} (200 \text{ kg} \times 373 \text{ K} + 4 \times 10^3 \text{ kg} \times 293 \text{ K})}{4190 \text{ J/(kg}\cdot\text{K)} (4 \times 10^3 \text{ kg} + 200 \text{ kg})} \simeq 322 \text{ K}$$

or $\Theta = 49°\text{C}$.

Figure 10 shows the temperature versus the amount of heat, $t = f(Q)$. The temperature remains unchanged over segment AB (the evolution of heat during condensation occurs due to a decrease in the potential energy of interaction between the molecules). The temperature of condensate falls from T_{steam} to Θ over segment BC (cooling the condensate liberates heat). The temperature of the cold water increases from T to Θ over segment CD (the process involves the absorption of heat).

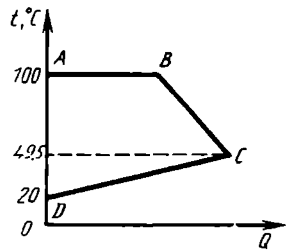

Fig. 10

Answer. The final temperature Θ is approximately 49°C.

Problem 29. 1.5 l of water at 20°C are poured into an aluminium pot whose mass is 600 g and put on an electric hot plate whose efficiency is 75%. In 35 min the water boils and 20% of it is converted into steam. What is the power of the hot plate?

Given: $m_a = 0.6$ kg is the mass of the aluminium pot, $V = 1.5 \text{ l} = 1.5 \times 10^{-3} \text{ m}^3$ is the volume of the water, $t_0 = 20°\text{C}$ is the initial temperature of the water and the pot, $\eta = 75\% = 0.75$ is the efficiency of the hot plate, $t = 35 \times 60$ s is the duration of the process, and $m_{\text{steam}} = 0.2m_w$ is the mass of the steam formed. From tables, we take the boiling point of water, $t_b = 100°\text{C}$, the specific heat of aluminium, $c_a = 880 \text{ J/(kg}\cdot\text{K)}$, the density of water, $\rho = 10^3 \text{ kg/m}^3$, the specific heat of water, $c_w \simeq 4190 \text{ J/(kg}\cdot\text{K)}$, and the specific latent heat of vaporization for water, $r = 2.26 \times 10^6$ J/kg.

Find: the power P of the hot plate.

Solution. The amount of heat required to heat the water in the pot and to convert part of it into steam (which will be

regarded as the useful heat Q_{useful}) is

$$Q_{useful} = c_a m_a (t_b - t_0) + c_w m_w (t_b - t_0) + m_{steam} r.$$

The amount of heat liberated by the hot plate will be regarded as the spent heat Q_{spent}.

Proceeding from the energy conservation law and taking into account the efficiency of the hot plate, we can write $\eta Q_{spent} = Q_{useful}$. Hence $Q_{spent} = Q_{useful}/\eta$.

The required power of the hot plate can be determined by dividing the spent energy by the time:

$$P = \frac{(c_a m_a + c_w m_w)(t_b - t_0) + 0.2 m_w r}{\eta t}.$$

The unknown mass of the water can be determined from the formula for density: $\rho = m_w/V$:

$$m_w = \rho V = 10^3 \text{ kg/m}^3 \times 1.5 \times 10^{-3} \text{ m}^3 = 1.5 \text{ kg}.$$

Substituting the numerical values, we obtain

$$P = \frac{[880 \text{ J/(kg·K)} \times 0.6 \text{ kg} + 4190 \text{ J/(kg·K)} \times 1.5 \text{ kg}] \times 80 \text{ K}}{0.75 \times 35 \times 60 \text{ s}}$$
$$+ \frac{0.2 \times 1.5 \text{ kg} \times 2.26 \times 10^6 \text{ J/kg}}{0.75 \times 35 \times 60 \text{ s}} \simeq 780 \text{ W}.$$

Answer. The power of the hot plate is approximately 780 W.

Problem 30. The relative humidity of air in a $5 \times 4 \times 3$ m^3 room at 20°C is 70%. Determine the dew point. How much moisture will be liberated from the air in the form of dew if its temperature is reduced to 11°C? What will be the relative humidity of the air?

Given: $V = 60$ m^3 is the volume of the air in the room, $t_1 = 20$°C is the initial temperature of the air, $B_1 = 70\% = 0.7$ is the relative humidity of the air, and $t_2 = 11$°C is the final temperature of the air. From tables, we take the saturated vapour density $\rho_{s1} = 17.3 \times 10^{-3}$ kg/m^3 for t_1, and the saturated vapour density $\rho_{s2} = 10 \times 10^{-3}$ kg/m^3 for t_2.

Find: the dew point t_d, the mass Δm of vapour that has condensed, and the relative humidity of the air, B_2, after the dew has precipitated.

Solution. The temperature at which the water vapour in the air becomes saturated is known as the dew point. In order 'o determine this temperature, we must know the abso-

lute humidity of the air, ρ_a. Using the formula $B_1 = \rho_a/\rho_{s1}$ for relative humidity, we obtain

$$\rho_a = B_1\rho_{s1} = 0.7 \times 17.3 \times 10^{-3}\ \mathrm{kg/m^3} = 12.1 \times 10^{-3}\ \mathrm{kg/m^3}.$$

Using Table 6, we find that a vapour having a density of 12.1×10^{-3} kg/m³ is saturated at 14°C. Consequently, the dew point is 14°C.

In order to answer the second question, we must use the same table. The saturated vapour density at 11°C, i.e. the maximum density, is $\rho_{s2} = 10 \times 10^{-3}$ kg/m³. Before the dew precipitates, the mass of the air in the room is $m_1 = \rho_a V$, and after that, $m_2 = \rho_{s2}V$. Consequently, the mass of the vapour precipitated in the form of dew is

$$\Delta m = m_1 - m_2,\quad \Delta m = \rho_a V - \rho_{s2} V = V(\rho_a - \rho_{s2}),$$
$$\Delta m = 60\ \mathrm{m^3}\,(12.1 \times 10^{-3}\ \mathrm{kg/m^3} - 10 \times 10^{-3}\ \mathrm{kg/m^3})$$
$$= 126 \times 10^{-3}\ \mathrm{kg}.$$

Since the absolute humidity at 11°C is the maximum admissible density at this temperature, the relative humidity is $B_2 = 100\%$.

Answer. The dew point is 14°C, the mass of the condensed vapour is 126 g, and the relative humidity is 100%.

Problem 31. The relative humidity of air at a temperature of 16°C is 54%. What is the reading of the wet-bulb thermometer of a psychrometer? What is the absolute humidity of the air?

Given: $B = 54\% = 0.54$ is the relative humidity of the air and $t = 16$°C is the air temperature. From tables, we take $\rho_{s16} = 13.6 \times 10^{-3}$ kg/m³.

Find: the reading t_w of the wet-bulb thermometer and the absolute humidity of the air, ρ_a.

Solution. We shall use Table 20. In the first column, we find 16°C (the reading of the dry-bulb thermometer). From the same row, we take a relative humidity of 54%. It lies in the column for which the temperature difference between the readings of the dry-bulb and the wet-bulb thermometers is 5°C. Consequently, $t - t_w = 5$°C, whence

$$t_w = t - 5^\circ\mathrm{C},\quad t_w = 16^\circ\mathrm{C} - 5^\circ\mathrm{C} = 11^\circ\mathrm{C}.$$

In order to find the absolute humidity, we write $B = \rho_a/\rho_{s16}$ from which

$$\rho_a = B\rho_{s16},$$
$$\rho_a = 0.54 \times 13.6 \times 10^{-3}\ \mathrm{kg/m^3} = 7.3 \times 10^{-3}\ \mathrm{kg/m^3}.$$

Answer. The reading of the wet-bulb thermometer is 11°C and the absolute humidity is approximately 7.3×10^{-3} kg/m³.

Problem 32. At a temperature of 30°C, the partial pressure of water vapour in air is 4.1 kPa. Determine the absolute humidity of the air.

Given: $T = 303$ K is the air temperature and $p = 4.1$ kPa $= 4.1 \times 10^3$ Pa is the partial pressure of the water vapour. From tables, we take the molar mass of water vapour, $M = 18 \times 10^{-3}$ kg/mol, and the molar gas constant $R = 8.31$ J/(mol·K).

Find: the absolute humidity of the air, ρ_a.

Solution. Since unsaturated vapours obey the gas laws, we can use the equation of state $pV = \frac{m}{M} RT$ to determine the density of the water vapour in air. From this equation, we express the ratio of the vapour mass to its volume, i.e. the absolute humidity of the air:

$$\frac{m}{V} = \rho_a = \frac{pM}{RT}.$$

Substituting the numerical values, we obtain

$$\rho_a = \frac{4.1 \times 10^3 \text{ Pa} \times 18 \times 10^{-3} \text{ kg/mol}}{8.31 \text{ J/(mol·K)} \, 303 \text{ K}} = 0.029 \text{ kg/m}^3.$$

Answer. 1 m³ of air contains about 29 g of water vapour.

Questions and Problems

5.1. Can water be made boil without heating it?

5.2. Equal masses of water and steam are at 100°C. Are their internal energies equal?

5.3. In the south, drinking water in summer is stored in vessels made of porous clay. Why?

5.4. At a pressure below 0.1 MPa (lower than atmospheric pressure), water boils at a temperature lower than 100°C. Explain why the steam temperature in a boiler, where a pressure gauge indicates a pressure of 0.07 MPa, is higher than 100°C.

5.5. A hermetically sealed vessel contains water and water vapour. How will the number density of the water vapour molecules change as a result of heating?

5.6. Why does a mist appear in lowlands after a hot summer day?

5.7. When petroleum is distilled, first petrol is liberated, followed by naphtha, kerosene, and other components. How can this be explained?

5.8. Which puts out a fire more quickly, boiling or cold water?

5.9. Is the temperature of the boiling water in a deep vessel the same at the surface and at the bottom?

5.10. When a gas is released from a cylinder, dew or even frost is formed at the valve. What is the reason behind this phenomenon?

5.11. It is easier to tolerate hot weather in Central Asia than at medium latitudes (at the same air temperature). Why?

5.12. Water and ether have different specific latent heats of vaporization. Is this always true?

5.13. How much heat is liberated during the condensation of 1 g of petrol vapour having a temperature of 150°C?

5.14. In which case is more energy liberated: during the condensation of 1 kg of water vapour or 1 kg of mercury vapour? What is the ratio of the amounts of heat? The vapours are both at their boiling points.

5.15. How much heat is required to vaporize 5 kg of water at 373 K? How much extra heat is needed if the temperature of the water is 0°C?

5.16. How much heat is required to vaporize 10 g of turpentine having a temperature of 100°C?

5.17. 100 g of steam at 100°C condense into water at 20°C. How much heat is liberated in the process?

5.18. 9.9×10^6 J of heat are spent to heat 2.24 l of water taken at 19°C. The water is heated to 100°C and some of it is converted into steam. Determine the mass of the steam.

5.19. What temperature becomes established in a bath containing 80 l of water at 20°C after 2.2 kg of steam at 373 K is added? The energy taken to heat the bath itself should be neglected.

5.20. Stripped steam at 100°C is mixed with 2 t of water whose temperature is 293 K. How much steam is required to raise the temperature of the water to 309 K?

5.21. 0.3 kg of steam at 100°C are mixed with 6 kg of water. After the steam condenses, the temperature of the

water rises to 40°C. Determine the initial temperature of the water.

5.22. 150 g of steam at 373 K are mixed with 1.65 l of water whose temperature is 20°C. Assuming that heat exchange occurs without energy dissipation, determine the final temperature.

5.23. A copper body at 993 K is immersed in 1.75 kg of water at 291 K. As a result, the water is heated to 100°C, and 75 g of it are vaporized. Determine the mass of the body, neglecting energy losses. Plot the graph of $T = f(Q)$.

5.24. The temperature in a refrigerator is maintained by vaporizing a refrigerant freon-12. What will be the change in the internal energy of the air in the refrigerator chamber if 50 g of freon-12 are vaporized? The freon is at its boiling point.

5.25. 21 g of dry water vapour at 100°C is let into a copper calorimeter having a mass of 200 g and containing 400 g of water at 283 K. As a result, the temperature of the water in the calorimeter rises to 40°C. Determine the specific latent heat of vaporization of the water and compare the result with the tabulated value. Calculate the absolute and relative errors of measurement.

5.26. A 5-kg iron weight at 500°C is placed in a boiler containing 10 l of water at 20°C. Some of the water is vaporized, and the final temperature in the boiler is 25°C. The heat spent during the heat exchange to heat the boiler is 21.85 kJ. Determine the mass of the steam formed.

5.27. A 3-kg steel bar at 450°C is immersed in a vessel containing 3 l of water at 20°C. As a result, 70 g of the water are vaporized, and the temperature in the vessel becomes 50°C. Determine the heat losses during the heat exchange.

5.28. 2 l of water are heated in a 600-g aluminium teapot from 20 to 100°C. 50 g of the water are converted into steam. How much natural gas was burnt if the burner efficiency is 60%?

5.29. 1.2 l of water are heated from 283 to 373 K on an electric hot plate. 3% of the water is converted into steam. How long does the heating last if the power of the hot plate is 800 W and its efficiency is 65%?

5.30. 48 l of water at 277 K were poured in a still. Determine the amount of wood burnt in the furnace to obtain 20 l of distilled water. The efficiency of the still is 15%.

5.31. Water boils at 470.4 K under a pressure of 1.47 MPa (see Table 8). How much Donetsk coal must be burnt in the furnace of a boiler to obtain 50 kg of steam under these conditions? The initial temperature of the water is 10°C, and the efficiency of the boiler is 80%.

5.32. A 1.2-kg aluminium pot containing 2 l of water at 15°C is heated on a gas burner. The water in the pot is heated to 373 K, and 200 g of it are vaporized. What is the efficiency of the burner if 0.1 m^3 of natural gas was burnt?

5.33. 1 m^3 of air at 15°C contains 10 g of water vapour. Determine the absolute and relative humidities of the air.

5.34. The temperature of air is 30°C, the relative humidity is 64%. Determine the absolute humidity and the dew point.

5.35. The relative humidity of air at 25°C is 75%. How much water vapour is contained in a cubic metre of the air?

5.36. The absolute humidity of air at 5°C is 5.2×10^{-3} kg/m^3. To what temperature should this air be cooled for dew to start to precipitate?

5.37. The relative humidity of air at 18°C is 50%. At what temperature will dew precipitate?

5.38. How much water vapour is contained in 1 l of air at 17°C if the dew point is 10°C? Determine the relative humidity of the air.

5.39. The relative humidity of air in a room at 17°C is 70%. To what temperature has the window glass been cooled if it is covered with moisture?

5.40. The temperature of air is 22°C and its dew point is 10°C. Determine the absolute and relative humidities of the air.

5.41. In which case will moisture be felt more: if 1 m^3 of air contains 10 g of water vapour at 25°C or 3.8 g at 4°C?

5.42. Fog was observed in the morning, when the air temperature was 12°C. What was the temperature during the night if the relative humidity of air did not change and was 70%?

5.43. The dry-bulb thermometer of a psychrometer shows 21°C, while the reading of the wet-bulb thermometer is 16°C. What is the relative humidity of air and how much water vapour is contained in 1 m^3 of the air?

5.44. The air temperature in a room is 23°C and its relative humidity is 55%. What is the reading of the wet-bulb

thermometer of a psychrometer? What is the dew point?

5.45. The relative humidity of air in the halls of a museum is 65%. The difference between the readings of the wet-bulb and dry-bulb thermometers is 4°C. What is the temperature in the halls? How will the relative humidity change if the psychrometric difference decreases?

5.46. A cylinder contains air at 15°C. The relative humidity of the air is 63%. After the air was dried by calcium chloride, the mass of the cylinder decreased by 3.243 g. Determine the volume of the cylinder.

5.47. The relative humidity of air at 12°C is 78%. What will the change in the relative humidity be if the temperature increases to 18°C?

5.48. The temperature in a room of size $8 \times 5 \times 2.5\ m^3$ is 20°C and the relative humidity is 70%. How much water vapour is contained in the room? How much vapour is condensed as a result of a temperature drop to 10°C?

5.49. The relative humidity of air in a room of volume 30 m^3 is 60% at 20°C. Determine the saturated vapour pressure at this temperature if the mass of water evaporated in the room is 310 g.

5.50. The temperature of air in a room is 27°C and the partial pressure of water vapour in it is 1.7 kPa. Determine the absolute and relative humidities of the air.

§ 6. PROPERTIES OF LIQUIDS

Basic Concepts and Formulas

Substances in the liquid state have constant volumes, are fluid, and hence acquire the shape of the vessel in which they are contained.

Since the molecules of a liquid are packed more densely than gas molecules, the intermolecular interaction forces are significant. The range of these forces does not exceed 10 nm.

The molecules in the surface layer of a liquid experience the action of forces whose resultant is directed inside the liquid. In order to bring a molecule from the bulk to the surface of a liquid, work must be done. Therefore, the molecules of the surface layer have excess potential energy,

which is responsible for the stretched state of the surface layer.

Surface tension σ is a quantity defined as the ratio of the work required for increasing the surface area to the increment of this area:

$$\sigma = \frac{A}{\Delta S}.$$

The unit of surface tension is the joule per metre squared (J/m^2). Surface tension can also be defined as the ratio of the force F acting in the surface (force of surface tension) to the length l of the boundary of the liquid surface:

$$\sigma = F/l.$$

In this case, σ is expressed in newtons per metre (N/m).

A liquid is termed wetting if the forces of intermolecular interaction between a solid and the liquid are stronger than the forces acting between the liquid molecules. The meniscus (curved surface of the liquid) is concave for wetting liquids, and the wetting angle θ (i.e. the angle between the meniscus and the surface of the solid) is acute. For nonwetting liquids, the meniscus is convex, and the wetting angle θ is obtuse.

The curved surface of a liquid produces an additional pressure (called Laplacian pressure)

$$p_{ad} = \pm \frac{2\sigma}{R},$$

where R is the radius of the spherical surface. The additional pressure in capillaries causes wetting liquids to rise and nonwetting liquids to fall by height

$$h = \frac{2\sigma}{\rho g R} \cos\theta.$$

For complete wetting, we have $\theta = 0$, $\cos\theta = 1$, and

$$h = \frac{2\sigma}{\rho g R}.$$

Worked Problems

Problem 33. A frame in the form of equilateral triangle with sides 4 cm long is carefully placed on the surface of water. What force keeps the frame on the surface? What

force must be applied to separate the frame from the water surface? The mass of the frame is 2 g.

Given: $r = 4 \times 10^{-2}$ m is the length of a side of the frame, and $m = 2 \times 10^{-3}$ kg is the mass of the frame. From tables, we take the surface tension of water, $\sigma = 0.072$ N/m, and the free fall acceleration $g = 9.81$ m/s^2.

Find: the force F_1 keeping the frame on the surface and the force F_2 required to separate the frame from the water surface.

Solution: The frame is kept on the water surface by the force of surface tension, F_1, which can be determined from the formula $\sigma = F_1/l$:

$$F_1 = \sigma l,$$

where l is the length of the inner and outer boundaries of the liquid surface, the length being equal to twice the perimeter of the triangle: $l = 2 \times 3r$. This gives

$$F_1 = 6\sigma r, \; F_1 = 6 \times 0.072 \text{ N/m} \times 4 \times 10^{-2} \text{ m} = 1.73 \times 10^{-2} \text{ N}.$$

In order to separate the frame from the water surface, we must overcome the force of gravity acting on the frame in addition to the force of surface tension, i.e.

$$F_2 = F_1 + mg,$$

$$F_2 = 1.73 \times 10^{-2} \text{ N} + 2 \cdot 10^{-3} \text{ kg} \times 9.81 \text{ m/s}^2 = 3.7 \times 10^{-2} \text{ N}.$$

Answer. The force keeping the frame on the surface of water is 1.73×10^{-2} N. The force required to separate the frame from the surface is 3.7×10^{-2} N.

Problem 34. The surface tension of water was determined in a laboratory by using the drop weight method. 100 drops were released from a burette the inner diameter of whose opening is 1.8 mm. The mass of the droplets was 3.78 g. Using these results, determine the surface tension of the water and, comparing it with the tabulated value, calculate the relative error of the measurements.

Given: $d_0 = 1.8 \times 10^{-3}$ m is the inner diameter of the burette opening, $n = 100$ is the number of drops, and

$m = 3.78 \times 10^{-3}$ kg is the mass of 100 drops. From tables, we take the free fall acceleration $g = 9.81$ m/s^2 and the tabulated value of the surface tension for water, $\sigma_t = 0.072$ N/m.

Find: the surface tension σ of water and the relative error δ_σ of measurement.

Solution. Water drips from a burette when its valve is not open completely. The diameter d_n of the neck of a drop immediately before separation is smaller than the diameter d_0 of the burette opening (Fig. 11). This ratio is usually $d_n/d_0 = 0.9$. The drop separates only when the force of gravity becomes equal to or slightly larger than the force of surface tension: $m_1 g = \sigma l$, where $m_1 = m/n$ and $l = \pi d_n = 0.9\pi d_0$. Consequently,

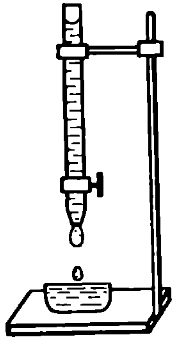

Fig. 11

$$\sigma = \frac{mg}{0.9\pi d_0 n},$$

$$\sigma = \frac{3.78 \times 10^{-3}\ \text{kg} \times 9.8\ \text{m/s}^2}{0.9 \times 3.14 \times 1.8 \times 10^{-3}\ \text{m} \times 100}$$

$$= 0.0728\ \text{N/m} = 72.8 \times 10^{-3}\ \text{N/m}.$$

The absolute error Δ_σ is equal to the difference $\sigma - \sigma_t$:

$$\Delta_\sigma = 72.8 \times 10^{-3}\ \text{N/m} - 72.0 \times 10^{-3}\ \text{N/m}$$

$$= 0.8 \times 10^{-3}\ \text{N/m}.$$

The relative error is equal to the ratio of the absolute error to the tabulated value of the surface tension:

$$\delta_\sigma = \frac{\Delta_\sigma}{\sigma_t}\ 100\%, \quad \delta_\sigma = \frac{0.8 \times 10^{-3}\ \text{N/m}}{72.0 \times 10^{-3}\ \text{N/m}}\ 100\% \simeq 1.1\%.$$

Answer. The surface tension of water is 72.8×10^{-3} N/m, and the relative error of measurement is about 1.1%.

Problem 35. To what height will water rise in a capillary whose inner diameter is 3.0 mm? What will be the height of mercury column in the same capillary? The capillaries are made of glass.

Given: $d = 3.0 \times 10^{-3}$ m is the inner diameter of the capillary. From tables, we take the surface tension of water, $\sigma_1 = 0.072$ N/m, the surface tension of mercury, $\sigma_2 = 0.47$ N/m, the density of water, $\rho_1 = 10^3$ kg/m^3, the density

of mercury, $\rho_2 = 1.36 \times 10^4$ kg/m³, and the free fall acceleration $g = 9.81$ m/s².

Find: the heights h_1 and h_2 of the water and mercury columns in the capillaries.

Solution. Water is a liquid that wets glass, and hence it has a concave meniscus. For wetting liquids, the Laplacian pressure is directed upwards, and equals $p_L = 2\sigma/r$ for complete wetting. The Laplacian pressure causes water to rise in the capillary until the hydrostatic and Laplacian pressures equalize: $p_L = p_h$, $p_h = \rho g h$, $2\sigma_1/r = \rho_1 g h_1$, where $r = d/2$. This gives

$$h_1 = \frac{2\sigma_1}{\rho_1 g r},$$

$$h_1 = \frac{2 \times 0.072\ \text{N/m}}{10^3\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 \times 1.5 \times 10^{-3}\ \text{m}} \simeq 9.8 \times 10^{-3}\ \text{m}.$$

Mercury is a liquid which does not wet glass. Therefore, the Laplacian pressure is directed downwards, into the bulk of the liquid. Hence the mercury is forced down the capillary.

The height to which a nonwetting liquid is forced down is determined by using the same formula

$$h_2 = \frac{2\sigma_2}{\rho_2 g r},$$

$$h_2 = \frac{2 \times 0.47\ \text{N/m}}{1.36 \times 10^4\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 \times 1.5 \times 10^{-3}\ \text{m}} \simeq 4.7 \times 10^{-3}\ \text{m}.$$

Answer. The water rises in the capillary by about 9.8 mm, while the mercury is lowered by 4.7 mm.

Questions and Problems

6.1. Touch the surface of water between two floating matches with a piece of soap. Repeat the experiment with a piece of sugar. Why do the matches move apart in the first case and approach each other in the second case? How do sugar and soap affect the surface tension of water?

6.2. Small pellets are made by pouring molten lead into a vessel containing water. Why do the pellets become spherical?

6.3. A capillary tube is immersed in a vessel of hot water. Will the level of the water in the capillary be affected by a fall in the temperature of the water?

6.4. Why can oily spots not be washed away by water?

6.5. Why can tin solder copper and not aluminium?

6.6. Why is an area to be soldered thoroughly cleaned from grease, dirt, and oxides?

6.7. What liquid can be poured into a tumbler to above the brim?

6.8. Why are the foundations of buildings covered with tar paper?

6.9. Why does caked soil during a drought dry more than plowed soil?

6.10. It is expedient to break up the soil between rows of crops regularly. Why is this type of cultivation sometimes called "dry irrigation"?

6.11. A hollow metal cube with a capacity of 1 dm^3 is filled with kerosene. Determine the force of the surface tension.

6.12. A cylindrical tumbler 9 cm in height contains 250 cm^3 of milk. What is the force of surface tension?

6.13. Water and petrol are poured into a test tube with a diameter of 1.5 cm so that the height of the petrol column is 10 cm. Determine the force of surface tension and the force of the pressure exerted by the petrol on the water. The curvature of the surface of the petrol should be neglected.

6.14. A match 4 cm long floats on the surface of water whose temperature is 20°C. If castor oil is poured to one side of the match, it starts moving. Determine the force acting on the match and the direction of its motion.

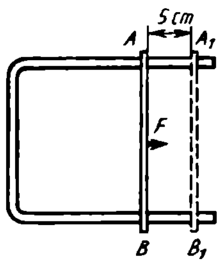

Fig. 12

6.15. A wire ring having a radius of 6.0 cm is brought in contact with the surface of blue vitriol. The mass of the ring is 5 g. What force should be applied to separate the ring from the surface of the solution?

6.16. Determine the potential energy of the surface layer of water having an area of 20 cm^2.

6.17. A soap film is formed on a wire frame with a movable side $AB = 10$ cm (Fig. 12). What work must be done to stretch the film by moving AB by 5 cm? The friction between AB and the frame should be neglected.

6.18. 532 drops of castor oil were dripped from a pipette with a tip diameter of 1.2 mm into a measuring glass. The volume of castor oil was found to be 7 cm^3. Determine the surface tension of castor oil.

6.19. Calculate the surface tension of a liquid and identify it if a force of 0.035 N is required to separate a square frame with a side of 8.75 cm from the liquid surface. The mass of the frame is 2 g.

6.20. How high can water rise in a capillary tube with an inner diameter of 10^{-3} m?

6.21. Determine the mass of alcohol rising in a capillary tube immersed in a vessel containing alcohol. The inner diameter of the capillary is 0.4 mm. The surface tension of ethyl alcohol should be taken to be 0.02 N/m.

6.22. Determine the mass of mercury forced down a capillary having an inner diameter of 0.1 mm and immersed in mercury.

6.23. 100 drops of pure water and the same number of drops of water mixed with ether are poured into two test tubes. In which test tube will the liquid level be higher?

6.24. A liquid rises 4.25 cm up a capillary with an inner diameter of 0.6 mm. Determine the density of the liquid if its surface tension is 0.071 N/m.

6.25. Determine the difference between the levels of mercury in two communicating capillaries having inner diameters of 1 and 2 mm.

§ 7. PROPERTIES OF SOLIDS. DEFORMATIONS

Basic Concepts and Formulas

All solids are elastic[3] and retain their volume and shape.

Solids are characterized by the long-range order in the arrangement of the particles of which they are composed. These particles are atoms, molecules, or ions.

The crystalline structure of a solid is a result of the ordered arrangement of its particles. The physical properties of a crystal depend on the orientation of the symmetry axis in the crystal (anisotropy).

Under the action of external forces, solids are deformed. The deformations disappearing after the forces stop acting on

[3] Amorphous bodies are treated as supercooled liquids.

a body are known as elastic. There are a large number of different deformations, but extension (compression) and shear are important.

Extension is characterized by an absolute deformation Δl:

$$\Delta l = l - l_0$$

and a relative deformation (strain)

$$\varepsilon = \frac{\Delta l}{l_0}.$$

Mechanical stress σ is defined as the ratio of the internal force emerging in a body as a result of deformation to the cross-sectional area of the body: $\sigma = F/S$. The unit of mechanical stress is the pascal (Pa).

Hooke's law establishes a relation between elastic deformations and the internal forces. The mechanical stress σ is directly proportional to the strain ε:

$$\sigma = k\varepsilon, \quad \text{or} \quad \sigma = E\frac{\Delta l}{l_0},$$

where E is Young's modulus. Young's modulus is measured in the same units as stress, viz. in pascals.

The elastic limit is the maximum stress emerging in a material for which Hooke's law remains in force.

The safety factor is defined as the ratio of the maximum (ultimate) stress σ_u of a construction to the admissible stress σ_{ad}:

$$n = \frac{\sigma_u}{\sigma_{ad}}.$$

For an elastic deformation, the potential energy E_p of a body is equal to the work done during the deformation (extension or compression) of a body:

$$E_p = \frac{F\,\Delta l}{2} = \frac{ES}{2l}(\Delta l)^2.$$

When a solid changes its state of aggregation (when it melts), the separation between the particles in the crystal lattice increases, and the lattice is destroyed. The potential energy of the interaction between molecules (particles) increases.

To melt 1 kg of a solid at its melting point, a certain amount of heat λ is required, which is known as the specific

latent heat of fusion:

$$\lambda = Q/m.$$

The specific latent heat of fusion is measured in joules per kilogram (J/kg).

In order to melt a crystalline substance, heat must be spent to heat it to its melting point and to convert it into a liquid:

$$Q = cm\,(T_m - T) + \lambda m.$$

Worked Problems

Problem 36. Determine the elongation of a copper rod having a length of 6 m and a cross-sectional area of 0.4 cm^2 under the action of a force of 2 kN.

Given: $l = 6$ m is the length of the rod, $S = 0.4 \times 10^{-4}$ m^2 is its cross-sectional area, and $F = 2 \times 10^3$ N is the applied force. From tables, we take Young's modulus for copper $E = 130 \times 10^9$ Pa.

Find: the elongation Δl of the rod.

Solution. We shall solve the problem using Hooke's law $\frac{\Delta l}{l} = \frac{1}{E}\sigma$, or $\frac{\Delta l}{l} = \frac{F}{ES}$, whence

$$\Delta l = \frac{Fl}{ES}, \quad \Delta l = \frac{2 \times 10^3 \text{ N} \times 6 \text{ m}}{130 \times 10^9 \text{ Pa} \times 0.4 \times 10^{-4} \text{ m}^2} \simeq 2.3 \times 10^{-3} \text{ m}.$$

Answer. The copper rod elongates by approximately 0.23 cm.

Problem 37. A chandelier of mass 250 kg is suspended on an aluminium rod with an ultimate stress of 0.11 GPa. What must the cross-sectional area of the rod be for the safety factor to be 4? What is the strain in the rod?

Given: $m = 250$ kg is the mass of the chandelier, $n = 4$ is the safety factor, and $\sigma_u = 1.1 \times 10^8$ Pa is the ultimate stress. From tables, we take the free fall acceleration $g = 9.81$ m/s^2 and Young's modulus for aluminium, $E = 7 \times 10^{10}$ Pa.

Find: the cross-sectional area S of the rod and the strain ε.

Solution. The deformation of the rod is caused by the force of gravity $G = mg$. The cross-sectional area of the rod is chosen depending on the mechanical stress σ emerging in the rod: $\sigma = G/S = mg/S$, whence

$$S = mg/\sigma.$$

Given the safety factor n and the ultimate stress σ_u, we can determine the admissible stress: $n = \sigma_u/\sigma$, $\sigma = \sigma_u/n$. Finally,

$$S = \frac{mg}{\sigma_u} n,$$

$$S = \frac{250 \text{ kg} \times 9.81 \text{ m/s}^2 \times 4}{1.1 \times 10^8 \text{ Pa}} \simeq 8.9 \times 10^{-5} \text{ m}^2.$$

The strain in the rod is

$$\varepsilon = \frac{\sigma_u}{nE}, \quad \varepsilon = \frac{1.1 \times 10^8 \text{ Pa}}{4 \times 7 \times 10^{10} \text{ Pa}} \simeq 4 \times 10^{-4}.$$

Answer. The cross-sectional area of the rod is 0.89 cm^2 and the strain is about 4×10^{-4}.

Problem 38. How much heat is required to convert 0.8 kg of ice at —10°C into steam at 100°C? Plot the graph of $t = f(Q)$.

Given: $m = 0.8$ kg is the mass of the ice, $t_1 = -10$°C is the initial temperature of the ice and $t_2 = 100$°C is the temperature of the steam. From tables, we take the melting point for ice, $t_0 = 0$°C, the boiling point for water, $t_b = 100$°C, the specific heat for ice, $c_i = 2090$ J/(kg·K), the specific heat for water, $c_w = 4187$ J/(kg·K), the specific latent heat of fusion for ice, $\lambda = 3.35 \times 10^5$ J/kg, and the specific latent heat of vaporization of water, $r = 2.26 \times 10^6$ J/kg.

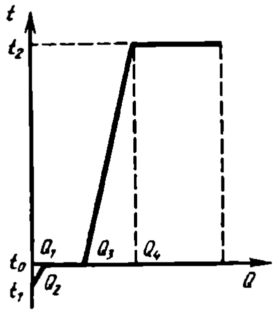

Fig. 13

Find: the amount of heat Q required to convert the ice into steam.

Solution. The required amount of heat Q is determined by summing the amounts of heat $Q = Q_1 + Q_2 + Q_3 + Q_4$ (Fig. 13), where Q_1 is the heat required to heat the ice to the melting point,

$$Q_1 = c_i m (t_0 - t_1),$$

$$Q_1 = 2090 \text{ J/(kg·K)} \times 0.8 \text{ kg} \times 10 \text{ K}$$
$$= 16720 \text{ J} = 16.72 \text{ kJ},$$

Q_2 is the amount of heat required to melt the ice,

$$Q_2 = \lambda m, \ Q_2 = 3.35 \times 10^5 \text{ J/kg} \times 0.8 \text{ kg}$$
$$= 2.6 \times 10^5 \text{ J} = 260 \text{ kJ},$$

Q_3 is the amount of heat required to heat the water obtained from the ice to the boiling point,

$$Q_3 = c_w m\,(t_2 - t_0),$$
$$Q_3 = 4187 \text{ J/(kg}\cdot\text{K)} \times 0.8 \text{ kg} \times 100 \text{ K} = 334960 \text{ J}$$
$$\simeq 335 \text{ kJ},$$

and Q_4 is the amount of heat required to evaporate the water

$$Q_4 = rm, \ Q_4 = 2.26 \times 10^6 \text{ J/kg} \times 0.8 \text{ kg} = 1.808$$
$$\times 10^6 \text{ J} \simeq 1810 \text{ kJ}.$$

The total amount of heat is

$$Q = 16.72 \text{ kJ} + 260 \text{ kJ} + 335 \text{ kJ} + 1810 \text{ kJ}$$
$$\simeq 2420 \text{ kJ}.$$

Answer. In order to convert 0.8 kg of ice into steam, an energy of 2420 kJ = 2.42 MJ is required.

Problem 39. A certain amount of steam at 100°C is introduced into a vessel containing 0.5 kg of water and 20 g of ice at 0°C. As a result, all the ice is melted, and the water is heated to 19°C. Find the mass of the steam. The heat capacity of the vessel should be neglected. Plot the graph of $t = f\,(Q)$.

Given: $m_w = 0.5$ kg is the mass of the water, $m_i = 0.02$ kg is the mass of the ice, $t_0 = 0$°C is the initial temperature of the water and ice, $\theta = 19$°C is the temperature established in the vessel, and $t_{steam} = 100$°C is the temperature of the steam. From tables, we take the specific latent heat of vaporization (condensation) for water, $r = 2.26 \times 10^6$ J/kg, the specific heat for water, $c = 4187$ J/(kg·K), and the specific latent heat of fusion for ice, $\lambda = 3.35 \times 10^5$ J/kg.

Find: the mass m_{steam} of the steam.

Solution. To solve the problem, we must write the heat balance equation. The steam delivered to the vessel containing water and ice condenses, liberating heat $Q_1 = rm_{steam}$. The water obtained from the steam is cooled from $t_{steam} = 100$°C to θ, liberating heat $Q_2 = cm_{steam} \times (t_{steam} - \theta)$. Thus, the heat given away is $Q_{giv} = Q_1 + Q_2$.

The received heat Q_{rec} is the sum of the heat spent to melt the ice, $Q_3 = \lambda m_i$, and the heat spent to heat the cold water and the water obtained from melting ice: $Q_4 = c\,(m_i + m_w) \times (\theta - t_0)$. Consequently, $Q_{rec} = Q_3 + Q_4$.

According to the energy conservation law, $Q_{giv} = Q_{rec}$. Let us write the heat balance equation and determine the mass of the steam:

$$rm_{steam} + cm_{steam}\,(t_{steam} - \theta) = \lambda m_i + c\,(m_i + m_w)\,(\theta - t_0),$$

$$m_{steam} = \frac{\lambda m_i + c\,(m_i + m_w)\,(\theta - t_0)}{r + c\,(t_{steam} - \theta)},$$

$$m_{steam} = \frac{3.35 \times 10^5\ \text{J/kg} \times 0.02\ \text{kg} + 4187\ \text{J/(kg·K)} \times 0.52\ \text{kg} \times 19\text{K}}{2.26 \times 10^6\ \text{J/kg} + 4187\ \text{J/(kg·K)} \times 81\ \text{K}}$$

$$= 1.85 \times 10^{-2}\ \text{kg}.$$

The graph of $t = f\,(Q)$ is presented in Fig. 14. Over region *1*, heat Q_1 is liberated during the condensation of the steam at constant temperature (the potential energy of interaction between the particles decreases). In region *2*, heat Q_2 is liberated because the water obtained from the steam cools (the kinetic energy of the particles decreases). In region *3*, heat Q_3 is absorbed to melt the ice (the potential energy of the interaction between the particles increases). In region *4*, heat Q_4 is absorbed by the cold water and the ice water (the kinetic energy of the particles increases), their temperature increasing to θ.

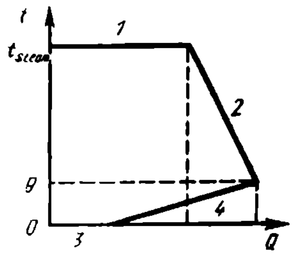

Fig. 14

Answer. About 19 g of steam are required to melt the ice and heat the water.

Problem 40. How much gray cast iron taken at 20°C can be melted in a furnace with an efficiency of 20% by burning 1.94 t of A-1 grade coal?

Given: $\eta = 20\% = 0.2$ is the efficiency of the furnace, $m_c = 1.94 \times 10^3$ kg is the mass of coal spent, and $t_1 = 20$°C is the initial temperature of the cast iron. From tables, we take the specific heat of combustion of coal, $q = 2.05 \times 10^7$ J/kg, the specific heat of cast iron, $c = 550$ J/(kg·K).

the melting point for cast iron, $t_2 = 1150°C$, and the specific latent heat of fusion for cast iron, $\lambda = 9.7 \times 10^4$ J/kg.

Find: the mass m_1 of the molten cast iron.

Solution. The amount of heat Q_{useful} required to heat the cast iron and melt it can be determined from the formula $Q_{\text{useful}} = cm_1(t_2 - t_1) + \lambda m_1$. On the other hand, the useful heat is just 20% of the spent heat, i.e. 20% of the heat liberated by burning the coal: $\eta = \frac{Q_{\text{useful}}}{Q_{\text{spent}}}$, where $Q_{\text{spent}} = qm_c$. Hence $Q_{\text{useful}} = \eta q m_c$.

We write the heat balance equation and determine the mass of the cast iron:

$$m_1 [c(t_2 - t_1) + \lambda] = \eta q m_c, \quad m_1 = \frac{\eta q m_c}{c(t_2 - t_1) + \lambda},$$

$$m_1 = \frac{0.2 \times 2.05 \times 10^7 \text{ J/kg} \times 1.94 \times 10^3 \text{ kg}}{550 \text{ J/(kg}\cdot\text{K)} \times 1130 \text{ K} + 9.7 \times 10^4 \text{J/kg}} = 11070 \text{ kg} \simeq 11.1 \text{ t}.$$

Answer. About 11.1 t of cast iron can be melted in the furnace.

Problem 41. What must be the minimum velocity of an iron meteorite for it to be completely evaporated in the Earth's atmosphere? Assume that the initial temperature of the meteorite before it enters the atmosphere is 3 K and that 50% of its kinetic energy is converted into internal energy.

Given: $T = 3$ K is the initial temperature of the meteorite, $\eta = 50\% = 0.5$. From tables, we take the specific heat of iron in the solid state, $c = 460$ J/(kg·K), the melting point of iron, $T_m = 1803$ K, the specific latent heat of fusion of iron, $\lambda = 2.7 \times 10^5$ J/kg, the specific heat of iron in the liquid state, $c_l = 830$ J/(kg·K), the boiling point of iron, $T_b = 3323$ K, and the specific latent heat of vaporization for iron, $r = 5.8 \times 10^4$ J/kg.

Find: the minimum velocity v of the meteorite at which it enters the Earth's atmosphere.

Solution. The meteorite is made of iron, so all extra data needed are taken to be the tabulated values for iron.

When the meteorite enters the atmosphere, its kinetic energy is spent on heating, fusing, and evaporating it. The problem states that these processes consume 50% of the ki-

netic energy of the meteorite. Therefore, we can write

$$0.5\,\frac{mv^2}{2} = cm\,(T_m - T) + \lambda m + c_1 m\,(T_b - T_m) + rm.$$

Eliminating the mass of the meteorite from the equation (we take it out of the parentheses on the right-hand side of the equation and cancel it out), we obtain

$$v^2 = 4\,[c\,(T_m - T) + \lambda + c_1\,(T_b - T_m) + r].$$

We write the solution in the general form and then carry out the calculations:

$$v = 2\sqrt{c\,(T_m - T) + \lambda + c_1\,(T_b - T_m) + r},$$

$$v = 2\sqrt{(460 \times 1800 + 2.7 \times 10^5 + 830 \times 1520 + 5.8 \times 10^4)\,\mathrm{m^2/s^2}}$$

$$\simeq 3100\ \mathrm{m/s}.$$

Answer. The minimum velocity of the meteorite must be approximately 3.1 km/s.

Questions and Problems

7.1. Why should a tool being sharpended be periodically cooled?

7.2. Two cubes, one cut from a single crystal and one from glass, are placed in a vessel containing hot water. Will they retain their shape?

7.3. The hardness of glass and tool steel is the same. Why are cutting tools not made of glass?

7.4. Why is lead wire used for fuses rather than some other metal?

7.5. Under what conditions can lead be melted in water?

7.6. Why does precipitation in summer have the form of rain or hail and not snow?

7.7. Why is salt sprinkled over pavements covered with ice in winter?

7.8. What is the diameter of a rod if the mechanical stress caused in it by a force of 2×10^3 N is 160 MPa?

7.9. Tensile forces of 150 N each act on the two ends of an iron wire of 2-m length and 1-mm^2 cross-sectional area. Determine the absolute elongation and strain of the wire.

7.10. Determine the elongation of a 4-m long steel wire

having a diameter of 2 mm and loaded with 70 kg. Determine the energy of elastic deformation of the wire. The mass of the wire should be neglected.

7.11. What will be the difference in the elongation of a wire if it is replaced by the same length of wire made of the same material and with the same load but with twice the diameter?

7.12. A wire having a cross-sectional area of 1 mm^2 and a length of 1 m is elongated by 1 mm under a load of 200 N. What will the elongation be for a wire made from the same material, 3 m long, with a cross-sectional area of 0.5 mm^2, and loaded by a force of 300 N?

7.13. A residual deformation appears in a copper wire having a cross-sectional area of 1.5 mm^2 when loaded by a tensile force of 45 N. What is the elastic limit of the wire material?

7.14. A brass wire has a diameter of 1 mm and a length of 3.6 m. Calculate Young's modulus for brass if a 19-kg load elongates the wire by 8 mm.

7.15. What must the minimum diameter of a steel rope be for it to be able to hold a load of 2.45×10^4 N? The ultimate stress for steel is 1.25 GPa.

7.16. What is the maximum load that can be borne by a steel rope with a cross-sectional area of 12 mm^2? The ultimate stress should be taken as 785 MPa.

7.17. How many piles having a diameter of 10 cm each are required to support a platform having a mass of 2×10^5 kg if the admissible compressive stress is 10^7 Pa?

7.18. Determine the cross-sectional area of the steel rope of a crane uniformly lifting a 6-t load if the ultimate stress of a material is 780 MPa and the safety factor is 10.

7.19. What must the minimum length of a steel rope suspended at one end be for it to rupture at the point of suspension under the force of gravity? The ultimate stress of the rope is 320 MPa.

7.20. Under a 120-N force, a spring is elongated by 5 cm. What is the potential energy of elastic deformation of the spring?

7.21. What must the elongation be of an iron rod having a length of 1.8 m, a cross-sectional area of 7.8 mm^2, and a potential energy of elastic deformation of 3.9×10^{-2} J?

7.22. A flask containing ice at 0°C is placed in a vessel

containing water at the same temperature. Will the ice melt?

7.23. Why are alcohol and not mercury thermometers used to measure temperatures in the north?

7.24. How much heat is required to melt 2.6 kg of lead at 300 K?

7.25. How much heat must be spent to convert 125 g of ice at 268 K into steam?

7.26. 0.15 kg of molten lead at its melting point are poured into 300 g of water at 285 K. Determine the temperature at thermal equilibrium, neglecting heat losses.

7.27. Equal volumes of lead and tin are taken at their melting points. What is the ratio between the amounts of heat required to liquefy them? The temperature dependence of their densities should be neglected.

7.28. To make shot, 50 kg of lead at its melting point are sprayed in jets into water. Determine the final temperature in 19 l of water taken at 283 K. Plot the graph $t = f(Q)$.

7.29. 58 g of molten lead at 357°C were poured into an indentation in a piece of ice at 273 K. How much ice is melted if the lead is cooled to 273 K? Plot the graph $t = f(Q)$.

7.30. An aluminium vessel whose mass is 240 g contains 360 g of water at 25°C. After 20 g of ice at its melting point are placed in the water and melted, the temperature in the vessel becomes 293 K. Using these data, determine the specific latent heat of fusion for ice.

7.31. An aluminium bar at 520°C is placed on ice at 273 K. Determine the mass of the aluminium bar if 26 kg of ice were melted to cool the bar.

7.32. In the far north, fresh water is obtained by melting snow. How much firewood has to be burnt to melt 1500 kg of snow at 263 K if the temperature of the obtained water is 5°C? The efficiency of the set-up is 30%.

7.33. The mass of fuel oil burnt in a smelting furnace constitutes 12% of the mass of fused metal. Determine the efficiency of the furnace in which 200 kg of liquid aluminium are obtained in a smelting process at the melting point of aluminium. The initial temperature of the metal is 283 K.

7.34. Determine the mass of metallurgical coke burnt to smelt 1.5 t of scrap iron taken at 293 K. The efficiency of the blast cupola is 60%.

7.35. A refrigerator operates for 5 h to produce ice at

271 K from 200 g of water taken at 15°C. Determine the power of the refrigerator.

7.36. Calculate the efficiency of a smelting furnace in which 1.1 t of Donetsk coal are burnt to obtain 7.2 t of white cast iron. The initial temperature of the white cast iron is 0°C.

7.37. In frictional welding, aluminium rods at 17°C are pressed tightly together, and then one is rotated by a spindle. As a result, the temperature of the aluminium where the rods are in contact rises. What mass of the aluminium is fused in 10 s if the force of the pressure is 1260 N at a rotational speed of the spindle of 90 m/min?

7.38. A skidding car moves over a snow field. The skidding consume 5 kW of power. How much snow will melt under the wheels in 1 min if the temperature of snow is 0°C?

7.39. A lead bullet having a velocity of 450 m/s and a temperature of 117°C strikes a steel beam. What fraction of the bullet fuses if 40% of its kinetic energy before the impact is spent on heating the bullet?

7.40. While making ice in a frige, the temperature of water is reduced from 278 to 273 K in 5 min and it takes 1 h 20 min then for the water to be converted into ice. Using these data, determine the specific latent heat of fusion for ice assuming that the specific heat of water is 4190 J/(kg·K).

7.41. A 300-g lump of wet snow is placed into 1.2 l of water at 21°C. After all the snow has melted, the temperature of the water drops to 6°C. How much water was contained in the lump of snow? The specific heat of water should be taken as 4190 J/(kg·K).

§ 8. THERMAL EXPANSION OF BODIES

Basic Concepts and Formulas

Solids, liquids and gases all expand upon heating. Solids expand much less than liquids and gases which expand the most.

Solid bodies with one dominant dimension are characterized by the coefficient of linear expansion α.

If the length of a solid body at $t_0 = 0$°C is l_0 and becomes l_t as a result of it being heated to a temperature t,

the coefficient of linear expansion is defined as

$$\alpha = \frac{\Delta l}{l_0 \Delta t},$$

where $\Delta l = l_t - l_0$ and $\Delta t = t - t_0$. The coefficient of linear expansion is measured in reciprocal kelvins (K^{-1}).

If the initial temperature is 0°C, $\Delta t = t$.

The length of a body at different temperatures can be determined from the formula

$$l_t = l_0 (1 + \alpha t).$$

Since the value of α is very small, we can use the formula

$$l_2 = l_1 (1 + \alpha \Delta t),$$

where $\Delta t = t_2 - t_1$, and l_2 and l_1 are the lengths of the body at these temperatures.

Solid three-dimensional bodies (like a cube or a sphere) and liquids are characterized by a coefficient of volume expansion β:

$$\beta = \frac{\Delta V}{V_0 \Delta t}.$$

The coefficient of volume expansion is also measured in reciprocal kelvins (K^{-1}).

By analogy with linear expansion, the volume of a body at any temperature can be expressed as follows:

$$V_t = V_0 (1 + \beta t).$$

For two nonzero temperatures, we can write

$$V_2 = V_1 (1 + \beta \Delta t),$$

where $\Delta t = t_2 - t_1$.

The following relation can be established between the coefficient of linear expansion and the coefficient of volume expansion: $\beta \simeq 3\alpha$.

The densities of solids and liquids also change as a result of heating. As the volume increases due to heating, the density decreases:

$$\rho_t = \frac{\rho_0}{1 + \beta \Delta t}.$$

At constant pressure, the coefficients of volume expansion for all gases are the same and equal to $\beta = 0.00\,361\ \mathrm{K}^{-1} = \frac{1}{273.15}\ \mathrm{K}^{-1}$.

Worked Problems

Problem 42. A guitar is tuned in a room at 293 K, the length of its steel string being 0.7 m. What will be the change in the length of the string outdoors where the temperature is 263 K? What will the additional mechanical stress be? What are the elastic force and the potential energy of the elastically deformed string? The cross-sectional area of the string is 0.85 mm^2.

Given: $l_1 = 0.7$ m is the initial length of the string, $T_1 = 293$ K is the initial temperature, $T_2 = 263$ K is the temperature outdoors, and $S = 8.5 \times 10^{-7}\ \mathrm{m}^2$ is the cross-sectional area of the string. From tables, we take the coefficient of linear expansion for steel $\alpha = 1.2 \times 10^{-5}\ \mathrm{K}^{-1}$, and Young's modulus for steel $E = 2.2 \times 10^{11}$ Pa.

Find: the absolute decrease Δl in the length of the string, the additional mechanical stress σ, the elastic force F, and the potential energy E_p of elastically deformed string.

Solution. As the temperature decreases, the length of the string decreases by Δl. The value of Δl can be determined from the formula

$$\Delta l = l_1 \alpha \Delta T,$$

where $\Delta T = T_2 - T_1$,

$$\Delta l = 0.7\ \mathrm{m} \times 1.2 \times 10^{-5}\ \mathrm{K}^{-1}\,(-30\ \mathrm{K})$$
$$= -2.52 \times 10^{-4}\ \mathrm{m} \simeq -0.252\ \mathrm{mm}.$$

The minus sign in front of Δl in this case indicates that the string has become shorter.

The measure of the stressed state of a deformed body is the mechanical stress σ, which can be determined using Hooke's law:

$$\sigma = \frac{E\Delta l}{l_1}, \quad \sigma = \frac{2.2 \times 10^{11}\ \mathrm{Pa} \times 2.52 \times 10^{-4}\ \mathrm{m}}{0.7\ \mathrm{m}} = 7.92 \times 10^{7}\ \mathrm{Pa}.$$

The elastic force F emerging during the compression of the string is proportional to the absolute deformation:

$F = \frac{ES}{l_1} \Delta l$. Since $\frac{E \Delta l}{l_1} = \sigma$, we have

$$F = \sigma S, \quad F = 7.92 \times 10^7 \text{ Pa} \times 8.5 \times 10^{-7} \text{m}^2 \simeq 67 \text{ N}.$$

Knowing the elastic force and the decrease in the string length, we can determine the energy of the elastic deformation of the string:

$$E_p = \frac{F \Delta l}{2}, \quad E_p = \frac{67 \text{ N} \times 2.52 \times 10^{-4} \text{ m}}{2} \simeq 8.4 \times 10^{-3} \text{ J}.$$

Answer. As a result of the decrease tintemperature, the string contracts by about 0.25 mm. The additional stress emerging thereby is 7.92×10^7 Pa. The elastic force is approximately 67 N, and the potential energy of the elastic deformation is about 8.4 mJ.

Problem 43. A tank car contains 20 t of petrol at 298 K. How much smaller will the volume of the petrol at the terminal be if the ambient temperature is 248 K? The change in the capacity of the tank car as a result of the change in temperature should be neglected.

Given: $T_1 = 298$ K is the initial temperature of the petrol, $m = 20 \text{ t} = 2 \times 10^4$ kg is the mass of the petrol, $T_2 = 248$ K is the final temperature of the petrol. From tables, we take the coefficient of volume expansion of petrol, $\beta = 10^{-3} \text{ K}^{-1}$, and the density of petrol at 273 K, $\rho_0 = 7 \times 10^2 \text{ kg/m}^3$.

Find: the change ΔV in the volume of the petrol.

Solution. As the temperature falls, the volume of the petrol decreases by $\Delta V = V_2 - V_1$, where V_1 is the volume of the petrol at T_1, and V_2 is its volume at T_2. The volumes V_1 and V_2 can be found from the formulas

$$V_1 = V_0 [1 + \beta (T_1 - T_0)], \quad V_2 = V_0 [1 + \beta (T_2 - T_0)].$$

The unknown volume V_0 at $T_0 = 273$ K can be calculated from the formula $V_0 = m/\rho_0$. Then

$$\Delta V = \frac{m}{\rho_0} [1 + \beta (T_2 - T_0) - 1 - \beta (T_1 - T_0)],$$

or

$$\Delta V = \frac{m\beta}{\rho_0} (T_2 - T_1),$$

$$\Delta V = \frac{2 \times 10^4 \text{ kg} \times 10^{-3} \text{ K}^{-1} (248 \text{ K} - 298 \text{ K})}{7 \times 10^2 \text{ kg/m}^3} = -1.43 \text{ m}^3.$$

The minus sign indicates that the volume has decreased as a result of the fall in the temperature.

Answer. The volume of the petrol is reduced by 1.43 m^3 due to the temperature drop.

Problem 44. A steel ingot has a volume of 2.8 dm^3 at 0°C. Determine its volume at 525°C. Find the density of steel at this temperature. How much heat is spent to heat it?

Given: $V_0 = 2.8 \times 10^{-3}$ m^3 is the volume of the ingot at 0°C, $t_0 = 0°C$ is the initial temperature of the ingot, $t = 525°C$ is the temperature of the hot ingot. From tables, we take the coefficient of linear expansion for steel, $\alpha = 1.2 \times 10^{-5}$ K^{-1}, the density of steel at 0°C, $\rho_0 = 7.8 \times 10^3$ kg/m^3, and the specific heat of steel, $c = 460$ J/(kg·K).

Find: the volume V of the hot ingot, the density ρ of steel at 525°C, and the amount of heat Q spent to heat the ingot.

Solution. The volume of the ingot heated to 525°C can be determined from the formula

$$V = V_0 (1 + \beta \Delta T),$$

where $\Delta T = t - t_0$.

Since the coefficient of volume expansion for solids is $\beta = 3\alpha$ (with a small error), we can find the volume:

$$V = 2.8 \times 10^{-3}\ m^3 (1 + 3 \times 1.2 \times 10^{-5}\ K^{-1} \times 525°C)$$
$$= 2.85 \times 10^{-3}\ m^3.$$

The density of steel decreases with increasing temperature. It can be determined from the formula

$$\rho = \rho_0/(1 + \beta \Delta T),$$
$$\rho = \frac{7.8 \times 10^3\ kg/m^3}{1 + 3.6 \times 10^{-5}\ K^{-1} \times 525\ K} = 7.65 \times 10^3\ kg/m^3.$$

Pay attention to the fact that the temperature difference can be expressed either in degrees Celsius or in kelvins.

The heat spent to heat the ingot is $Q = cm\ (t - t_0)$. Since mass can be expressed in terms of density, $m = \rho_0 V_0 = \rho V$, we have

$$Q = \rho_0 V_0 c\ (t - t_0),$$
$$Q = 7.8 \times 10^3\ kg/m^3 \times 2.8 \times 10^{-3}\ m^3 \times 460\ J/(kg \cdot K) \times 525\ K$$
$$= 5.3 \times 10^6\ J.$$

Answer. The volume of the hot ingot is 2.85×10^{-3} m^3, the density of steel at 525°C is 7.65×10^3 kg/m^3, and the amount of heat spent to heat it is 5.3 MJ.

Remark. The volume of the hot ingot can be found in a different way: first we can determine $\Delta V = V_0\beta\Delta T$, and then $V = V_0 + \Delta V$.

Questions and Problems

8.1. Why is Invar used in clockwork mechanisms?

8.2. Why are the holes in the cover plates connecting rail sections together oval?

8.3. The filaments fused into the glass parts of vacuum tubes or incandescent lamps must have the same thermal expansion coefficients as that of glass. Why? Can copper wire be used?

8.4. A copper ring is heated in a bunsen flame. What will be the change in the inner diameter of the ring?

8.5. How can a steel rod be extracted from a tight-fitting brass sleeve?

8.6. Prove that the coefficients of volume expansion for solids are three times as large as the coefficients of linear expansion. Is it an exact or approximate relation?

8.7. Cold water can be poured into a red-hot flask made of quartz glass without breaking it. How can this be explained?

8.8. Why should tanks for petrol, kerosene or petroleum not be filled to the brim in summer time?

8.9. Dentists do not recommend that we eat very hot food. Why?

8.10. What will happen to a bimetallic strip[4] when it is heated? Where are such plates used?

8.11. A steel measuring tape 100 m long is used for geodesic surveying. How much will it be elongated if the temperature increases by 10 K?

8.12. What is the maximum increase in temperature of a 1-km long aluminium wire for it to be elongated by only 230 mm?

8.13. As a result of heating through 100 K, each metre of a wire elongates by 0.4 mm. What is the material from which the wire is made?

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[4] A bimetallic strip is made from two different metals rivetted together.

8.14. The Ostankino television tower in Moscow is made of reinforced concrete and is 533 m high at 273 K. How high will it be at + 20°C and at − 20°C? Assume that $\alpha = 1.2 \times 10^{-5}$ K^{-1} for reinforced concrete.

8.15. An iron rod 60 cm long at 273 K is placed into a furnace. As a result, it elongates by 6.5 mm. Determine approximately the temperature in the furnace.

8.16. The diameter of an iron tyre at 0°C is 6 mm less than the diameter of the wheel (equal to 1.2 m) on which it has to be set. By how many kelvins should the temperature of the tyre be raised to set it onto the wheel?

8.17. The inner diameter of a copper ring at 273 K is 5 cm. To what temperature should it be heated for it to accommodate a ball 5.01 cm in diameter?

8.18. The transmission line between the Volzhskaya hydroelectric station and Moscow is 1000 km long. The line is made of steel-and-aluminium wire. What will be the difference in the length of the steel and aluminium components if the temperature increases by 30 K?

8.19. The rod antenna of the Luna-20 probe, which is operating on the surface of the Moon, is 2 m long at noon when the temperature at the Moon's surface is 393 K. How long will the antenna be at midnight when the temperature on the Moon drops to 123 K? The antenna is made of brass.

8.20. A steel beam is fastened on two supports which prevent it from expanding. The cross-sectional area of the beam is 140 cm^2. What will be the force of the pressure exerted by the beam on the supports if the temperature increases by 20 K?

8.21. An aluminium rod whose cross-sectional area is 4 cm^2 is tightly clamped between two jams. What must be the increase in temperature for the force of pressure exerted by the rod on the jams to be 9.7 kN?

8.22. What forces should be applied to the ends of a steel rod of a cross-sectional area 10 cm^2 to prevent it from elongating when heated from 273 to 303 K?

8.23. Two rulers, one made from aluminium and one from steel, have the same length of 1 m at 0°C. At what temperature will the difference in their lengths be 5.5 mm?

8.24. How much heat is required to make a copper rod whose cross-sectional area is 2 cm^2 elongate by 0.1 mm when heated?

8.25. A steel tyre is set on a wheel at 300 K. What force will emerge when the tyre is cooled to 293 K if its cross-sectional area is 20 cm^2?

8.26. A cylinder is cut from quartz so that its axis is parallel to the symmetry axis of the quartz crystal. At 18°C, the radius of the cylinder base is 10 mm and its height is 50 mm. Determine the volume of the cylinder at 573 K. The coefficient of linear expansion is 7.2×10^{-5} K^{-1} for the longitudinal axis and 1.32×10^{-5} K^{-1} for the transverse axis.

8.27. The coefficient of surface thermal expansion is twice the coefficient of linear expansion. What must be stipulated in this case?

8.28. A copper sheet having a size of 0.6×0.5 m^2 is heated from 293 K to 600°C. Determine the area of the hot sheet.

8.29. What is the increase in the temperature of an aluminium sheet if its area has increased by 3200 mm^2 by heating? The area of the sheet at 0°C is 1.5 m^2.

8.30. A vessel whose volume is 5 l is filled to the brim with kerosene at 0°C, brought into a room where the temperature is 18°C, and placed on a tray. How much kerosene (in litres) will overflow if the thermal expansion of the vessel is not taken into account? How much will overflow if it is taken into account, the coefficient of volume expansion for the vessel material being $\beta = 3.6 \times 10^{-5}$ K^{-1}?

8.31. A 2-l aluminium kettle is filled with water at 4°C. How much water (in litres) will overflow from the kettle if it is heated to 353 K?

8.32. What is the density of tungsten at its melting point?

8.33. At what temperature is the density of concrete 2.19×10^3 kg/m^3? Assume that the coefficient of linear expansion for concrete is 1.2×10^{-5} K^{-1}.

8.34. The volume of a brass cylinder at 325 K is 425 cm^3. Determine the mass of the cylinder.

8.35. The mass of a copper bar is 10 kg. At what temperature will the bar have a volume of 1.125 dm^3?

8.36. Alcohol at 273 K is poured into a 10-l glass bottle. Neglecting the thermal expansion of glass, calculate the volume and mass of alcohol that can be poured into it so that it does not overflow at 50°C.

8.37. Solve Problem 8.36 taking into account the expansion of the glass bottle.

8.38. How much petrol can be poured at 273 K into a 60-m^3 iron tank so that it does not overflow when heated to 40°C? Determine the mass of the petrol.

8.39. What volume is occupied by water in a steam boiler at 100°C if the mass of the water is 2000 kg?

8.40. What is the minimum capacity of the cooling system of a transformer if the temperature of the oil in it does not exceed 363 K during operation of the transformer? The mass of the oil is 3000 kg.

8.41. A 500-cm^3 flask made of quartz glass is filled with mercury. Determine the coefficient of volume expansion for mercury if 8.91 cm^3 of the mercury overflows as a result of heating the flask from 273 to 373 K.

8.42. How much heat is spent if the volume of a mass of mercury increases when heated by 4.5 cm^3? Heat losses should be neglected.

Chapter II

Fundamentals of Electrodynamics

§ 9. ELECTRIC FIELD

Basic Concepts and Formulas

Under normal conditions, any physical body is electrically neutral, i.e. contains equal numbers of elementary electric charges of opposite signs. The unit of electric charge is the coulomb (C). When a body is electrostatically charged, the electric charges are redistributed so that the body has an excess charge of one sign; it becomes electrically charged. For example, if a body has an excess number of electrons, it is negatively charged.

The algebraic sum of the charges in an isolated system remains constant (the *law of charge conservation*).

Two bodies bearing like charges repel each other, while oppositely charged bodies attract each other. The force F with which two point charges Q_1 and Q_2 interact is given by Coulomb's law

$$F=\frac{1}{4\pi\varepsilon_0}\frac{|Q_1|\,|Q_2|}{\varepsilon r^2},$$

where r is the separation between the charges, ε is the permittivity of the medium, and ε_0 is the electric constant. It is good to remember that $1/(4\pi\varepsilon_0) = 9\times10^9$ m/F.

Electric charges interact through electric fields, which are characterized by a vector quantity known as the electric field strength $\mathbf{E}$:

$$\mathbf{E}=\frac{\mathbf{F}}{Q_t},$$

where Q_t is a test charge in the field.

The electric field strength produced by a point charge is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\varepsilon r^2},$$

where Q is the electric charge producing the field.

When a charge moves in an electric field, work is done. Therefore, the energy characteristic of electric field is the electric potential

$$\varphi = \frac{A}{Q_t}.$$

The unit of potential is the volt (V).

The electric potential of the field produced by a point charge or electrically charged sphere is

$$\varphi_{sph} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\varepsilon r}.$$

The work A done in an electric field to move a charge depends on the potential difference for the points between which the charge Q moves:

$$A = Q(\varphi_1 - \varphi_2).$$

It does not depend on the shape of the path along which the point moves.

For a uniform field,

$$E = \frac{\varphi_1 - \varphi_2}{d} = \frac{U}{d},$$

where d is the separation between points *1* and *2* measured along the field line. This formula gives the dimensions of the electric field strength, viz. volt per metre (V/m).

When an isolated conductor is electrostatically charged, its potential increases in proportion to the charge Q imparted to it: $Q = C\varphi$. Here C is a proportionality factor known as the capacitance of the conductor:

$$C = \frac{Q}{\varphi}.$$

The unit of capacitance is the farad (F).

The electric capacitance of an isolated sphere is

$$C_{sph} = 4\pi\varepsilon_0\varepsilon r.$$

Electric energy can be stored in capacitors. The capacitance of a parallel-plate capacitor is calculated by the formula

$$C = \frac{\varepsilon_0 \varepsilon S}{d},$$

while the energy accumulated in it is given by

$$W = \frac{CU^2}{2}.$$

When capacitors are connected in parallel to form a bank, the total capacitance is equal to the sum of the capacitances of individual capacitors:

$$C = C_1 + C_2 + C_3 + \ldots.$$

The potential difference (voltage) across all the capacitors is then the same.

When a bank is formed by series-connected capacitors, the total capacitance is determined from the formula

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots.$$

Worked Problems

Problem 45. A conducting sphere bearing a charge of 1.8×10^{-8} C is brought in contact with two similar spheres, one of which has a charge of -0.3×10^{-8} C and the other is neutral. How will the charge be distributed among the spheres? What will be the force of interaction between two such spheres in vacuum at a distance of 5 cm from each other?

Given: $Q_1 = 1.8 \times 10^{-8}$ C, $Q_2 = -0.3 \times 10^{-8}$ C, and $Q_3 = 0$ are the charges of the spheres before they are brought in contact, and $r = 5 \times 10^{-2}$ m is the distance at which two spheres interact. From tables, we take the electric constant $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m and the permittivity of vacuum, $\varepsilon = 1$.

Find: the electric charges Q_1', Q_2', and Q_3' of the spheres after they have been brought in contact and the force F of electrostatic interaction between two spheres.

Solution. When the spheres are brought in contact, some of the charges neutralize each other since they have opposite signs. The remaining charge will be equally distributed between the three spheres:

$$Q_1' = Q_2' = Q_3' = \frac{Q_1 + Q_2 + Q_3}{3}, \quad Q_1' = \frac{1.8 \times 10^{-8}\ \text{C} - 0.3 \times 10^{-8}\ \text{C}}{3}$$
$$= 0.5 \times 10^{-8}\ \text{C}.$$

The force of electrostatic interaction between two identical charges Q_1' and Q_2' in vacuum can be determined from Coulomb's law:

$$F=\frac{1}{4\pi\varepsilon_0}\frac{(Q_1')^2}{r^2},\ \frac{1}{4\pi\varepsilon_0}=9\times10^9\ \mathrm{m/F}=9\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}},$$

$$F=9\times10^9\ \mathrm{N\cdot m^2/C^2}\frac{(0.5\times10^{-8}\ \mathrm{C})^2}{25\times10^{-4}\ \mathrm{m^2}}=9\times10^{-5}\ \mathrm{N}.$$

Answer. The charge on each sphere after they have been brought in contact is 0.5×10^{-8} C and the force of their interaction is 9×10^{-5} N.

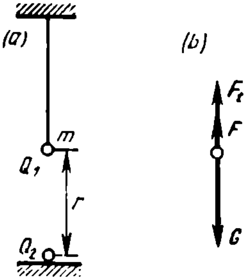


Fig. 15

Problem 46. Two charged metal spheres negligibly small in size are placed in transformer oil where they interact with a force of 2.5×10^{-4} N. Determine the separation between the spheres if their charges are 6 and 60 nC.

Given: $Q_1=6\times10^{-9}$ C and $Q_2=6\times10^{-8}$ C are the electric charges on the spheres and $F=2.5\times10^{-4}$ N is the force of their electrostatic interaction. From tables, we take the permittivity of transformer oil, $\varepsilon=2.5$.

Find: the distance r between the spheres.

Solution. We have point charges and hence Coulomb's law is applicable. Then the distance between the charges is

$$r=\sqrt{\frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon F}}.$$

Substituting the numerical values, we obtain

$$r=\sqrt{\frac{9\times10^9\ \mathrm{m/F}\times6\times10^{-9}\times6\times10^{-8}\ \mathrm{C^2}}{2.5\times2.5\times10^{-4}\ \mathrm{N}}}=7.2\times10^{-2}\,\mathrm{m}.$$

Answer. The separation between the spheres is 7.2 cm.

Problem 47. A 2-g sphere bearing a charge of 3×10^{-8} C is suspended in air on a silk thread. Determine the tension in the thread when another sphere bearing a like charge of 2.4×10^{-7} C is placed 10 cm beneath the first (Fig. 15*a*).

Given: $m=2\times10^{-3}$ kg is the mass of the sphere, $Q_1=3\times10^{-8}$ C is the electric charge of the sphere, $r=10^{-1}$m is

the distance between the spheres, and $Q_2 = 2.4 \times 10^{-7}$ C is the electric charge of the second sphere. From tables, we take the permittivity of air, $\varepsilon = 1$, and the free fall acceleration $g = 9.81$ m/s^2.

Find: the tension F_t of the thread.

Solution. The sphere on the thread experiences three forces: the force of gravity $G = mg$, the Coulomb force of repulsion

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{\varepsilon r^2},$$

and the tension F_t in the thread (Fig. 15*b*). Since the spheres

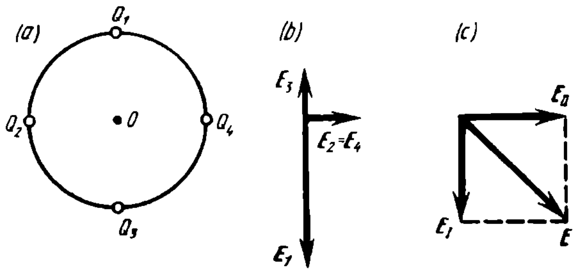


Fig. 16

are at rest, the equation for the forces acting on the first sphere has the form $F_t + F - G = 0$. Hence

$$F_t = G - F, \quad F_t = mg - \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{\varepsilon r^2} \left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ m/F}\right).$$

Substituting the numerical values, we obtain

$$F_t = 2 \times 10^{-3} \text{ kg} \times 9.81 \text{ m/s}^2 - 9 \times 10^9 \text{ m/F} \times \frac{3 \times 10^{-8} \text{ C} \times 2.4 \times 10^{-7} \text{ C}}{10^{-2} \text{ m}^2} \simeq 1.31 \times 10^{-2} \text{ N}.$$

Answer. The tension in the thread is approximately 1.3×10^{-2} N.

Problem 48. Electric charges $Q_1 = 4.8 \times 10^{-7}$ C, $Q_2 = Q_3 = 1.6 \times 10^{-7}$ C, and $Q_4 = -1.6 \times 10^{-7}$ C (Fig. 16*a*) are arranged in a circle of radius 2 cm at equal distances from one another. Determine the field strength and potential of the electric field formed by all the charges at the centre O of the circle.

Given: $Q_1 = 4.8 \times 10^{-7}$ C, $Q_2 = Q_3 = 1.6 \times 10^{-7}$ C, and $Q_4 = -1.6 \times 10^{-7}$ C are the charges producing the field, $r = 2 \times 10^{-2}$ m is the radius of the circle. From tables, we take the permittivity of vacuum, $\varepsilon = 1$.

Find: the electric field strength E and the electric potential φ at point O.

Solution. Since the medium is not specified in the problem, we solve it for vacuum. Each of the charges produces at point O a field of strength $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, and $\mathbf{E}_4$ respectively. The electric field vector $\mathbf{E}$ at O is equal to the geometric sum of the field strengths due to the individual charges:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4.$$

The electric field strength E_1 due to charge Q_1 can be calculated by the formula

$$E_1 = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{\varepsilon r^2} \left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ m/F} \right),$$

$$E_1 = 9 \times 10^9 \text{ m/F} \frac{4.8 \times 10^{-7} \text{ C}}{4 \times 10^{-4} \text{ m}^2} = 10.8 \times 10^6 \text{ V/m}.$$

Since the charges Q_2, Q_3, and Q_4 are equal in magnitude and are at the same distance from point O, we can write

$$E_2 = E_3 = E_4 = \frac{1}{4\pi\varepsilon_0} \frac{Q_2}{\varepsilon r^2},$$

$$E_2 = E_3 = E_4 = 9 \times 10^9 \text{ m/F} \frac{1.6 \times 10^{-7} \text{ C}}{4 \times 10^{-4} \text{ m}^2} = 3.6 \times 10^6 \text{ V/m}.$$

In order to determine the resultant field strength, we first add the vectors directed along the same straight line (Fig. 16*b*)

$$E_{\text{I}} = E_1 - E_2 = 10.8 \times 10^6 \text{ V/m} - 3.6 \times 10^6 \text{ V/m} = 7.2 \times 10^6 \text{ V/m},$$

$$E_{\text{II}} = E_2 + E_4 = 3.6 \times 10^6 \text{ V/m} + 3.6 \times 10^6 \text{ V/m} = 7.2 \times 10^6 \text{ V/m}.$$

The required resultant vector $\mathbf{E}$ can be found using the parallelogram rule (Fig. 16*c*). In the case under consideration, we can use the Pythagorean theorem since we have a right-angled triangle:

$$E = \sqrt{2(7.2 \times 10^6 \text{ V/m})^2} = 10.2 \times 10^6 \text{ V/m},$$

The electric potential is a scalar quantity. Therefore, the potential of the resultant field produced by the charges Q_1, Q_2, Q_3, and Q_4 is equal to the algebraic (and not geometric as for the field strength) sum of the potentials of the fields produced by all the charges:

$$\varphi_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{\varepsilon r}, \quad \varphi_1 = 9\times 10^9\ \text{m/F}\ \frac{4.8\times 10^{-7}\ \text{C}}{2\times 10^{-2}\ \text{m}} = 21.6\times 10^4\ \text{V},$$

$$\varphi_2 = \varphi_3 = 9\times 10^9\ \text{m/F}\ \frac{1.6\times 10^{-7}\ \text{C}}{2\times 10^{-2}\ \text{m}} = 7.2\times 10^4\ \text{V},$$

$$\varphi_4 = -7.2\times 10^4\ \text{V}.$$

The potential at point O is

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3 - \varphi_4.$$

Since φ_3 and φ_4 are equal in magnitude, we have

$$\varphi = \varphi_1 + \varphi_2, \quad \varphi = 21.6\times 10^4\ \text{V} + 7.2\times 10^4\ \text{V} = 2.88\times 10^5\ \text{V}.$$

Answer. The field strength at the centre of the circle is approximately 10^7 V/m, and the field potential is 2.9×10^5 V.

Problem 49. Two point charges of 2.64×10^{-8} and 3.3×10^{-9} C are in vacuum at a distance of 0.6 m from each other. What work must be done to bring the charges closer to 25 cm?

Given: $Q_1 = 2.64\times 10^{-8}$ C, $Q_2 = 3.3\times 10^{-9}$ C are the electric charges, $r_1 = 0.6$ m is the initial distance between the charges and $r_2 = 0.25$ m is the separation between the charges after they are brought closer. From tables, we take the electric constant $\varepsilon_0 = 8.85\times 10^{-12}$ F/m and the permittivity of vacuum, $\varepsilon = 1$.

Find: the work A required to bring the charges closer together.

Solution. We assume that the charge Q_1 produces an electric field and the charge Q_2 is moved in the field. Then the work of external forces done to bring the second charge closer to the first is

$$A = Q_2(\varphi_1 - \varphi_2),$$

where φ_1 and φ_2 are the electric potentials of the points between which charge Q_2 is moved:

$$\varphi_1 = \frac{Q_1}{4\pi\varepsilon_0\varepsilon r_1} = \frac{2.64\times 10^{-8}\ \mathrm{C}}{4\pi 8.85\times 10^{-12}\ \mathrm{F/m}\times 0.6\ \mathrm{m}} = 396\ \mathrm{V},$$

$$\varphi_2 = \frac{Q_1}{4\pi\varepsilon_0\varepsilon r_2} = \frac{2.64\times 10^{-8}\ \mathrm{C}}{4\pi 8.85\times 10^{-12}\ \mathrm{F/m}\times 0.25\ \mathrm{m}} = 950\ \mathrm{V}.$$

We can now find the work:

$$A = 3.3\times 10^{-9}\ \mathrm{C}\,(396\ \mathrm{V} - 950\ \mathrm{V}) \simeq -1.83\times 10^{-6}\ \mathrm{J}.$$

The minus sign indicates that the work is done against the field forces.

Answer. The work done to bring the charges closer is 1.83×10^{-6} J.

Problem 50. A dust particle with mass 10^{-7} g is suspended between the plates of a parallel-plate air capacitor to which a voltage of 500 V is applied (Fig. 17). The separation between the plates is 5 cm. Determine the electric charge on the dust particle.

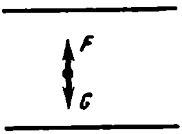


Fig. 17

Given: $m = 10^{-10}$ kg is the mass of the dust particle, $U = 500$ V is the voltage across the capacitor plates, and $d = 5\times 10^{-2}$ m is the distance between the plates. From tables, we take the free fall acceleration $g = 9.8\ \mathrm{m/s^2}$.

Find: the charge Q of the dust particle.

Solution. The dust particle in the uniform field of the capacitor is acted upon by the force of gravity $G = mg$ directed downwards and by the force $F = QE$ exerted by the electric field in the upward direction. The particle is in equilibrium provided that these forces are equal: $G = F$, or $mg = QE$. Hence

$$Q = mg/E.$$

Using the well-known relation $E = U/d$ between the electric field strength and voltage, we obtain

$$Q = \frac{mgd}{U},\quad Q = \frac{10^{-10}\ \mathrm{kg}\times 9.8\ \mathrm{m/s^2}\times 5\times 10^{-2}\ \mathrm{m}}{500\ \mathrm{V}} \simeq 10^{-13}\ \mathrm{C}.$$

Answer. The charge on the dust particle is approximately 10^{-13} C.

Problem 51. A voltage of 90 V is applied to a parallel-plate air capacitor with a plate area of 60 cm^2. The charge on the capacitor becomes 10^{-9} C. Determine the capacitance of the capacitor, the energy stored in it, and the separation between the plates.

Given: $S = 6 \times 10^{-3}$ m^2 is the area of a plate, $U = 90$ V is the voltage across the plates, and $Q = 10^{-9}$ C is the charge on the capacitor. From tables, we take the electric constant $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, and the permittivity of air $\varepsilon = 1$.

Find: the capacitance C of the capacitor, the energy W stored in the capacitor, and the separation d of the plates.

Solution. Using the formula $C = Q/U$, we determine the capacitance of the capacitor:

$$C = \frac{10^{-9}\ \mathrm{C}}{90\ \mathrm{V}} \simeq 1.1 \times 10^{-11}\ \mathrm{F} \simeq 11\ \mathrm{pF}.$$

In order to determine the energy stored in the capacitor, we can use any of the following three formulas:

$$W = \frac{QU}{2},\quad W = \frac{CU^2}{2},\quad W = \frac{Q^2}{2C}.$$

This gives

$$W = \frac{10^{-9}\ \mathrm{C} \times 90\ \mathrm{V}}{2} = 4.5 \times 10^{-8}\ \mathrm{J}.$$

The separation between the capacitor plates can be determined from the formula for the capacitance of a parallel-plate capacitor, $C = \varepsilon\varepsilon_0 S/d$. Since $\varepsilon = 1$, we obtain $C = \varepsilon_0 S/d$. Hence

$$d = \frac{\varepsilon_0 S}{C},\quad d = \frac{8.85 \times 10^{-12}\ \mathrm{F/m} \times 6 \times 10^{-3}\ \mathrm{m}^2}{1.1 \times 10^{-11}\ \mathrm{F}} \simeq 5 \times 10^{-3}\ \mathrm{m}.$$

Answer. The capacitance of the capacitor is approximately 11 pF, its energy is 4.5×10^{-8} J, and the separation between the plates is about 5 mm.

Problem 52. Determine the capacitance of the capacitor bank shown in Fig. 18*a* if $C_1 = 1.5\ \mu$F, $C_2 = 2\ \mu$F, $C_3 = 3\ \mu$F, $C_4 = 4\ \mu$F, and $C_5 = 2\ \mu$F. What energy is stored in the bank if the voltage applied to it is 500 V?

Given: $C_1 = 1.5\ \mu$F, $C_2 = 2\ \mu$F, $C_3 = 3\ \mu$F, $C_4 = 4\ \mu$F, and $C_5 = 2\ \mu$F are the capacitances of the capacitors, and $U = 500$ V is the voltage across the bank.

Find: the capacitance C of the bank and the energy W of the electric field of the capacitor bank.

Solution. In the problem, it is convenient to express the capacitances of the capacitors in microfarads. The capacitors

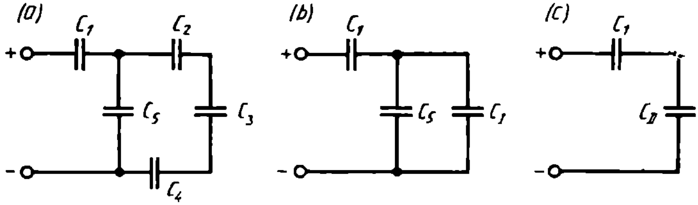


Fig. 18

C_2, C_3, and C_4 are connected in series. They can be replaced by a single capacitor having an equivalent capacitance (Fig. 18*b*). For series-connected capacitors, we have

$$\frac{1}{C_I}=\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4},$$

$$\frac{1}{C_I}=\frac{1}{2\,\mu F}+\frac{1}{3\,\mu F}+\frac{1}{4\,\mu F},\quad C_I=0.92\,\mu\mathrm{F}.$$

The capacitors C_5 and C_I are connected in parallel. Therefore, their equivalent capacitance is

$$C_{II}=C_5+C_I,\; C_{II}=2\,\mu\mathrm{F}+0.92\,\mu\mathrm{F}=2.92\,\mu\mathrm{F}\simeq 3\,\mu\mathrm{F}.$$

As a result, we obtain the capacitors C_1 and C_{II} in series (Fig. 18*c*).

The capacitance of the bank can be found from the formula

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_{II}},$$

$$C=\frac{C_1C_{II}}{C_1+C_{II}},\quad C=\frac{1.5\,\mu\mathrm{F}\times 3\,\mu\mathrm{F}}{1.5\,\mu\mathrm{F}+3\,\mu\mathrm{F}}=1\,\mu\mathrm{F}=1\times 10^{-6}\,\mathrm{F}.$$

The energy stored in the bank can be determined from the formula

$$W=\frac{CU^2}{2},\quad W=\frac{1\times 10^{-6}\,\mathrm{F}\times 25\times 10^{4}\,\mathrm{V}^2}{2}=1.3\times 10^{-3}\,\mathrm{J}.$$

Answer. The capacitance of the bank is 1 μF and the energy is approximately 1.3×10^{-3} J.

Problem 53. An electron flies from point A to point B, the potential difference between these points being 100 V.

What is the velocity acquired by the electron at point B if the velocity at point A is zero? The charge-to-mass ratio for electrons is 1.76×10^{11} C/kg.

Given: $U = 100$ V is the potential difference between points A and B, $e/m_e = 1.76 \times 10^{11}$ C/kg is the charge-to-mass ratio for the electron.

Find: the velocity v of the electron at point B.

Solution. The work done by the field in displacing the electron is $A = eU$. This work is converted into the kinetic energy of the electron $E_k = m_e v^2/2$. From the energy conservation law, we have $eU = m_e v^2/2$, whence

$$v = \sqrt{2U \frac{e}{m_e}},$$

$$v = \sqrt{2 \times 100\,\text{V} \times 1.76 \times 10^{11}\,\text{C/kg}} = 5.9 \times 10^6\,\text{m/s}.$$

Answer. The velocity of the electron at point B is approximately 5900 km/s.

Questions and Problems

Conservation of electric charge. Coulomb's law

9.1. A drop of oil has an electric charge of -3.2×10^{-19} C. Determine the number of excess electrons on the drop.

9.2. Three electrons are missing from an electrostatically charged body. Determine the magnitude and sign of the charge on the body.

9.3. Can an electric charge of one sign be obtained as a result of electrostatic charging by friction?

9.4. Under what conditions can a brass rod be charged?

9.5. An electric charge on a conducting sphere has to be divided into three equal parts. How can this be done?

9.6. Why is a metal chain that reaches the ground fixed to a lorry for transporting petrol?

9.7. Can a positive charge be obtained on an electroscope using a negatively charged ebonite rod?

9.8. What will happen to the surface charge density on a metal sheet rolled into a cylinder?

9.9. An elder ball is tied to a silk thread. What will happen when an electrostatically charged rod is brought close to it?

9.10. Will the force of interaction between two point charges change if the magnitude of each charge and the separation between charges are halved?

9.11. Two identical conducting spheres bearing electric charges of 3.2×10^{-19} and -3.2×10^{-19} C are brought in contact. What are the new values of charge on the spheres? How many electrons have passed from one sphere to another?

9.12. What is the force of interaction between two point charges of 12 nC each separated in vacuum by 3 cm? By what factor will the force of interaction be reduced if the charges are placed in water?

9.13. Two identical point charges in glycerol 9.0 cm apart interact with a force of 1.3×10^{-5} N. Determine the magnitudes of the charges.

9.14. What is the force of interaction between two 1-C charges 1 m apart (a) in vacuum and (b) in kerosene?

9.15. At what distance would two 1-C charges interact in vacuum with a force of 1 N?

9.16. Using Table 14, answer the following questions: (1) What is the ratio of the forces of interaction between electric charges in vacuum and in mica? (2) In what media is the force of interaction between charges equal to half the force of their interaction in vacuum? (3) To what medium should charges be transferred from vacuum so that their interaction is reduced by a factor of 81?

9.17. Will the electrostatic interaction force between two charges change if they are set in ice instead of vacuum?

9.18. Two mercury drops on glass of mass 20 g each and 4.0 cm apart receive charges of -6.0×10^{-8} and 2.0×10^{-7} C. In what direction and with what acceleration will they start to move? Will the motion be uniformly accelerated? Gravitational forces should be neglected.

9.19. With what force is an electron moving in an orbit of radius 5.0×10^{-11} m attracted to a helium nucleus? The charge of the nucleus is 3.2×10^{-19} C. The orbit should be assumed to be circular.

9.20. Two little spheres having a mass of 1 g each and suspended in vacuum from the same point on silk threads

have equal negative charges. They repel each other and move apart to 12 cm, forming an angle of 22°. Determine the number of electrons on each ball and the tension in the threads. Show the forces acting on the balls on a diagram. Will the tension in the threads change if the charges interact in zero-gravity?

9.21. Two point charges of 5.0×10^{-9} and 1.5×10^{-8} C are 4.0 cm apart in vacuum. Determine the force with which these charges will act on a third charge of 1.0×10^{-9} C, located at the midpoint of the line connecting the charges.

9.22. A charge of 1.57×10^{-8} C is transferred to a metal sphere of radius 5 cm. What is the surface charge density on the sphere?

9.23. The surface charge density of a conducting sphere is 5×10^{-5} C/m^2. Determine the magnitude of the charge on the sphere if its radius is 8 cm.

9.24. Two electric charges, one of which is twice the other, interact at a distance of 0.60 m in vacuum with a force of 2.0 mN. Calculate the magnitudes of charges. At what distance in kerosene will the interaction between the charges be the same?

9.25. What is the ratio of the electrostatic force of repulsion between two electrons to their gravitational attraction?

9.26. Determine the permittivity of kerosene if two equal like charges interact in vacuum with the same force at a distance of 0.283 m as in kerosene at 0.20 m. Assuming that the force of interaction in kerosene is 3.0×10^{-2} N, determine the magnitudes of the charges.

Electric field strength

9.27. Do the electric field vector and the vector of the force exerted on a charge by an electric field always have the same directions?

9.28. Why is a metal cap sometimes put on a vacuum tube?

9.29. Can electric charges be separated on (a) a conductor, (b) a dielectric?

9.30. A cylindrical conductor is attached to a conical conductor with the same base area. What can be said about the electric field strength near various points on the surface of the resultant conductor?

9.31. Why is a ball usually fixed to the rod of an electroscope?

9.32. Two electric charges of the same magnitude and sign produce an electric field. What is the field strength at the midpoint of the straight line connecting the conductors?

9.33. The electric field strength at a given point is 300 V/m. What does this mean?

9.34. Determine the electric field strength at a point where a force of 5 mN acts on a charge of 0.7×10^{-6} C.

9.35. What is the force exerted by an electric field on a charge of 3.2×10^{-8} C at a point where the field strength is 500 V/m?

9.36. The electric field strength near the surface of the Earth before a stroke of lightning is 2×10^5 V/m. What is the force exerted by the field on an electron?

9.37. An electric field is produced in vacuum by a point charge of 7.5×10^{-8} C. Determine the field strength 15 cm away from the charge.

9.38. A point charge of 2.2 nC produces a field with a strength of 2.5 kV/m at 6.0 cm from the charge. Determine the permittivity of the medium.

9.39. A vessel of kerosene contains a conducting sphere with a negligibly small size and a charge of 8.0×10^{-8} C. Determine the electric field strength 5.0 cm from the charge. What will be the change in the field strength at this point if the kerosene is let out of the vessel?

9.40. The electric field strength 5.0 cm from a charge is 1.5×10^5 V/m. What is the electric field strength 10.0 cm from the charge? Determine the magnitude of the charge.

9.41. How many excess electrons are contained on a dust particle acted upon by a force of 2.4×10^{-10} N in an electric field of strength 1.5×10^5 V/m?

9.42. An electric field is produced by a point charge. What are the loci of the points where the magnitude of the electric field strength is the same?

9.43. Will an electric field remain uniform if a conducting sphere is placed in it?

9.44. Two identical point charges of 1.0×10^{-8} and 2.0×10^{-8} C are in vacuum 20 cm apart and produce an electric field. Determine the field strength at the midpoint of the line connecting the charges. What will the electric field strength be if the charges are unlike?

9.45. Two conducting spheres having diameters of 10.0 and 4.0 cm are 120.0 cm apart and bear charges of 3.0×10^{-6} and 2.0×10^{-6} C respectively. Determine the electric field strength at the midpoint O of the straight line connecting the spheres (Fig. 19).

9.46. Two like electric charges of 7.0×10^{-8} C each are at points A and B (Fig. 20). Determine the electric field strength at point O which is the apex of the right angle AOB; $AO = BO = 5.0$ cm.

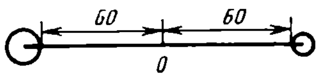


Fig. 19

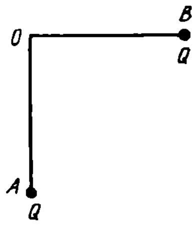


Fig. 20

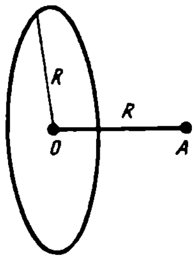


Fig. 21

9.47. Determine the electric field strength of the field at a point three radii from the surface of a charged conducting sphere. The surface charge density on the sphere is 1.6×10^{-7} C/m^2.

9.48. A conducting sphere of radius R is electrostatically charged to a surface density σ. What will be the field strength (a) at the centre of the sphere, (b) at half the radius of the sphere, (c) on the surface of the sphere?

9.49. A drop bearing a charge of 2×10^{-8} C is in equilibrium in a uniform electric field of strength 49 V/m. Determine the mass of the drop.

9.50. With what acceleration will a 10-g ball with charge 1.0×10^{-5} C fall in the electric field of the Earth? The electric field strength near the surface of the Earth is 130 V/m.

9.51. A thin conducting ring of radius R has an electric charge Q. Determine the electric field strength at the centre O of the ring and at point A (Fig. 21).

9.52. Why is the uniformity of the electric field of a charged parallel-plate capacitor violated if an uncharged metal sphere is placed between its plates?

Electric potential. Potential difference. Work done in an electric field

9.53. Considering the Earth to be a sphere of radius 6400 km, determine its electric charge and potential if the electric field strength produced by the Earth near the surface is 130 V/m.

9.54. The work done in displacing a 2.0×10^{-8}-C charge from infinity to a point in a field is 1.13×10^{-4} J. What is the electric potential at this point?

9.55. Determine the potential difference between two points in a field if a work of 3.0×10^{-5} J has to be done in moving an 8.0×10^{-7}-C charge between these points.

9.56. An electric field moves a positive charge of 3.0×10^{-7} C between two points with potentials 200 V and 1200 V. What is the work done by the field in this case?

9.57. An electric field is produced by a point charge of 4×10^{-8} C. What is the electric potential 6 cm away from this charge? What work must be done against the field in order to bring a positive charge of 1 C from infinity to this point?

9.58. Two point electric charges of 1.0×10^{-5} and 6.0×10^{-6} C are 20 cm apart in air. Determine the electric potential at the midpoint of the straight line connecting the charges.

9.59. A 1.6×10^{-7}-C charge is moved 3.0 cm along a field line in a uniform electric field having a strength of 5.0×10^{3} V/m. Determine the work done and the potential difference for two points between which the charge is moved.

9.60. As an electron passes between two points in an electric field, its velocity increases from 2.0×10^{6} to 3.0×10^{7} m/s. What is the potential difference between these points? What is the increase in the kinetic energy of the electron?

9.61. Two 8-nC charges are located at two apexes of an equilateral triangle having a side of 6.0 cm. Determine the field strength and the potential at the third apex.

9.62. Two conducting spheres of radii 2.0 and 3.0 cm are charged to 30 and 40 V respectively. What will be the electric potential of the spheres after they have been connected by a wire? Assume that the separation between the spheres is large compared to their radii.

9.63. An electron having a velocity of 6.0×10^7 m/s flies into a parallel-plate air capacitor midway between the plates. What must the voltage across the plates be for the deviation of the electron to be maximum? The plates are 10.0 cm long and 3.0 cm apart.

9.64. An electron with a velocity of 1.6×10^6 m/s flies into a uniform electric field of strength 90 V/m and moves along a field line until it comes to a halt. How long and how far does it fly in the field? Assume that the electron mass is 9.0×10^{-31} kg.

Capacitance. Capacitors. The electrical energy in a capacitor

9.65. Can the potential of a charged conductor be changed without changing its charge?

9.66. A parallel-plate air capacitor of capacitance C is immersed in a medium with a permittivity of 2. What will the capacitance of the capacitor be?

9.67. The capacitance of a spherical conductor is proportional to its radius. What must the radius of a conducting sphere be for its capacitance in vacuum to be 1 F? What is the ratio between the radius of such a sphere and the Earth's radius?

9.68. A parallel-plate air capacitor is connected to a constant-voltage source. What will be the change in the capacitance and the energy of the capacitor, the voltage across it, and the charge on the capacitor plates if the separation between the plates is reduced?

9.69. Determine the capacitance of an isolated conducting sphere of radius 5.0 cm immersed in kerosene. Express the answer in farads, microfarads, and picofarads.

9.70. Assuming the Earth to be a sphere of radius 6400 km, determine its capacitance.

9.71. An isolated conducting sphere whose capacitance is 5.0 pF is electrostatically charged to a potential of 1570 V. Determine the radius of the sphere and the surface charge density on it.

9.72. Why do electrolytic capacitors have a large capacitance?

9.73. Two conducting spheres having a charge of $1.0 \times$

10^{-8} C each are arranged a long way apart. The capacitance of the larger sphere is 2.2×10^{-11} F, and that of the smaller sphere is 5.6×10^{-12} F. What are their potentials? What will happen when they are connected by a conductor?

9.74. Calculate the capacitance of a parallel-plate capacitor made from tin foil plates 15 cm^2 in area with a mica dielectric layer 0.8 mm thick. The permittivity of mica is $\varepsilon = 6$.

9.75. A parallel-plate air capacitor consists of two plates 100 cm^2 each in area. When a charge of 6.0×10^{-9} C is transferred to one of the plates, the capacitor is charged to a voltage of 120 V. Determine the separation between the plates.

9.76. The plates of a parallel-plate air capacitor have an area of 62.3 cm^2 each and their separation is 5 mm. Determine the charge of the capacitor if the potential difference across its plates is 60 V.

9.77. The separation between the plates of a parallel-plate air capacitor is 0.1 cm and the area of each plate is 200 cm^2. The potential difference across the plates is 600 V. What charge is stored in the capacitor? What will be the change in the voltage if the space between the plates is filled with mica, whose permittivity is 6?

9.78. A parallel-plate air capacitor with a plate separation of 1.5 mm is charged to a voltage of 150 V. How far apart should the plates be moved in order to increase the voltage to 600 V?

9.79. Determine the capacitance of a capacitor in which nine mica plates having a thickness of 0.12 mm and an area of 12.56 cm^2 are interleaved between tin foil plates. The permittivity ε of mica is 6.

9.80. Determine the energy of a parallel-plate paper capacitor with a plate area of 600 cm^2. The charge on the capacitor is 2×10^{-7} C and the dielectric is wax paper 2.0 mm thick.

9.81. Three capacitors whose capacitances are 4, 2, and 6 μF are connected to form a bank and to a constant-voltage source of 200 V. Determine the capacitance and the energy of the bank when the capacitors are connected (1) in series and (2) in parallel.

9.82. Determine the capacitance of a capacitor bank connected as shown in Fig. 22 if $C_1 = 1.2$ μF and $C_2 = C_3 = 0.6$. μF.

9.83. Determine the capacitance of a bank of capacitors connected as shown in Fig. 23. The capacitance of all the capacitors is the same and equal to 0.6 μF. Determine the electric charge stored in the bank if a voltage of 100 V is applied to it.

9.84. Capacitors are connected as shown in Fig. 24. Given: $C_1 = C_2 = 2$ μF, $C_3 = C_4 = C_5 = 6$ μF. Determine

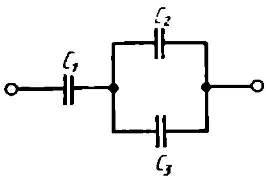


Fig. 22

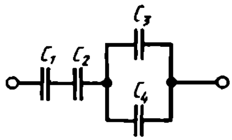


Fig. 23

the potential difference between points A and B if the energy stored in the bank is 1.35×10^{-1} J.

9.85. An uncharged 100-μF capacitor is connected in parallel to a 50-μF capacitor charged to a voltage of 300 V.

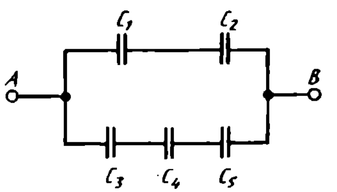


Fig. 24

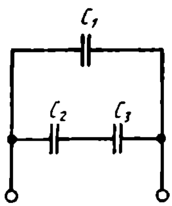


Fig. 25

What is the voltage across the capacitors? What is the charge distribution between them?

9.86. After a capacitor having an unknown capacitance and charged to a voltage of 600 V is connected in parallel to an uncharged 5-μF capacitor, the voltage across the bank drops to 100 V. What is the capacitance of the first capacitor?

9.87. Two capacitors having different capacitances are connected in parallel and to a constant-voltage source. Which of the capacitors will accumulate more energy?

9.88. A pulsed flash for photography is obtained by discharging a 800-μF capacitor for 2.8 ms at a voltage of

300 V across the capacitor. Determine the energy of the flash and the mean power.

9.89. Three capacitors are connected to make a bank as shown in Fig. 25 and then to a 200-V constant-voltage source. Determine the capacitance of capacitor C_1 and its charge if the capacitance of the bank is $C = 6$ μF and the capacitances of the remaining capacitors are $C_2 = 2$ μF and $C_3 = 3$ μF.

9.90. The force with which the plates of a parallel-plate capacitor attract each other is determined from the formula $F = Q^2/(2\varepsilon_0 \varepsilon S)$. Is the energy required to move the plates apart higher (1) when the capacitor remains connected to the source or (2) when it has been charged and then before the plates are moved apart disconnected from the source?

9.91. A spherical capacitor consists of two conducting concentric hollow spheres. If the difference between the radii of the spheres is small, the capacitance of the capacitor can be calculated using the formula for a parallel-plate capacitor. Write this formula in a general form and determine the capacitance of an air capacitor formed by spheres of radii 5.00 and 4.95 cm.

9.92. Determine the volume energy density of the uniform electric field in a mica capacitor charged to a voltage of 90 V. The separation between the plates is 1.0 mm, and the permittivity of mica should be taken to be 6.

§ 10. ELECTRIC CURRENT IN METALS. OHM'S LAW. ELECTRIC RESISTANCE

Basic Concepts and Formulas

An electric current is an ordered (directional) motion of charged particles (in metals, these particles are free electrons).

The current I in a conductor is the amount of electricity Q flowing per second through a cross section of the conductor:

$$I = Q/t, \text{ or } I = en\bar{v}S,$$

where n is the number density of the charge carriers (e), $\bar{v}$ is the mean velocity of the charges, and S the cross-sectional area of the conductor. Current is a base quantity in SI, its unit being the ampere (A). The direction of current is assumed to be opposite to the direction of motion of electrons.

In order to create a direct current in metals, an electric field must act on the free electrons. The field must ensure a constant potential difference across the ends of the conductor (circuit). In a current source, the extraneous forces produce an excess of electrons at the negative terminal and an electron shortage at the positive terminal. In other words, a potential difference is created. Each current source is characterized by an electromotive force (emf) $\mathscr{E}$ which is equal to the work done by extraneous forces in moving a positive charge of 1 C along the circuit:

$$\mathscr{E} = A_{\mathrm{ext}}/Q.$$

The emf unit is the volt (V).

Ohm's law for a conductor establishes a relation between the current flowing in it and the voltage across its ends:

$$I = \frac{1}{R} U,$$

where the proportionality factor $1/R$ is called the electric conductance and R the electric resistance of the conductor.

The unit of resistance is the ohm (Ω).

The resistance of a conductor depends on its size, material, and temperature:

$$R = \rho \frac{l}{S}, \quad R_t = R_0 (1 + \alpha \Delta T),$$

where ρ is the resistivity in $\Omega \cdot \mathrm{m}$, and α is the temperature resistance coefficient in K^{-1}:

$$\alpha = \frac{\Delta R}{R_0 \Delta T}.$$

For metal conductors, α is positive. At temperatures close to absolute zero, superconductivity is observed for some conductors. This is a state in which the resistance abruptly drops to zero.

Ohm's law for a circuit establishes a relation between the current, electromotive force, and the total resistance of the circuit:

$$I = \frac{\mathscr{E}}{R + r},$$

where R and r are the resistances of the external part of the circuit and of the current source.

The voltage U across the terminals of a current source with a closed circuit is smaller than the emf by the voltage drop within the source itself:

$$U = \mathcal{E} - Ir.$$

Short circuiting is observed when the resistance of the external part of the circuit is negligibly small, while the current attains its maximum value. Using Ohm's law for a closed circuit, we can determine the current during short circuiting:

$$I_{sh.c} = \mathcal{E}/r.$$

Individual parts of a circuit (resistors) can be connected in series or in parallel.

For a connection in series, the resistors are connected one after another so that the current in all the subcircuits is the same, while the total, or equivalent, resistance of the circuit is given by

$$R_{scr} = R_1 + R_2 + \ldots + R_n.$$

If $R_1 = R_2 = \ldots = R_n = R$, we have

$$R_{ser} = Rn.$$

The voltage drop for a series connection is proportional to the resistances:

$$\frac{U_1}{U_2} = \frac{R_1}{R_2}.$$

For a connection in parallel, the voltage across all the parallel branches is the same, while the current in a branch is determined by its resistance:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

The total, or equivalent, resistance is then given by the formula

$$\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}.$$

If $R_1 = R_2 = \ldots = R_n = R$, we have

$$\frac{1}{R_{par}} = \frac{1}{R} n, \text{ or } R_{par} = \frac{R}{n}.$$

To measure the current, an ammeter is connected in series to the circuit. The resistance of the instrument should be very low. In order to change the value of a scale division on the ammeter, a bypass (shunt) resistor R_{sh} is connected in parallel with the instrument. The resistance of the shunt is lower than the resistance of the ammeter by a factor of $n-1$:

$$R_{sh}=\frac{R_A}{n-1},$$

where n is the number by which the scale of the instrument is multiplied (Fig. 26).

To measure voltage, a voltmeter is connected in parallel to the region of the circuit across which the voltage is being

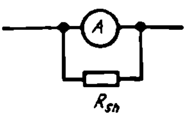


Fig. 26

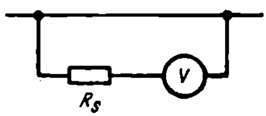


Fig. 27

measured. The resistance of the voltmeter must be very high. If the voltage to be measured exceeds the range of a voltmeter, a multiplying coil (series resistor) R_s is connected in series with the instrument (Fig. 27):

$$R_s = R_V (n - 1),$$

where n is the number by which the range of the voltmeter is increased, and R_V is the resistance of the voltmeter.

Sources of electric power (current sources) can be connected to form batteries. For a series connection, the positive terminal of the first source is connected to the negative terminal of the second source, and so on. The current in this case is

$$I=\frac{n\mathscr{E}}{R+nr},$$

where n is the number of identical current sources in the battery.

For a parallel connection, all the positive terminals are connected in one junction and the negative terminals form

the other junction. The current in such a battery is

$$I = \frac{\mathscr{E}}{R + r/n}.$$

While solving problems in which Ohm's law is applied to branched circuits, it is necessary

(1) to choose arbitrarily the directions of the currents and the direction of circumvention of subcircuits and indicate them on a circuit diagram;

(2) to write equations for the currents at the junctions, the number of equations being one less than the number of junctions, the algebraic sum of currents at a junction is always equal to zero if the currents flowing to a junction and from it are taken with opposite signs; and

(3) to write the equations for all the closed subcircuits considering that the algebraic sum of emf's in any closed subcircuit is equal to the algebraic sum of the voltage drops; if the potential increases in the direction of circumvention (we move from "minus" to "plus"), the emf is assumed to be positive, otherwise, the emf is assumed to be negative. The voltage drop is assumed to be positive if the direction of the current coincides with the chosen direction of circumvention; otherwise, it is taken with the minus sign. The total number of equations must be equal to the number of unknowns.

Worked Problems

Problem 54. A voltage of 3.6 V is applied to the ends of a steel conductor 20 m long. Determine the mean velocity of ordered motion of the charge carriers in the conductor if their number density is 4.0×10^{28} m^{-3}.

Given: $U = 3.6$ V is the voltage drop across the conductor, $n = 4.0 \times 10^{28}$ m^{-3} is the number density of mobile charge carriers, and $l = 20$ m is the length of the conductor. From tables, we find the electron charge $e = 1.6 \times 10^{-19}$ C and the resistivity of steel, $\rho = 1.2 \times 10^{-7}$ $\Omega \cdot$m.

Find: the mean velocity $\bar{v}$ of the charge carriers.

Solution. The mobile charge carriers in metals are free, or conduction, electrons. Consequently, we must find the mean velocity $\bar{v}$ of the ordered motion of free electrons.

In the electron theory of conductivity, Ohm's law is formulated as follows: $I = en\bar{v}S$, whence

$$\bar{v} = \frac{I}{enS}.$$

The current can be determined from the formula $I = U/R$, where R should be expressed in terms of the length and the cross-sectional area: $R = \rho l/S$. Substituting the expression for current into the formula for velocity, we obtain

$$\bar{v} = \frac{US}{\rho lenS}, \quad \bar{v} = \frac{U}{\rho len},$$

$$\bar{v} = \frac{3.6\ \text{V}}{1.2\times 10^{-7}\ \Omega\cdot\text{m}\times 20\ \text{m}\times 1.6\times 10^{-19}\ \text{C}\times 4.0\times 10^{28}\text{m}^{-3}}$$

$$\simeq 2.343\times 10^{-4}\ \text{m/s}.$$

Answer. The mean velocity of ordered motion of the free electrons is approximately 0.23 mm/s.

Problem 55. A telegram is sent from Leningrad to Moscow (the distance between the cities is 650 km) via a steel telegraph line carrying a current of 1.7 mA at a voltage of 150 V. The cross-sectional area of the wire is 5.0 mm². Determine the voltage drop in the wires and the voltage at the receiving end (Fig. 28).

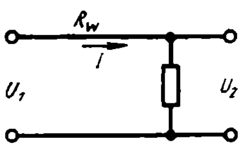


Fig. 28

Given: $l = 6.5\times 10^5$ m is the distance between Leningrad and Moscow, $I = 1.7\times 10^{-3}$ A is the current in the wires, $U_1 = 150$ V is the voltage at the beginning of the transmission line, and $S = 5\times 10^{-6}$ m² is the cross-sectional area of the wire. From tables, we find the resistivity of steel, $\rho = 1.2\times 10^{-7}\ \Omega\cdot$m.

Find: the voltage drop U_w in the wires and the voltage U_2 at the end of the line.

Solution. The length of the wire forming the electric circuit (which must be closed) is twice the distance between the cities.

The voltage drop in the wires is $U_w = IR_w$, where $R_w = \rho \frac{2l}{S}$. Then

$$U_w = I\rho \frac{2l}{S},$$

$$U_w = \frac{1.7 \times 10^{-3}\,\text{A} \times 1.2 \times 10^{-7}\,\Omega\cdot\text{m} \times 2 \times 6.5 \times 10^{5}\,\text{m}}{5 \times 10^{-6}\,\text{m}^2} \simeq 53\ \text{V}.$$

The voltage at the end of the transmission line is

$$U_2 = U_1 - U_w, \quad U_2 = 150\ \text{V} - 53\ \text{V} = 97\ \text{V}.$$

Answer. The voltage drop in the wires is approximately 53 V, and the voltage at the end of the line is 97 V.

Problem 56. What is the change in the resistance of a telegraph line due to the change from winter temperatures to summer temperatures if the line is made from a steel wire having a cross-sectional area of 5.0 mm²? The temperature changes from —30 to +30°C. The length of the wire at 0°C is 200 km. The linear expansion of the wire should be neglected.

Given: $S = 5.0 \times 10^{-6}$ m² is the cross-sectional area of the wire, $t_1 = -30$°C is the winter temperature, $t_2 = +30$°C is the summer temperature, and $l = 2.0 \times 10^5$ m is the length of the wire. From tables, we find the resistivity of steel at 0°C, $\rho_0 = 1.2 \times 10^{-7}\ \Omega\cdot$m, and the temperature resistance coefficient for steel, $\alpha = 0.004$ K^{-1}.

Find: the change ΔR in the resistance of the telegraph line due to the seasonal change in temperature.

Solution. The resistance of the line at 0°C is $R_0 = \rho_0 l/S$, while the resistances at t_1 and t_2 are $R_1 = \rho_0 \frac{l}{S}(1 + \alpha t_1)$ and $R_2 = \rho_0 \frac{l}{S}(1 + \alpha t_2)$. The change in the resistance is given by

$$\Delta R = R_2 - R_1, \ \Delta R = \rho_0 \frac{l}{S}(1 + \alpha t_2 - 1 - \alpha t_1) = \rho_0 \frac{l\alpha}{S}(t_2 - t_1).$$

Since the absolute values of the temperatures are equal ($|t_1| = |t_2|$), we can write

$$\Delta R = \rho_0 \frac{l}{S} 2\alpha t_1,$$

$$\Delta R = 1.2 \times 10^{-7}\ \Omega\cdot\text{m}\,\frac{2.0\times 10^5\ \text{m}}{5\times 10^{-6}\ \text{m}^2} \times 2 \times 0.004\ \text{K}^{-1} \times 30\ \text{K}$$

$$\simeq 1.2\ \text{k}\Omega.$$

Answer. The resistance of the wire has increased by about 1.2 kΩ as a result of the change from winter to summer temperatures.

Problem 57. Calculate the resistance of the circuit in Fig. 29*a*.

Given: the resistances $R_1 = 6\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 12\ \Omega$, and $R_5 = 8\ \Omega$ of the resistors.

Find: the total (equivalent) resistance R of the circuit.

Solution. While solving problems where loads are connected in series and in parallel, it is expedient to replace

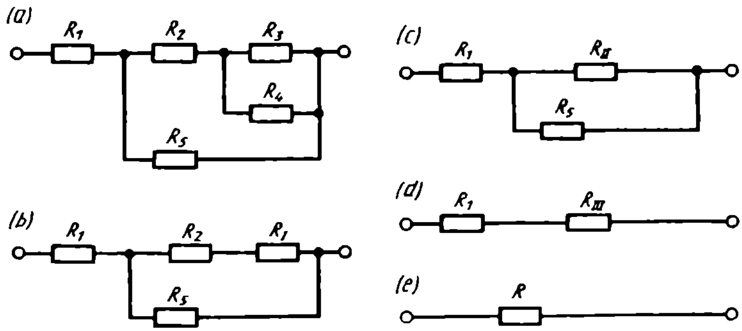


Fig. 29

a branched circuit by equivalent circuits as shown in Figs. 29*b-e*.

Since resistors R_3 and R_4 are connected in parallel, their equivalent resistance is

$$R_{\text{I}} = \frac{R_3 R_4}{R_3 + R_4}, \quad R_{\text{I}} = \frac{4\,\Omega \times 12\,\Omega}{4\,\Omega + 12\,\Omega} = 3\,\Omega$$

(Fig. 29*b*).

Resistors R_{I} and R_2 are connected in series, and their equivalent resistance is $R_{\text{II}} = R_2 + R_{\text{I}}$, $R_{\text{II}} = 5\ \Omega + 3\ \Omega = 8\ \Omega$ (Fig. 29*c*).

Resistors R_{II} and R_5 are connected in parallel, and hence their equivalent resistance is

$$R_{III}=\frac{R_{II}R_5}{R_{II}+R_5},\quad R_{III}=\frac{8\ \Omega\times 8\ \Omega}{8\ \Omega+8\ \Omega}=4\ \Omega$$

(Fig. 29*d*).

Resistance R_{III} can be determined in a different way: $R_{III} = R_{II}/2 = 4\ \Omega$.

The required resistance R is

$$R = R_1 + R_{III},\quad R = 6\ \Omega + 4\ \Omega = 10\ \Omega$$

(Fig. 29*e*).

Answer. The total resistance of the circuit is 10 Ω.

Problem 58. When a current source with an emf of 4.2 V is connected to a nickeline wire 10 m long and having a diameter of 1.0 mm, the current in the circuit is 0.6 A. Determine the internal resistance of the current source.

Given: the emf $\mathscr{E} = 4.2$ V of the current source, $l = 10$ m is the length of the nickeline wire, $d = 1.0 \times 10^{-3}$ m is the diameter of the wire cross section, and $I = 0.6$ A is the current in the circuit. From tables, we find the resistivity of nickeline, $\rho = 4.2 \times 10^{-7}\ \Omega\cdot\mathrm{m}$.

Find: the internal resistance r of the current source.

Solution. We shall solve the problem by using Ohm's law for a closed circuit:

$$I=\frac{\mathscr{E}}{R+r},$$

whence

$$r=\frac{\mathscr{E}}{I}-R.$$

The external resistance R can be determined from the formula $R = \rho l/S$. Considering that $S = \pi d^2/4$, we obtain

$$R=\frac{4\rho l}{\pi d^2}=\frac{4\times 4.2\times 10^{-7}\ \Omega\cdot\mathrm{m}\times 10\ \mathrm{m}}{3.14\times 1.0\times 10^{-6}\ \mathrm{m}^2}=5.4\ \Omega.$$

Then

$$r=\frac{4.2\ \mathrm{V}}{0.6\ \mathrm{A}}-5.4\ \Omega=1.6\ \Omega.$$

Answer. The internal resistance of the current source is 1.6 Ω.

Problem 59. A battery of cells with an emf of 3 V and an internal resistance of 0.25 Ω supplies power to a circuit

consisting of four resistors $R_1 = 1.0\ \Omega$, $R_2 = 3.0\ \Omega$, $R_3 = 1.5\ \Omega$, and $R_4 = 0.75\ \Omega$ connected as shown in Fig. 30*a*. Determine the current in the unbranched subcircuit and the voltage drop in the battery.

Given: $\mathscr{E} = 3$ V is the emf of the battery, $r = 0.25\ \Omega$ is the internal resistance of the battery, and $R_1 = 1.0\ \Omega$, $R_2 = 3.0\ \Omega$, $R_3 = 1.5\ \Omega$, and $R_4 = 0.75\ \Omega$ are the resistances of the resistors.

Find: the current I in the circuit and the voltage drop U_{int} in the battery.

Solution. We shall solve the problem with the help of Ohm's law for a closed circuit: $I = \mathscr{E}/(R + r)$. Therefore,

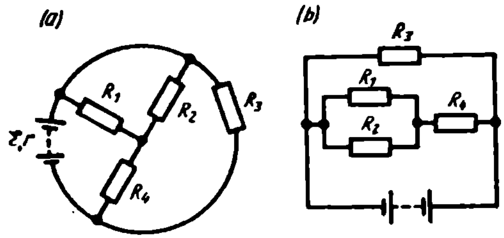


Fig. 30

we must determine the external resistance R. For simplicity, we represent the circuit as shown in Fig. 30*b*. It is clear from the circuit diagram that the resistors R_1 and R_2 are connected in parallel. Their equivalent resistance R_{I} can be found from the formula

$$R_{\text{I}} = \frac{R_1 R_2}{R_1 + R_2}, \quad R_{\text{I}} = \frac{1.0\,\Omega \times 3.0\,\Omega}{1.0\,\Omega + 3.0\,\Omega} = 0.75\,\Omega.$$

The subcircuit with resistance R_{I} and resistor R_4 are connected in series. Their equivalent resistance R_{II} can be determined as follows:

$$R_{\text{II}} = R_{\text{I}} + R_4, \quad R_{\text{II}} = 0.75\ \Omega + 0.75\ \Omega = 1.5\ \Omega.$$

Resistances R_{II} and R_3 can be replaced by an equivalent resistance R which can be determined from the rules governing parallel connection of loads. Since these resistances are equal, we have

$$R = R_{\text{II}}/2 = 0.75\ \Omega.$$

The current in the unbranched circuit is

$$I = \frac{3.0\ \text{V}}{0.75\ \Omega + 0.25\ \Omega} = 3\ \text{A}.$$

The voltage drop in the current source can be determined from Ohm's law for a conductor:

$$U_{\text{int}} = Ir, \quad U_{\text{int}} = 3\ \text{A} \times 0.25\ \Omega = 0.75\ \text{V}.$$

Answer. The current in the unbranched circuit is 3 A and the voltage drop in the current source is 0.75 V.

Problem 60. A galvanometer with an internal resistance of 19.8 Ω can be used to measure currents up to 10 mA. What must be done to extend its range to 1.0 A? How can it be used to measure voltages up to 10 V?

Given: $R_g = 19.8\ \Omega$ is the resistance of the galvanometer, $I_g = 0.01$ A is the current through the galvanometer, $I = 1.0$ A is the current in the circuit, and $U = 10$ V is the voltage to be measured.

Find: the resistance of the shunt R_{sh} and the resistance R_s of the series resistor.

Solution. If a galvanometer is used as an ammeter (in this case it is connected in series to a circuit), a shunt (resistor) R_{sh} is connected in parallel to it as shown in Fig. 26. The resistance of the shunt can be calculated by using the rules of parallel connection:

$$I_{sh} = I - I_g, \quad \frac{R_g}{R_{sh}} = \frac{I_{sh}}{I_g}, \quad \frac{R_g}{R_{sh}} = \frac{I - I_g}{I_g} = n - 1,$$

$$\text{where } n = \frac{I}{I_g},$$

$$R_{sh} = \frac{R_g}{n-1}, \quad R_{sh} = \frac{19.8\ \Omega}{99} = 0.2\ \Omega.$$

If a galvanometer is used as a voltmeter (in this case it is connected in parallel to the subcircuit in which the voltage drop U has to be measured), a resistor R_s is connected in series to it as shown in Fig. 27.

The voltage drop U is distributed in proportion to the resistances R_g and R_s:

$$\frac{U - U_g}{U_g} = \frac{R_s}{R_g}, \quad \frac{U}{U_g} - 1 = \frac{R_s}{R_g},$$

$$R_s = (n - 1) R_g, \text{ where } n = \frac{U}{U_g} = \frac{U}{I_g R_g},$$

$$n = \frac{10\ \text{V}}{0.01\ \text{A} \times 19.8\ \Omega} = 50,$$

$$R_s = 49 \times 19.8\ \Omega \simeq 970\ \Omega.$$

Answer. In order to measure current, the galvanometer must be shunted by a resistor with $R_{sh} = 0.2\ \Omega$. To measure voltage, a series resistor with resistance $R_s \simeq 970\ \Omega$ should be connected to the galvanometer.

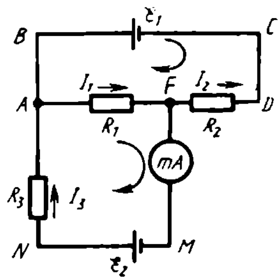


Fig. 31

Problem 61. How should two galvanic cells having an emf of 1.45 V each and an internal resistance of 0.4 Ω each be connected in order to obtain the maximum current when the circuit is closed with an external resistance of 0.65 Ω?

Given: $n = 2$ is the number of cells in the battery, $\mathscr{E} = 1.45$ V is the emf of a cell, $r = 0.4\ \Omega$ is the internal resistance of a cell, and $R = 0.65\ \Omega$ is the resistance of the external circuit.

Find: I_{ser}, the current for the series connection and the current I_{par} for the parallel connection of the cells.

Solution. In order to find out which is the best way to connect the circuit, we determine the currents for the cells in series and in parallel and compare them:

$$I_{ser} = \frac{n\mathscr{E}}{R + nr}, \quad I_{ser} = \frac{2 \times 1.45\ \text{V}}{0.65\ \Omega + 2 \times 0.4\ \Omega} = 2\ \text{A},$$

$$I_{par} = \frac{\mathscr{E}}{R + r/n}, \quad I_{par} = \frac{1.45\ \text{V}}{0.65\ \Omega + 0.4\ \Omega/2} = 1.7\ \text{A}.$$

Answer. It is better to connect the cells in series.

Problem 62. The circuit shown in Fig. 31 contains three resistors $R_1 = 100\ \Omega$, $R_2 = 50\ \Omega$, and $R_3 = 20\ \Omega$ and galvanic cells with emf's $\mathscr{E}_1 = 2$ V and $\mathscr{E}_2$. The ammeter indicates a current of 50 mA. Determine the currents in the resistors and the emf of the second cell. The internal resistance of the ammeter and of the cells should be neglected.

Given: $R_1 = 100\ \Omega$, $R_2 = 50\ \Omega$, and $R_3 = 20\ \Omega$ are the resistances of the resistors, $\mathscr{E}_1 = 2$ V is the emf of the first cell, $I_A = 0.05$ A is the reading of the ammeter.

Find: the currents I_1, I_2, and I_3 in the resistors and the emf $\mathscr{E}_2$ of the second cell.

Solution. We choose the direction of the current arbitrarily and indicate it by arrows on the circuit diagram. Since the algebraic sum of the currents flowing to and from a junction is zero (Kirchhoff's first rule), we can write $I_1 - I_2 - I_3 = 0$, whence

$$I_3 = I_1 - I_2. \tag{1}$$

The current indicated by the ammeter is $I_A = I_3$.

Let us go around the circuit clockwise and indicate this direction on the circuit diagram.

Since the algebraic sum of voltages in a closed circuit is equal to the algebraic sum of emf's (Kirchhoff's second rule), we can write

for the circuit *ABCDFA*

$$-I_1R_1 - I_2R_2 = -\mathscr{E}_1, \text{ or } I_1R_1 + I_2R_2 = \mathscr{E}_1, \tag{2}$$

for the circuit *AFMNA*

$$I_1R_1 + I_3R_3 = \mathscr{E}_2. \tag{3}$$

We determine I_1 from Eq.(1) and substitute it into Eq.(2):

$$I_1 = I_3 + I_2, \; (I_3 + I_2)\,R_1 + I_2R_2 = \mathscr{E}_1.$$

This gives

$$I_2 = \frac{\mathscr{E}_1 - I_3R_1}{R_1 + R_2}, \quad I_2 = \frac{2\ \text{V} - 0.05\ \text{A} \times 100\ \Omega}{100\ \Omega + 50\ \Omega} = -0.02\ \text{A}.$$

The minus sign indicates that the current I_2 flows in the opposite direction to that indicated on the diagram.

The current is

$$I_1 = I_3 + I_2 = 0.05\ \text{A} - 0.02\ \text{A} = 0.03\ \text{A}.$$

Substituting I_1 into Eq. (3), we obtain $\mathscr{E}_2$:

$$\mathscr{E}_2 = 0.03\ \text{A} \times 100\ \Omega + 0.05\ \text{A} \times 20\ \Omega = 4\ \text{V}.$$

Answer. The currents in resistors R_1, R_2, and R_3 are 0.03, —0.02, and 0.05 A respectively, the emf of the second cell is 4 V.

Questions and Problems

Current. Resistance. Ohm's law for a conductor

10.1. The anode current in a vacuum tube is 12 mA. How many electrons arrive at the anode per second?

10.2. How many electrons pass through the cross section of the contact wire of a tram in 2 s if the current is 500 A?

10.3. Determine the current in the contact copper wire of a trolleybus circuit if the number density of conduction electrons in copper is 3×10^{23} cm^{-3}, and the mean velocity of their ordered motion is 0.25 mm/s. The cross-sectional area of the wire is 85 mm^2.

10.4. Determine the number density of conduction electrons in copper if the current in a copper wire with a cross-sectional area of 105 mm^2 is 500 A, and the mean velocity of the ordered motion of conduction electrons is 0.1 mm/s.

10.5. A voltage of 18 V is applied across the ends of a copper wire 200 m long. Determine the mean velocity of the ordered motion of electrons in the conductor if the number density of conduction electrons in it is 3.0×10^{23} cm^{-3}.

10.6. What is the electric field strength in an aluminium wire of cross-sectional area 6 mm^2 for a current of 9 A if the potential difference between the ends of the wire is 21 V, the mobility of conduction electrons in aluminium is 7×10^{-3} m^2/(V·s), and the electron number density in it is 3.7×10^{22} cm^{-3}? How long is the conductor?

10.7. The coil of a hot plate carries a current of 2.7 A with an electric field strength of 3.1 V/m. Determine the mobility of the conduction electrons if their number density in the coil is 10^{23} cm^{-3}, and the cross-sectional area of the wire is 0.10 mm^2.

10.8. Determine the current density in a conductor if 0.15 C of electricity pass per second through a cross-sectional area of 6 mm^2. Is the admissible current density important when choosing the wire for a residential building?

10.9. A constant current of 3.6 mA is maintained when gold plating an article 1.2×15 cm^2 in area. Determine the current density.

10.10. What must the diameter of a copper wire be if it is to carry a current of 1000 A and the admissible current density is 2.5 A/mm^2?

10.11. Determine the current density in the excitation winding of the traction motor of a locomotive if the cross-sectional area of the winding is 4.7×25 mm^2 and the nominal current is 725 A.

10.12. What is the resistance of an aluminium wire 2 m long and having a cross-sectional area of 1 mm^2? What will the resistance of the same length of wire with twice the cross-sectional area be?

10.13. Two wires, one of nickeline and the other of nichrome, have the same length and diameter. Which of them has a higher resistance? What is the ratio between their resistances?

10.14. An iron wire and a tungsten wire of the same length and diameter are connected in turn to an accumulator. In which wire will the current be stronger? What is the ratio of the currents?

10.15. A long conductor was cut into two halves which were then twisted along the whole length. How does the resistance of the wire change?

10.16. Determine the capacitance of a capacitor the voltage across whose plates is 1 V as a result of charging for 12 s with a current of 10^{-6} A.

10.17. An electric circuit whose conductance is 2.4×10^{-2} S is connected to a d.c. source of 50 V. Determine the current in the circuit and its resistance.

10.18. Determine the length of a manganin wire required to make a potentiometer rated at a maximum resistance of 1500 Ω if the diameter of the wire is 0.3 mm.

10.19. A potentiometer is made of 2.25 m of constantan wire having a diameter of 0.15 mm. Determine the resistance of the potentiometer.

10.20. A bundle of copper wire having a mass of 3.6 kg has a resistance of 22.5 Ω. Determine the length of the wire in the bundle.

10.21. A coil made of 75 m of constantan wire whose diameter is 0.1 mm is connected to a current source supplying a voltage of 12 V. Determine the current in the coil.

10.22. The admissible current density of the wiring in a room is 6 A/mm^2. What is the voltage drop in the copper wire if its length is 30 m?

10.23. Determine the voltage drop and the current density in the winding of the magnetic pole of a locomotive motor.

The winding is made of copper strip having a cross-sectional area of 28×4.7 mm^2, the average length of a turn being 1.5 m, and the number of turns is 40. The current in the winding is 352 A.

10.24. An electric appliance is 0.5 km from a current source and is connected to it through a wire having a cross-sectional area of 5 mm^2. The appliance is moved 1 km further away from the source. What should the cross-sectional area of the wire be for the voltage drop to remain unchanged?

10.25. How much copper is required to manufacture the contact wire of a tram circuit having a resistance of 0.2 Ω if the cross-sectional area of the wire is 85 mm^2?

10.26. Determine the resistance of 1 km of tram steel rails if the mass of the rails is 55 kg/m.

10.27. Two wires, one of copper and one of nichrome, of the same length and diameter are heated through 5 K. The resistance of which wire changes more? What is the ratio of the resistances?

10.28. The series resistance connected to the ignition coil of a "Moskvich" motor car is 1.35 Ω at 293 K and 2 Ω at 373 K. Determine the material of which it is made.

10.29. The starter rheostat of an electric train is made of a cast iron plate and a ferro-aluminium high-resistance ribbon alloy. Its resistance is 4 Ω at 298 K. What will the change in its resistance be when the temperature is 723 K?

10.30. What must the temperature resistance coefficient of the material used for potentiometers be?

10.31. The tungsten filament of an incandescent lamp has a resistance of 20 Ω when cold. What will its operating resistance be, i.e. when the filament is heated to 2100°C?

10.32. What is the increase in the temperature of the copper winding of a d.c. motor armature as a result of prolonged operation during which the resistance has increased from 2 to 2.2 Ω?

10.33. An electric arc rated for a voltage of 45 V and a current of 10 A is connected to a circuit at a voltage of 110 V. Determine the required resistance of the series resistor if the resistance of the leads is 0.5 Ω.

10.34. What is the increase in the temperature of a potentiometer made of iron wire if its resistance has doubled?

10.35. The voltage at an electric substation is 600 V (Fig. 32). The two copper cables *AB* and *CD* connecting the

substation with a trolleybus' contacts are each 440 m long and 400 mm² in cross-sectional area. The contact wires are made of copper and have a cross-sectional area of 85 mm². The current in the trolleybus motor is 200 A. Determine the voltage across the current collectors of the trolley.

10.36. The experimental d.c. transmission line between Kashira and Moscow operates at a voltage of 200 kV and

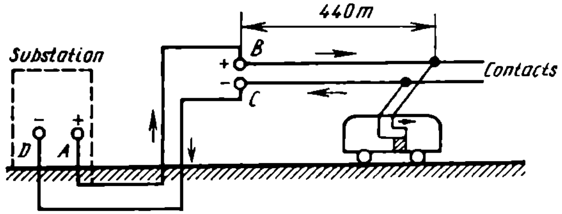


Fig. 32

is made of a single-core aluminium cable 112 km long. Determine the cross-sectional area of the cable if the voltage drop in it is 3.1 % of the nominal value at a current of 150 A.

10.37. A d.c. transmission line under a voltage of 1500 kV is 2500 km long. What will the voltage at the end of the line be if it is made of an aluminium wire with a cross-sectional area of 600 mm²? The admissible current load in the wire is 1070 A.

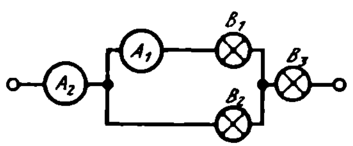


Fig. 33

10.38. Which of the ammeters in Fig. 33 will show a higher reading? What will the change in the resistance of the circuit be when bulb B_1 fuses? Will the reading of ammeters A_1 and A_2 change in this case?

10.39. The starter rheostat of a railway engine consists of 80 plates having a resistance of 0.05 Ω each and connected in series. Determine the resistance of the starter rheostat.

10.40. The armature winding of a traction motor consists of 132 sections connected in series in two parallel branches. The resistance of one section is 0.001 Ω. Determine the resistance of the whole winding.

10.41. Determine the total resistance, the current, and the voltage distribution in a circuit (Fig. 34), if the applied voltage is 220 V.

10.42. What series resistor should be connected to the heating element of an electric iron having a resistance $R = 24\ \Omega$ and rated at a voltage of 120 V in order to use it at a voltage of 220 V?

10.43. A circuit is constructed as shown in Fig. 34 and connected to a d.c. source of 120 V. It is known that $R_1 =$

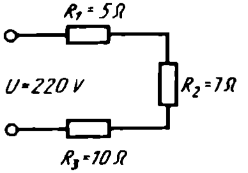


Fig. 34

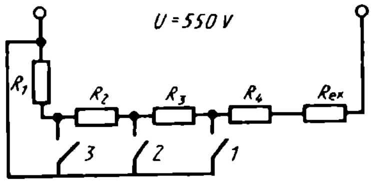


Fig. 35

$6\ \Omega$, $R_2 = 15\ \Omega$, and $I = 5$ A. Determine the total resistance of the circuit and the resistance of resistor R_3.

10.44. Figure 35 shows the excitation circuit for the traction motor of a trolleybus. The resistances of the sections of the rheostatic controller are $R_1 = 1040\ \Omega$, $R_2 = 200\ \Omega$, $R_3 = 90\ \Omega$, and $R_4 = 10\ \Omega$. The resistance of the excitation winding is $R_{ex} = 160\ \Omega$. Determine the excitation current with one of the switches *1*, *2*, or *3* closed and without the switches.

10.45. The magnetic field in the motor of an electric locomotive is produced by four series-connected windings. Determine the resistance of the windings if the number of turns in one of them is 39, the average length of a turn is 1.5 m, and the current in the windings is 352 A, the admissible current density being 2.7 A/mm^2.

10.46. How many equal pieces should a conductor of resistance $4\ \Omega$ be cut into and how should these pieces be connected to obtain a resistance of $1\ \Omega$?

10.47. What resistor should be connected in parallel to a resistor of $100\ \Omega$ for their total resistance to be $20\ \Omega$?

10.48. Determine the total resistance of a lamp resistor, the current in the circuit and through each lamp if the re-

sistor contains 10 parallel-connected lamps having a resistance of 440 Ω each, and the voltage applied is 220 V.

10.49. Three resistors of 1, 2, and 16 Ω are connected in parallel to a d.c. source. The current in the unbranched circuit is 5 A. Determine the total resistance of the circuit and the current through each resistor.

10.50. When two conductors are connected in parallel, their equivalent resistance is 0.72 Ω. The equivalent resistance for the same conductors connected in series is 3 Ω. Determine the resistance of each conductor.

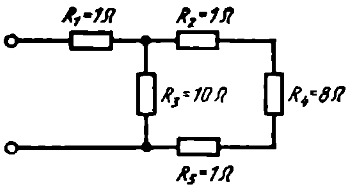


Fig. 36

10.51. Three incandescent lamps having a resistance of 400 Ω each are connected to a circuit at a voltage of 220 V. Determine the current passing through each lamp if they are connected in parallel and in series. Determine the total current in each case.

10.52. Five resistors are connected as shown in Fig. 36. They are connected to a 12-V d.c. source. Determine the total resistance of the circuit and the current in resistors R_1, R_3, and R_4.

10.53. A circuit is formed as shown in Fig. 36. All the resistors have the same resistance of 1.2 Ω. The current through the first resistor is 10 A. Determine the total resistance, the voltage across the terminals of the circuit, and the currents and voltages in each resistor.

10.54. The filament of an incandescent lamp is heated to about 2200°C and fuses more often when it is being switched on than when it is being switched off. Why? What conclusions can be drawn from a comparison of the currents through the lamp in the two cases? The material of the filament is tungsten.

10.55. Two resistors $R_1 = 12$ Ω and R_2 are connected in parallel to a current source. Determine the resistance R_2 and the current through it if the total current is 4 A and the voltage supplied to the resistors is 12 V.

10.56. Determine the voltage across the current collectors of trolleybuses if the contact wire is made of copper and has a cross-sectional area of 85 mm². The lengths of the

wires and the currents are indicated in Fig. 37. The generator voltage U_{BC} is 575 V.

10.57. Determine the voltage drop across a railway track if an electric locomotive is 800 m from the cable connecting

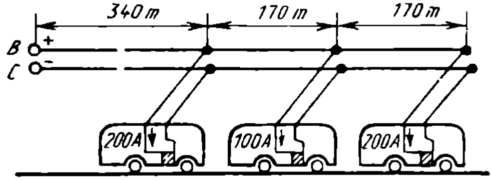


Fig. 37

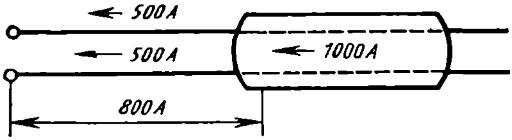


Fig. 38

the rails with the substation (Fig. 38). The cross-sectional area of the cable is 72 cm^2.

10.58. Figure 39 shows the circuit diagram of a starter rheostat consisting of three sections. Determine the re-

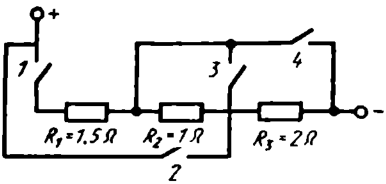


Fig. 39

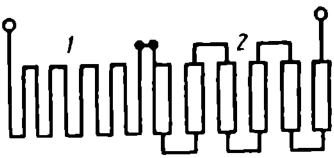


Fig. 40

sistance of the rheostat for the following three positions: (a) switch *1* is closed, (b) switch *2* is closed, (c) switches *1* and *4* are closed, (d) switches *2* and *4* are closed, (e) switches *1*, *2*, and *4* are closed, and (f) switches *2*, *3*, and *4* are closed.

10.59. Determine the resistance of two sections of a starter rheostat if the resistance of a cast iron plate is 0.06 Ω. The first section contains 10 such plates and the second 12 plates. The plates are connected as shown in Fig. 40.

10.60. Determine the resistance of a starter rheostat consisting of two series-connected sections. In the first section, 22 coils are connected in two parallel groups, each of which contains 11 series-connected coils. In the second section, 45 coils are connected into three parallel groups of 15 series-connected coils each. The coils are made of ferro-aluminium high-resistance alloy wire 1.6 m long and 3 mm in diameter.

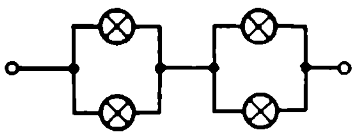


Fig. 41

10.61. The lighting circuit of a tram car consists of two parallel groups each containing five series-connected bulbs. Determine the total current in the circuit and in the groups if the resistance of each bulb is 220 Ω and the voltage in the circuit is 550 V.

10.62. Incandescent lamps, with a resistance of 440 Ω each, are connected to a circuit at a voltage of 220 V as shown in Fig. 41. Determine the total resistance of the lamps and the current and voltage in each lamp.

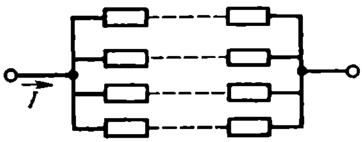


Fig. 42

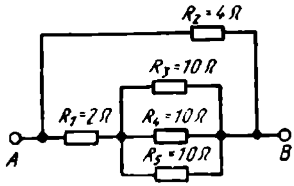


Fig. 43

10.63. What will the change in the voltage and current in the lamps (see Problem 10.62) be if one of the lamps fuses?

10.64. The armature winding in a locomotive motor consists of 924 copper rods 1 m long each. The rods are distributed equally among four parallel branches (Fig. 42). Determine the resistance of the armature winding for a current of 352 A, assuming that the current density is 5 A/mm^2.

10.65. Figure 43 presents a circuit diagram. The voltage U_{AB} is 120 V. Determine the resistance of the circuit, the unbranched current, and the current in each resistor.

10.66. Figure 44 shows a circuit diagram. The voltage U_{AB} is 220 V. Determine the total current and the current in parallel branches.

10.67. A current up to 50 A is to be measured with an ammeter whose scale is rated to 10 A. Determine the re-

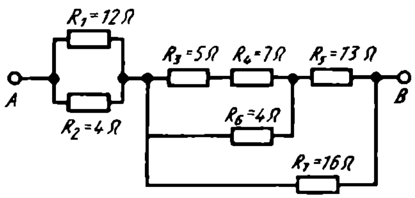


Fig. 44

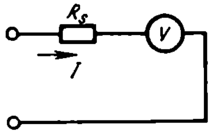


Fig. 45

sistance of the shunt if the resistance of the ammeter is 0.01 Ω (see Fig. 26).

10.68. An 0.18-Ω ammeter indicates a current of 6 A. The ammeter has a shunt whose resistance is 0.02 Ω. Determine the current in the mains (see Fig. 26).

10.69. A voltmeter with a scale from 0 to 150 V has to be used for measuring voltage from 0 to 250 V. Calculate the resistance of a series resistor if the resistance of the voltmeter is 600 Ω (Fig. 45).

10.70. A voltmeter rated for the maximum voltage of 20 V at a current of 8 mA has to be used for measuring a voltage of 100 V. What series resistance is required?

10.71. A voltmeter having an internal resistance of 1000 Ω is connected to a circuit with a constant voltage supply in series with a series resistor R_s and indicates 180 V. If another identical series resistor is connected to the circuit, the voltmeter indicates $U_2 = 150$ V. Determine the resistance of the series resistor R_s and the voltage U in the circuit.

10.72. The maximum current that can be measured with a milliammeter having an interval resistance $R = 150$ Ω is $I = 10$ mA. How long must a manganin wire of diameter $d = 0.1$ mm be if it is used as the series resistor to convert the instrument into a voltmeter with a scale from 0 to 10 V?

10.73. The internal resistance of a voltmeter is 300 Ω. A series resistor of 1200 Ω is connected to it in a measuring circuit. What is the ratio of the scale factors of the voltmeter with and without the series resistor?

Ohm's law for a closed circuit. Connection of batteries

10.74. A voltmeter is connected to the terminals of a current source. The external circuit is closed. What does the voltmeter measure in this case?

10.75. A cell whose internal resistance is 0.4 Ω is closed through an external circuit having a resistance of 2.1 Ω. The current in the circuit is 0.6 A. Determine the emf of the cell and the voltage drop across it.

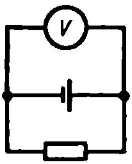


Fig. 46

10.76. Figure 46 shows a circuit diagram. When the resistance of the external circuit is 1.5 Ω, the voltmeter indicates 1.65 V, while for an external resistance of 3.6 Ω the reading of the voltmeter is 1.8 V. Determine the emf and the internal resistance of the cell.

10.77. A galvanic cell with an emf of 1.45 V and an internal resistance of 1.5 Ω is closed through an external resistance of 3.5 Ω. Determine the current in the circuit, the voltage across the cell terminals, and the efficiency of the cell in this circuit.

10.78. Determine the emf of a generator and the voltage across its terminals if its resistance is 0.05 Ω and the external resistance is 11.45 Ω. The current in the circuit is 20 A.

10.79. Determine the current in a circuit if the emf of the generator is 230 V, its resistance is 0.1 Ω, and the resistance of the external circuit is 22.9 Ω.

10.80. A resistor of 2.2 Ω is connected to a generator with an emf of 230 V. What is the resistance of the generator if the voltage across its terminals is 220 V?

10.81. A galvanic battery with an emf of 15 V and an internal resistance of 5 Ω is closed through a conductor whose resistance is 10 Ω. A 1-μF capacitor is connected to the battery terminals. Determine the electric charge on the capacitor.

10.82. A 2-μF capacitor and a 3-Ω resistor are connected in parallel to a current source with an emf of 10 V and an internal resistance of 1 Ω. What is the electric charge on the capacitor?

10.83. If a current source is closed through a 13-Ω resistor, the current in the circuit is 0.8 A. If the source is closed through an 8-Ω resistor, the current in the circuit is

1.2 A. Determine the emf and the internal resistance of the current source.

10.84. A d.c. generator with an emf of 150 V and an internal resistance of 0.3 Ω supplies voltage to 20 incandescent lamps having a resistance of 240 Ω each and connected in parallel. The resistance of the leads is 2.7 Ω. Determine the voltage across the generator terminals and across the lamps.

10.85. The emergency lamps of a tram car are fed by an accumulator battery having an emf of 48 V and an internal resistance of 0.2 Ω. Ten lamps having a resistance of 39.5 Ω each are connected as shown in Fig. 47. Determine the current in each lamp and in the leads.

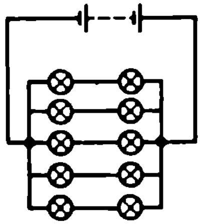


Fig. 47

10.86. A circuit contains 20 parallel-connected bulbs. The current through a bulb is 1 A. The resistance of the wires connecting the load with a generator is 0.2 Ω. What must the emf of the generator be for the voltage across the bulbs to be 220 V? The internal resistance of the generator is 0.05 Ω.

10.87. Three electric motors and ten parallel-connected incandescent lamps are connected to a generator with emf

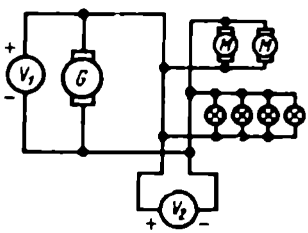


Fig. 48

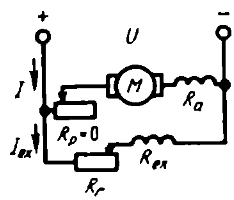


Fig. 49

240 V and resistance 0.025 Ω. A current of 50 A passes through each motor while the current through each lamp is 1 A. The resistance of the leads is 0.1 Ω. Determine the voltage across the generator terminals and across the loads.

10.88. A workshop receives electric power from a collective farm's electric power station. There are two electric motors (M) and four bulbs in the workshop connected as shown in Fig. 48. The current through each motor is 10 A, and through

each bulb 0.5 A. The distance l between the station and the workshop is 0.5 km. The internal resistance of the generator is 0.1 Ω and the voltage across its terminals is 220 V. Determine the emf of the generator and the cross-sectional area of the copper leads if the admissible voltage drop in them is 8%.

10.89. Determine the counter emf of a traction motor if the resistance of its windings is 0.1 Ω and the voltage in the circuit is 550 V at a current of 150 A.

10.90. Determine the emf of a generator with an internal resistance of 0.05 Ω and the counter emf of a motor if the current in the circuit is 100 A, the voltage across the generator terminals is 225 V, and the resistances of the motor winding and the leads are 0.2 and 0.1 Ω respectively.

10.91. The circuit diagram of a d.c. motor is shown in Fig. 49. The voltage in the circuit is $U = 550$ V, and the current is $I = 102$ A. The resistance of the armature circuit is $R_a = 0.1\ \Omega$, that of the parallel excitation winding $R_{ex} = 150\ \Omega$, and that of the rheostatic controller $R_r = 125\ \Omega$. Determine the current in the parallel excitation winding when the rheostatic controller is completely on, and the counter emf if the electric motor is started without a starter rheostat.

10.92. Four loads having a resistance of 10 Ω each are connected to an accumulator battery having an emf of 48 V and an internal resistance of 0.25 Ω. Determine the current through the battery if the loads are connected (a) in series, (b) in parallel, and (c) in two parallel branches containing two series-connected loads each.

10.93. One of two cells has an emf of 1.45 V and an internal resistance of 0.5 Ω and supplies voltage to a circuit with an efficiency of 90%, while the other cell has an emf of 2 V and an internal resistance of 0.5 Ω and operates with an efficiency of 80% in an identical circuit. Determine the current in the two circuits.

10.94. An accumulator with an emf of 1.45 V produces a current of 0.5 A in a conductor whose resistance is 2.5 Ω. Determine the short-circuit current.

10.95. During a short circuit, the current from a source of 1.8 V is 6 A. What must the external resistance be for the current to be 2 A?

10.96. Why should lead accumulators not be short circuited?

10.97. A galvanic cell is first connected to an external resistance of 1.9 Ω and the current in the circuit is 0.6 A. Then it is connected to an external resistance of 2.4 Ω and the current becomes 0.5 A. What is the short-circuit current for this cell?

10.98. Three galvanic cells, each having an emf of 1.5 V and an internal resistance of 0.6 Ω, are connected in series to a conductor having a resistance of 1.8 Ω. Determine the current in the circuit. What will the current in the same conductor be if the cells are connected in parallel?

10.99. Using the data in Problem 10.98, determine the connection for which the short-circuit current is the largest.

10.100. How should six galvanic cells, each having an emf of 1.5 V and an internal resistance of 0.5 Ω, be connected in parallel groups to obtain the maximum current when connected to a 1.0-Ω resistor?

10.101. How should 12 cells, each having an emf of 1.2 V and an internal resistance of 0.6 Ω, be connected to produce a current of 1.6 A in an external circuit whose resistance is 2.2 Ω?

10.102. A "Krona" battery is the power source for transistor radios. It contains seven series-connected galvanic cells. What is the emf of a cell if the emf of the battery is 9 V? What will the change in the emf of the battery be if the constituent cells are connected in parallel?

10.103. Two alkaline accumulators, each having an emf of 1.5 V and an internal resistance of 0.3 Ω, are connected first in series and then in parallel and used to supply voltage to a circuit with a resistance of 6 Ω. Which connection produces the higher current in the external circuit, i.e. which method is more advantageous?

10.104. Solve Problem 10.103 for the case when the external resistance of the circuit is 0.9 Ω. Using the results of the two problems, find the ratio between the external and internal resistances for which connecting the current sources in series is more advantageous.

10.105. A passenger carriage is lighted using a battery containing 26 series-connected lead accumulators. The emf of an accumulator is 2 V and the internal resistance is

0.004 Ω. Determine the emf and the voltage across the terminals of the battery if the current in the circuit is 20 A.

10.106. What is the reading of a voltmeter connected to the terminals of a battery consisting of three series-connected alkaline accumulators, each with an emf of 1.2 V and an internal resistance of 0.3 Ω? The external circuit consists

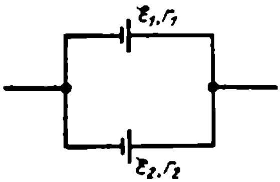


Fig. 50

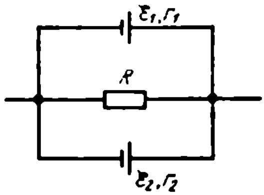


Fig. 51

of a bulb with a resistance of 16 Ω and 2-m long aluminium leads with a cross-sectional area of 0.2 mm². Determine the voltage on the bulb.

10.107. An accumulator battery, having an emf of 22 V and an internal resistance of 0.1 Ω, is charged from a rectifier the voltage across whose terminals is 24 V. Determine the resistance introduced into the circuit by a potentiometer connected in series with the battery if the charging current is 4 A.

10.108. Two accumulators, having emf's of 1.3 and 1.8 V and internal resistances of 0.1 and 0.15 Ω respectively, are connected in parallel. Determine the current in the circuit and the voltage across the terminals (Fig. 50).

10.109. Determine the current through a resistor $R = 2\ \Omega$ connected in a circuit as shown in Fig. 51. Given: $\mathscr{E}_1 = 2$ V, $r_1 = 0.50\ \Omega$, $\mathscr{E}_2 = 4.0$ V, and $r_2 = 0.70\ \Omega$.

§ 11. WORK, POWER, AND THE THERMAL EFFECT OF CURRENT

Basic Concepts and Formulas

Electric energy is transformed into other kinds of energy so that energy is conserved. The measure of the transformation is the work done by electric current:

$$A = QU = IUt,$$

If the current or voltage are substituted using Ohm's law for a conductor we obtain

$$A = IUt = I^2Rt = \frac{U^2}{R}t.$$

Power is the ratio of the work done by an electric current during time t to the time t:

$$P = \frac{A}{t}, \quad P = IU = I^2R = \frac{U^2}{R}.$$

The unit of work is the joule (J). In electrical engineering and everyday life, electrical work is measured in kilowatt-hours (1 kWh = 3.6 MJ). The unit of power is the watt (1 W = 1 J/s).

Joule's law: the amount of heat liberated in a conductor carrying a current is proportional to the square of the current, the time of its passage, and the resistance of the conductor:

$$Q = I^2Rt.$$

This formula is for series-connected loads. For a parallel connection, the formula is

$$Q = \frac{U^2}{R}t.$$

Worked Problems

Problem 63. An electric motor operating for 5 h is driven at the mains voltage of 380 V and a current of 35 A. The resistance of the motor winding is 0.5 Ω. Determine the amount of energy consumed, the amount of heat liberated in the winding during the operation, and the mechanical work done by the motor.

Given: $U = 380$ V is the voltage at the motor terminals, $I = 35$ A is the current, $R = 0.5\ \Omega$ is the resistance of the motor winding, and $t = 5$ h $= 5 \times 3600$ s is the operation time.

Find: the energy A consumed by the motor, the amount of heat Q liberated in the winding, and the mechanical work A_{mech}.

Solution. The energy consumed or the total work done by the current can be determined from the formula

$$A = IUt, \quad A = 35\ \text{A} \times 380\ \text{V} \times 5 \times 3600\ \text{s}$$
$$\simeq 2.4 \times 10^8\ \text{J}.$$

The amount of heat liberated in the winding of the motor can be determined from Joule's law:

$$Q = I^2Rt,$$

$$Q = (35\ \text{A})^2 \times 0.5\ \Omega \times 5 \times 3600\ \text{s} = 1.1 \times 10^7\ \text{J}.$$

The mechanical work done by the motor can be determined by subtracting the energy spent in heating the winding from the whole energy spent:

$$A_{\text{mech}} = A - Q,$$

$$A_{\text{mech}} = 2.4 \times 10^8\ \text{J} - 1.1 \times 10^7\ \text{J} = 2.29 \times 10^8\ \text{J}.$$

Answer. The energy spent by the motor is approximately 2.4×10^8 J, the amount of heat liberated in the winding is 11 MJ, and the mechanical work is 2.3×10^8 J.

Problem 64. A tower crane, whose efficiency is 70%, hoists a load of 49 kN at a constant velocity of 0.55 m/s. Determine the current in its electric motors if driven from a supply voltage of 380 V.

Given: $\eta = 70\%$ is the efficiency of the crane, $G = 4.9 \times 10^4$ N is the force of gravity acting on the load, $v = 0.55$ m/s is the velocity of hoisting, and $U = 380$ V is the voltage in the circuit.

Find: the current I.

Solution. We write the formula for the efficiency as the ratio of the useful power (Gv) and the power spent (IU):

$$\eta = \frac{Gv}{IU}\,100\%.$$

Hence we obtain the current:

$$I = \frac{Gv}{\eta U}\,100\%,$$

$$I = \frac{4.9 \times 10^4\ \text{N} \times 0.55\ \text{m/s} \times 100\%}{70\% \times 380\ \text{V}} \simeq 101\ \text{A}.$$

Answer. The current is approximately 101 A.

Problem 65. Two resistors of 40 and 80 Ω are connected in parallel to a source of constant voltage. The amount of heat liberated in the first resistor is 3.0×10^5 J. How much heat is liberated in the second resistor over the same time and in both resistors if they are connected in series?

Given: $R_1 = 40\ \Omega$ is the resistance of the first resistor, $R_2 = 80\ \Omega$ is the resistance of the second resistor, and $Q_1 =$

3.0×10^5 J is the amount of heat liberated in the first resistor.

Find: the amount of heat Q_2 liberated in the second resistor, and the amount of heat liberated in the series-connected resistors.

Solution. We use Joule's law

$$Q_1 = \frac{U^2}{R_1} t, \qquad (1)$$
$$Q_2 = \frac{U^2}{R_2} t.$$

Dividing these equations termwise, we obtain $Q_1/Q_2 = R_2/R_1$, whence we get Q_2:

$$Q_2 = \frac{R_1 Q_1}{R_2}, \quad Q_2 = \frac{40\ \Omega \times 3.0 \times 10^5\ \text{J}}{80\ \Omega} = 1.5 \times 10^5\ \text{J}.$$

If the resistors are connected in series, we have

$$Q = \frac{U^2 t}{R_1 + R_2}. \qquad (2)$$

Expressing $U^2 t$ from (1) and substituting it into Eq. (2), we obtain

$$Q = \frac{Q_1 R_1}{R_1 + R_2}, \quad Q = \frac{3.0 \times 10^5\ \text{J} \times 40\ \Omega}{120\ \Omega} = 1.0 \times 10^5\ \text{J}.$$

Answer. The amount of heat liberated by the second resistor is 1.5×10^5 J, and the heat liberated by the series-connected resistors is 1.0×10^5 J.

Problem 66. The air in a room loses 293 MJ of heat per day. Determine the diameter of a nichrome wire 10.2 m long from which the coil of an electric heater is made such that the temperature in the room is kept constant if supplied with a voltage of 220 V.

Given: $Q = 2.93 \times 10^8$ J is the amount of heat lost, $t = 24 \times 3600$ s is the time during which the heat is lost, $l = 10.2$ m is the length of the nichrome wire, $U = 220$ V is the supply voltage. From tables, we find the resistivity of nichrome, $\rho = 1.05 \times 10^{-6}\ \Omega \cdot \text{m}$.

Find: the diameter d of the wire.

Solution. Let us determine the amount of heat lost by the air in the room per second, i.e. the power of heat losses $P_1 = Q/t$.

The power of the electric heater can be determined from the formula $P_2 = U^2/R$.

In order to maintain the temperature in the room, the power of the heater must be the same as the power of heat losses: $P_1 = P_2$, $Q/t = U^2/R$. Hence we can find the resistance of the heater coil: $R = U^2t/Q$. It is well known that $R = \rho l/S$, where $S = \pi d^2/4$. Then $R = 4\rho l/\pi d^2$.

Equating the expressions for R, we obtain $\frac{U^2t}{Q} = \frac{4\rho l}{\pi d^2}$. Hence

$$d = \sqrt{\frac{4Q\rho l}{U^2 t\pi}},$$

$$d = \sqrt{\frac{4 \times 2.93 \times 10^8 \text{ J} \times 1.05 \times 10^{-6} \ \Omega\cdot\text{m} \times 10.2 \text{ m}}{220 \text{ V} \times 220 \text{ V} \times 24 \times 3600 \text{ s} \times 3.14}}$$
$$= 9.5 \times 10^{-4} \text{ m}.$$

Answer. The diameter of the wire is approximately 1 mm.

Problem 67. How long will it take to boil 1 l of water from 15°C using an electric immersion heater whose resistance is 25 Ω and efficiency is 85%? The applied voltage is 120 V.

Given: $V = 10^{-3}$ m^3 is the volume of the water, $t_1 = 15$°C is the initial temperature of the water, $t_2 = 100$°C is the boiling point of water, $R = 25$ Ω is the resistance of the coil, $U = 120$ V is the applied voltage, and $\eta = 0.85$ is the efficiency of the heater. From tables, we find the density of water, $\rho = 10^3$ kg/m^3, and the specific heat of water, $c = 4187$ J/(kg·K).

Find: the time t required for boiling water.

Solution. Problems involving a given efficiency should be solved starting with the formula for efficiency $\eta = \frac{Q_{rec}}{Q_{giv}}$, where $Q_{rec} = cm(t_2 - t_1)$, and $Q_{giv} = \frac{U^2}{R}t$. This gives $\eta = \frac{cm(t_2 - t_1)}{U^2 t}R$. Hence we can find the time: $t = \frac{cm(t_2 - t_1)R}{\eta U^2}$. Considering that the mass of water can be expressed in terms of density and volume, $m = \rho V$, we obtain

$$t = \frac{c\rho V(t_2 - t_1)R}{\eta U^2},$$

$$t = \frac{4187 \text{ J/(kg}\cdot\text{K)} \times 10^3 \text{ kg/m}^3 \times 10^{-3} \text{ m}^3 \times 85 \text{ K} \times 25 \ \Omega}{0.85 \times 120 \text{ V} \times 120 \text{ V}} = 720 \text{ s}$$
$$= 12 \text{ min}.$$

Answer. It takes 12 min to boil the water.

Questions and Problems

11.1. How much electricity will pass through the cross section of a conductor in 30 s if its resistance is 20 Ω and the voltage across it is 12 V? Determine the work done by the electric current.

11.2. What energy is supplied by a generator to an external circuit over 8 h if the readings of the ammeter and voltmeter remain unchanged at 50 A and 220 V respectively? Express the answer in joules and kilowatt-hours.

11.3. The reading of an electric meter is 0981 kWh. The load consists of three bulbs of 100 W, two bulbs of 60 W, and four bulbs of 15 W. What will the meter reading be in 30 days if the bulbs are switched on for 10 h every day? What will the electricity bill be at a rate of 4 kopecks per kilowatt-hour?

11.4. An electric motor driven by a current of 15 A at a constant voltage of 220 V develops a power of 3 kW. Determine the efficiency of the motor and the cost of 8 h of electric energy, if it is charged at a rate of 4 kopecks per kWh.

11.5. A d.c. electric transmission line between the Volzhskaya hydroelectric plant and the Donbas is rated for a voltage of 800 kV with a nominal current of 1000 A. Determine the transmitted power and the energy transferred per year under these nominal conditions.

11.6. Determine the electric energy expenditure over 8 h of operation of two parallel-connected electric motors if the current in one branch is 50 A and the supply voltage is 220 V.

11.7. How much energy is consumed by the motor of a tram in one hour of continuous operation if the voltage across the collector plates of the motor is 500 V and the current in the winding of the motor is 130 A? Express the answer in joules and kilowatt-hours.

11.8. Determine the power of the current in the circuit shown in Fig. 52.

11.9. Five resistors are connected as shown in Fig. 53 and to a voltage supply $U_{AB} = 24$ V. Determine the total resistance of the circuit, the potential difference between points C and D, the current in the fifth resistor, and the power of the current in the circuit.

11.10. An electric loader is driven by an accumulator battery consisting of 22 series-connected alkaline accumulators with an emf of 1.1 V each. The power developed by the motor during loading is 1.2 kW. Determine the current in the circuit if the voltage drop across the battery and in the wires is 2.2 V.

11.11. The battery of an electromobile consists of 42 series-connected accumulators having an emf of 2.0 V each.

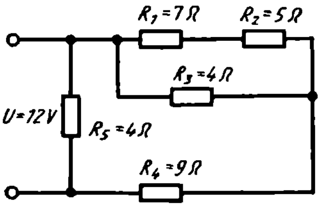


Fig. 52

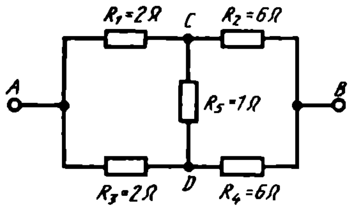


Fig. 53

Determine the total power of the battery if the current in each of two parallel-connected electric motors is 50 A.

11.12. Determine the power developed by an underground train consisting of six coaches if the motors in each coach are connected in two parallel branches consisting of two series-connected motors. The supply voltage is 750 V and the current in each branch is 150 A.

11.13. Determine the power developed by the motors of an electric locomotive if they are connected into three parallel branches of two series-connected motors, the supply voltage is 3 kV, and the current in each branch is 300 A.

11.14. Determine the energy consumption in 8 hours if four incandescent lamps are connected to a circuit at a voltage of 120 V (a) in parallel, (b) in series, and (c) in two parallel groups containing two series-connected lamps. The resistance of a lamp is 120 Ω.

11.15. Determine the power consumed and the useful power for the motor from Problem 10.89 if its efficiency is 90%.

11.16. Using the data of Problem 10.56, calculate the power consumed by the motors of trolleybus *1* and the power losses in the circuit.

11.17. A generator with an internal resistance of 0.01 Ω is connected to electric motors as shown in Fig. 54. The voltmeter indicates 220 V and the ammeter 500 A. The resistance of the leads is 0.05 Ω. Determine the total power, the power consumed by the electric motors, and the efficiency of the set-up.

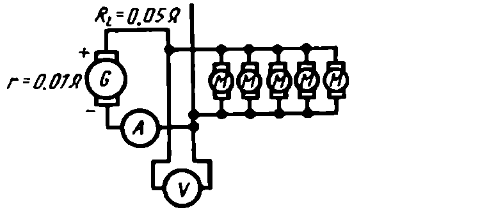


Fig. 54

11.18. Using the data of Problem 10.61, determine the power of the current in the bulbs, and the energy expenditure in ten hours of operation.

11.19. What must the cross-sectional area of a copper wire be to transmit a power of 1.0 kW at a voltage of 100 V over a distance of 50 m so that voltage losses are less than 6 V?

11.20. Two identical resistors are connected to a circuit at a constant voltage first in parallel and then in series. Is the electric power the same in both cases?

11.21. The resistance of a glowing incandescent lamp is 360 Ω. The supply voltage is 220 V. How many lamps are connected in parallel in the circuit if the power consumed by all the lamps is 2.15 kW? The resistance of the leads should be neglected.

11.22. An electric loader lifts a load of 500 kg to a height of 2 m at a constant velocity. The motor is driven by an accumulator battery with an emf of 24 V at a current of 41 A. The efficiency of the loader is 80%. Determine the velocity and duration of one operation, neglecting the internal resistance of the battery.

11.23. The motors of a tram moving uniformly along horizontal rails develop a tractive force of 2 kN. The voltage across the collector plates of the motors is 550 V and the current in the circuit is 80 A. The efficiency is 80%. Determine the velocity of the tram.

11.24. A tram starts to move at a constant acceleration of 1 m/s^2 for 8 s with a tractive force of 28 kN and an efficiency of 88%. What is the current at the end of the eight seconds if the supply voltage is 550 V?

11.25. Why is lead wire used in fuses?

11.26. The 8-A and 16-A fuses for the VAZ motor car have the same length. What is the difference between the fuses?

11.27. Given two hot plates one of which is rated for a voltage of 127 V and the other for 220 V. Which coil is thicker if the length and material of the coils are the same?

11.28. A carriage is heated with eight electric heaters having a resistance of 275 Ω each and connected in parallel. The voltage across the terminals of the heaters is 550 V. Determine the amount of heat given away by the heaters during 18 h of operation, neglecting heat losses.

11.29. Two bulbs rated for 110 V are connected in series and to a voltage supply of 220 V. Determine the voltage across each bulb, the power, and the amount of heat liberated by a bulb during 1 h if the powers of the bulbs are (a) 60 W each, (b) 60 and 40 W, and (c) 60 and 100 W.

11.30. An electric soldering iron operates at a voltage of 220 V and a current of 0.22 A. How much tin at 293 K can be melted by it per minute?

11.31. An electric samovar having a power of 600 W boils 1.5 l of water from 283 K in 20 min. Determine the efficiency of the samovar and the cost of the energy if electricity is charged at a rate of 4 kopecks per kWh.

11.32. Two resistors, having resistances of 1 and 4 Ω, are connected in turn to a current source and are found to consume the same power. Determine the internal resistance of the current source.

11.33. Determine the emf and the internal resistance of a current source if the power of an external circuit is 230 W at a current of 10 A and 337.5 W at a current of 15 A.

11.34. An electric motor is connected to a voltage supply of 220 V. The resistance of the motor winding is 1.8 Ω and the current in the motor is 12 A. Determine the power consumed and the efficiency of the motor.

11.35. Water boils in an electric boiler 12 min after it has been switched on. The heating element consists of 4.5 m of wire wound in a coil. What must be done to obtain boiling water in 8 min? The energy losses should be neglected.

11.36. The heating element of an electric boiler has two sections. If the two sections are connected in series, water boils in the boiler in 27 min, while if connected in parallel the water boils in 6 min. The resistance of one section is 40 Ω. What is the resistance of the other section? Energy losses should be neglected.

11.37. The distance between a generator with an emf of

240 V and an internal resistance of 0.1 Ω and a consumer is 50 m. How much copper is required for the cabling if a power of 22 kW is rated for a voltage of 220 V?

§ 12. ELECTRIC CURRENT IN ELECTROLYTES

Basic Concepts and Formulas

Liquid conductors mainly include solutions of salts, alkalis, and acids. The current carriers in liquid conductors are the ions formed by electrolytic dissociation. This is the decomposition of neutral molecules of salts, alkalis, and acids into positive and negative ions when they dissolve in water or other solvent. The large permittivity of water ($\varepsilon = 81$) and thermal motion lead to the decomposition of molecules.

The passage of a current through a liquid conductor (electrolyte) is accompanied by a chemical transformation of the substance and its deposition on the electrodes (this is known as electrolysis).

Electrolysis is governed by Faraday's two laws.

Faraday's first law of electrolysis. The mass of a substance liberated in electrolysis is proportional to the amount of electricity passing through the electrolyte:

$$m = kQ,$$

where k is the electrochemical equivalent and is the amount of substance liberated in the electrolysis by the passage of 1 C of electricity through the electrolyte.

Faraday's second law of electrolysis. Electrochemical equivalents are proportional to the ratio of the molar mass to the valency of the substance:

$$\frac{k_1}{k_2} = \frac{M_1}{n_1} : \frac{M_2}{n_2},$$

where M_1 and M_2 are the molar masses, and n_1 and n_2 are the valencies.

It is important to remember that the charge passing through an electrolyte to liberate 1 mol of a substance is $F = N_A e = 9.648456 \times 10^4$ C/mol, where F is the same number for all electrolytes and is known as the Faraday constant.

Faraday's generalized law is

$$m = \frac{1}{F}\frac{M}{n}Q.$$

The passage of a current through an electrolyte may cause the electrodes to polarize, which induces a counter emf, i.e. decreases the current through the circuit. In such a case, problems can be solved with the help of Ohm's law for a subcircuit containing an emf:

$$I = \frac{U - \mathscr{E}_{\text{pol}}}{R}.$$

Worked Problems

Problem 68. A metal article is electrolytically plated with a silver layer 20 μm thick. How long did the electrolysis require for a current density of 2.5×10^{-3} A/cm^2?

Given: $h = 20$ μm $= 2 \times 10^{-5}$ m is the thickness of the silver layer, and $j = 2.5 \times 10^{-3}$ A/cm^2 $= 25$ A/m^2 is the current density. From tables, we find the electrochemical equivalent of silver, $k = 1.118 \times 10^{-6}$ kg/C, the density of silver, $\rho = 10.5 \times 10^3$ kg/m^3, the valency of silver, $n = 1$, the molar mass of silver, $M = 108 \times 10^{-3}$ kg/mol, and the Faraday constant $F = 9.65 \times 10^4$ C/mol.

Find: the time t of the electrolysis.

Solution. 1st method. We solve the problem using Faraday's first law $m = kIt$. This gives $t = m/kI$. The mass and the current can be determined from the formulas $m = \rho Sh$ and $I = jS$. Substituting these quantities into the formula for time, we obtain

$$t = \frac{\rho h}{kj}, \quad t = \frac{10.5 \times 10^3 \text{ kg/m}^3 \times 20 \times 10^{-6} \text{ m}}{1.118 \times 10^{-6} \text{ kg/C} \times 25 \text{ A/m}^2} = 7500 \text{ s}.$$

2nd method. If the electrochemical equivalent is unknown, the problem can be solved using Faraday's generalized law $m = \frac{1}{F}\frac{M}{n}It$, whence $t = \frac{mFn}{MI}$. The mass and the current can be determined as before: $m = \rho V = \rho Sh$ and $I = jS$. This gives

$$t = \frac{\rho h F n}{Mj},$$

$$t = \frac{10.5 \times 10^3 \text{ kg/m}^3 \times 20 \times 10^{-6} \text{ m} \times 9.65 \times 10^4 \text{ C/mol}}{108 \times 10^{-3} \text{ kg/mol} \times 25 \text{ A/m}^2} = 7500 \text{ s}.$$

Answer. The time required for silver plating is about 2.1 h.

Problem 69. During the electrolysis of silver nitrate solution, 12 g of silver took an hour to deposit on the cathode. The voltage across the terminals of the electrolytic bath was 5.2 V, the resistance of the solution was 1.5 Ω, and the polarization emf was 0.7 V. Determine the valency of silver and the number of silver atoms liberated at the cathode.

Given: $t = 3600$ s is the duration of the electrolysis, $m = 1.2 \times 10^{-2}$ kg is the mass of liberated silver, $U = 5.2$ V is the voltage across the terminals of the bath, $R = 1.5\ \Omega$ is the resistance of the electrolyte solution, and $\mathscr{E} = 0.7$ V is the polarization emf. From tables, we find the molar mass of silver, $M = 108 \times 10^{-3}$ kg/mol, the Faraday constant $F = 9.65 \times 10^4$ C/mol, and the electron charge $e = 1.6 \times 10^{-19}$ C.

Find: the valency n of silver and the number N of silver atoms liberated at the cathode.

Solution. The valency of silver can be determined from Faraday's generalized law $m = \frac{1}{F}\frac{M}{n}It$, whence $n = \frac{MIt}{mF}$.

Since the polarization emf emerges in the circuit, the current can be determined from Ohm's law for a subcircuit with an emf: $I = (U - \mathscr{E})/R$. Then the expression for the valency becomes

$$n = \frac{M(U-\mathscr{E})t}{mFR}, \quad n = \frac{108 \times 10^{-3}\ \text{kg/mol}\,(5.2\ \text{V} - 0.7\ \text{V})\,3600\ \text{s}}{1.2 \times 10^{-2}\ \text{kg} \times 9.65 \times 10^4\ \text{C/mol} \times 1.5\ \Omega} = 1.$$

Since the valency of silver is unity, we can determine the number of silver atoms by dividing the charge passing through the electrolyte by the elementary charge: $N = Q/e$. Since $Q = It$, we obtain

$$N = \frac{(U-\mathscr{E})t}{Re}, \quad N = \frac{(5.2\ \text{V} - 0.7\ \text{V})\,3600\ \text{s}}{1.5\ \Omega \times 1.6 \times 10^{-19}\ \text{C}} = 6.75 \times 10^{22}.$$

Answer. Silver is monovalent, and 6.75×10^{22} silver atoms were liberated at the cathode.

Problem 70. The current through an electrolytic bath containing the solution of zinc sulphate ($ZnSO_4$) increases linearly $I = (2 + 0.02t)$. How much zinc will be liberated at the cathode 5 min after the current starts changing?

Given: $I = (2 + 0.02t)$ is the linear increase in current, and $t = 300$ s is the duration of the electrolysis. From tables, we find the electrochemical equivalent of zinc, $k = 3.4 \times 10^{-7}$ kg/C.

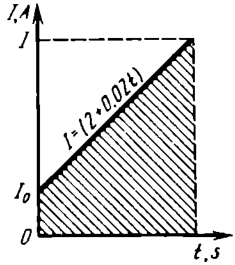


Fig. 55

Find: the mass m of the deposited zinc.

Solution. We use Faraday's first law of electrolysis

$$m = kQ,$$

where Q is the amount of electricity passing through the electrolyte. In order to determine Q, we plot the current versus time graph. The current can be determined from the given equation. At time $t_0 = 0$, we have $I_0 = 2$ A, while at $t = 300$ s, we have $I = (2 + 0.02 \times 300)$ A $= 8$ A. The time variation of current is shown in Fig. 55.

From the graph, we see that the amount of electricity passing through the electrolyte is equal to the area of the hatched region (trapezium):

$$Q = \bar{I}t = \frac{I_0 + I}{2}\, t, \quad Q = \frac{(2\text{ A} + 8\text{ A})}{2}\, 300\text{ s} = 1500\text{ C}.$$

The mass of the zinc is

$$m = 3.4 \times 10^{-7}\text{ kg/C} \times 1500\text{ C} = 51.0 \times 10^{-5}\text{ kg}.$$

Answer. 510 mg of zinc will be deposited on the cathode.

Problem 71. Determine the power consumed during the electrolysis of sulphuric acid solution if 150 mg of hydrogen are liberated over 25 min, and the resistance of the electrolyte is 0.4 Ω. The losses should be neglected.

Given: $t = 1500$ s is the duration of the electrolysis, $m = 0.15 \times 10^{-3}$ kg is the mass of the liberated hydrogen, $R = 0.4$ Ω is the resistance of the electrolyte. From tables, we find the electrochemical equivalent of hydrogen, $k = 1.044 \times 10^{-8}$ kg/C.

Find: the power P consumed in the electrolysis.

Solution. In order to determine the power of the current, we use the formula $P = I^2R$. The current can be determined from Faraday's first law of electrolysis: $I = m/kt$. Substi-

tuting this quantity into the formula for power, we obtain

$$P=\frac{m^2R}{(kt)^2},\quad P=\frac{(0.15\times10^{-3}\text{ kg})^2\,0.4\ \Omega}{(1.044\times10^{-8}\text{ kg/C}\times1500\text{ s})^2}\simeq 37\text{ W}.$$

Answer. The power consumed in the liberation of the hydrogen is approximately equal to 37 W.

Questions and Problems

12.1. A solution of the salt NaCl is electrically neutral. Can we state that there are no ions in the solution? Why?

12.2. Electrolytic capacitors are connected to a circuit, with their polarity observed. Can the polarity be ignored? Why?

12.3. Current reversal is used to polish the surface of a metal electrolytically. What effect does it produce? Can this method be used to sharpen cutting tools?

12.4. How much aluminium, silver, and copper will be deposited on the cathode as a result of the passage of 1 C of electricity through appropriate electrolytes? How many electrons pass through each electrolyte thereby?

12.5. The electrolysis of copper sulphate solution yields 1 g of copper. How much aluminium can be obtained electrolytically by passing the same amount of electricity through an appropriate electrolyte?

12.6. Two electrolytic baths are connected in series and to a current source. The first bath contains a solution of nickel sulphate $NiSO_4$ while the second contains a solution of chromium (II) chloride $CrCl_2$. How much chromium will be liberated in the second bath if 300 g of nickel are liberated in the first bath?

12.7. Two series-connected electrolytic baths contain copper sulphate $CuSO_4$ and copper chloride CuCl solutions respectively. How much copper will be liberated in each bath when 1 C of electricity is passed through them?

12.8. A student first calculated the electrochemical equivalent of copper using the formula $k = M/Fn$ and then experimentally determined it. What must the increase in the mass of the cathode be after 15 min if copper sulphate solution is electrolyzed at a current of 1 A? The valency of copper is 2.

12.9. To determine the electrochemical equivalent of copper, a student electrolyzed a copper sulphate solution for 20 min at a current of 1.5 A. The cathode increased in mass by 600 mg. What is the resultant electrochemical equivalent? What are the absolute and relative errors of measurement in comparison with the tabulated value?

12.10. A solution was electrolyzed for 20 min at a current of 1.5 A. 594 mg were liberated at the cathode. What was liberated?

12.11. During an electrolysis, 503 mg of a metal were liberated at the cathode. The process lasted 5 min at a current of 1.5 A. What was the metal and what is its valency?

12.12. What are the charges of mono-, bi-, and trivalent ions?

12.13. The valencies of silver and gold are 1 and 3 respectively. Determine their electrochemical equivalents.

12.14. How many silver atoms are liberated at the cathode when silver nitrate solution is electrolyzed for 1 h at a current of 1 A?

12.15. Two series-connected electrolytic baths contain solutions of copper sulphate and auric chloride respectively. As a result of the electrolysis, 2 g of copper was liberated at the cathode. How much trivalent gold was liberated in the other bath? How many copper and gold atoms were deposited on the cathodes?

12.16. Copper is refined at a voltage of 0.3 V across the bath terminals. How much copper is deposited on the cathode over 1 h if the resistance of the electrolyte is $3 \times 10^{-5}\ \Omega$? Copper is bivalent.

12.17. To obtain aluminium, a current of 50 A is passed through molten cryolite with alumina at a voltage of 6 V. Determine the energy consumption per ton of aluminium and the resistance of the electrolyte.

12.18. In a copper refinery the cathodes are replaced after 10 days of continuous operation of electrolytic baths. The amount of copper accumulated on each electrode over this period is 71 kg. Determine the current density if the area of a cathode is 0.9 m^2.

12.19. During an electrolysis, 5×10^4 C of electricity were passed through a ferric chloride ($FeCl_3$) solution. How much iron and chlorine are liberated in the process? At which electrode is the chlorine liberated? Why?

12.20. How long does it take to gold-plate a watch case electrolytically with a layer thickness of 12 μm at a current density of 0.1 A/dm^2?

12.21. The reflectivity of an automobile's headlight is increased by an electrolytic coating of a 10-μm thick silver layer. How long must the electrolysis of a silver nitrate solution with a current density of 0.3 A/dm^2 be to obtain the required thickness of the layer?

12.22. A melt of aluminium salts is electrolyzed at a voltage of 6.5 V. How much electricity is spent to obtain 1 t of aluminium and how much does it cost at a rate of 2 kopecks per kWh if the efficiency of the electrolyzer is 75%?

12.23. How much aluminium is obtained by electrolysis if 100 kWh of electricity has been spent? The electrolysis is at a voltage of 6 V with an efficiency of 80%.

12.24. How long does it take for a copper anode 50 $\times$ 10 $\times$ 1 mm^3 in size to completely dissolve if a copper sulphate solution is electrolyzed at a current of 0.3 A?

12.25. Electrolytic nickel plating of an article is carried out at a current density of 0.8 A/dm^2. Determine the rate of growth of the nickel layer. The valency of nickel in the compound is 2.

12.26. During the electrolysis of weakly acidified water, 0.5 l of hydrogen are obtained at a pressure of 0.13 MPa over 50 min. Determine the temperature of the hydrogen if the current in the circuit is 1.6 A.

12.27. A silver nitrate solution was electrolyzed for 5 min, and the amount of silver deposited on the cathode was 336 mg. An ammeter in the circuit indicated 0.9 A. Was this reading correct or has a correction to be made?

12.28. Given the electrochemical equivalent of oxygen, determine the electrochemical equivalent of hydrogen, sodium, and magnesium.

12.29. How much electric charge passes through a silver nitrate solution in 20 s if the current increases from 1 to 4 A during the process? How much silver is deposited on the cathode?

12.30. Determine the electric power spent if a weakly acidified water is electrolyzed for 25 min liberating 0.5 g of oxygen. The resistance of the electrolyte is 1.8 Ω and does not change with time.

12.31. During an electrolysis of a silver nitrate solution, the current in the bath varied thus: $I = 0.2 + 6 \times 10^{-3}t$. How much silver was deposited on the cathode 300 s after the current started to vary?

12.32. At what current density does the thickness of the silver layer in a silver nitrate solution grow at a rate of 3×10^{-3} μm/s?

12.33. How many atoms of a monovalent metal are deposited per square metre of cathode surface if an electrolysis is carried out for 10 min at a current density of 5 A/m²?

12.34. During the electrolysis of a nickel sulphate solution, 2.19 g of nickel are deposited after 40 min on the surface of the cathode. Determine the polarization emf if the voltage across the bath terminals is 5 V and the resistance of the solution is 1.4 Ω.

§ 13. ELECTRIC CURRENT IN GASES AND IN VACUUM

Basic Concepts and Formulas

Under normal conditions, all gases are nonconducting. They either do not contain charge carriers or the number of carriers in them is small.

Heating or irradiating a gas ionizes it: its atoms lose a valency electron and become positive ions. Some of the electrons that escape the atoms recombine with neutral molecules to form negative ions. Thus, positive and negative ions and electrons become the mobile charge carriers in the gas.

In order to detach an electron from an atom, i.e. to ionize it, the following work must be done:

$$A_i = \varphi_i e,$$

where φ_i is the ionization potential, which is different for different gases. It should be kept in mind that $1\ \text{eV} = 1.6 \times 10^{-19}$ J.

For an impact ionization to emerge, an electron must acquire a kinetic energy greater than or equal to the ionization work

$$\frac{mv^2}{2} \geqslant A_i.$$

An electron may acquire this energy at the expense of the work done by electric field forces:

$$mv^2/2 = eE\lambda,$$

when it travels a distance λ between two collisions.

Positive ions also take part in ionization.

The passage of an electric current through a gas is known as an electric discharge. There are several types of gas discharge.

Conduction in vacuum is determined by the presence of a source of charged particles. For example, in a vacuum tube, electrons emitted from the surface of a heated cathode acquire a kinetic energy from the field, i.e.

$$mv^2/2 = Ue,$$

where U is the voltage between the electrodes of the tube and is known as the accelerating voltage.

The number of electrons emitted by the cathode depends on its temperature. The saturation current density is given by

$$j = env,$$

where n and v are the number density and velocity of the electrons respectively.

Worked Problems

Problem 72. Under normal conditions, a spark discharge in air occurs at a field strength of 3×10^6 V/m. Determine the energy required for an electron to ionize air molecules if the mean free path of an electron is 5 μm. What must the minimum velocity of an electron capable of ionizing air molecules be?

Given: $E = 3 \times 10^6$ V/m is the electric field strength and $\lambda = 5 \times 10^{-6}$ m is the mean free path of an electron. From tables, we find the electron charge $e = 1.6 \times 10^{-19}$ C and the electron mass $m = 9.11 \times 10^{-31}$ kg.

Find: the ionization energy W_i and the minimum velocity v required for the ionization.

Solution. The energy W_i required to ionize air molecules is received by an electron at the expense of the work done by electric field forces: $A = e(\varphi_1 - \varphi_2)$, where the potential difference can be expressed in terms of the electric field

strength: $\varphi_1 - \varphi_2 = E\lambda$. Then the ionization energy is

$$W_1 = A = eE\lambda,$$

$$W_1 = 1.6 \times 10^{-19}\ \mathrm{C} \times 3 \times 10^6\ \mathrm{V/m} \times 5 \times 10^{-6}\ \mathrm{m}$$

$$= 2.4 \times 10^{-18}\ \mathrm{J}.$$

The electron moving in the electric field acquires a kinetic energy $mv^2/2$. The molecules may ionize when the kinetic energy of the electron is higher than or equal to the ionization energy. To determine the minimum velocity, we can use the equality $mv^2/2 = W_1$, from which v can be determined:

$$v = \sqrt{\frac{2W_1}{m}}, \quad v = \sqrt{\frac{2 \times 2.4 \times 10^{-18}\ \mathrm{J}}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 2.3 \times 10^6\ \mathrm{m/s}.$$

Answer. The ionization energy is 2.4×10^{-18} J, and the velocity of the electrons is 2.3×10^6 m/s.

Problem 73. The separation between the electrodes in an ionization chamber is 6.2 cm, and the area of each electrode is 100 cm^2. An ionizer produces 10^9 pairs of ions per second in 1 cm^3 of the chamber. The ion pairs have a mobility of 3.29×10^{-4} m^2/(V·s). Assuming that the ions are monovalent, determine the saturation current and the field strength in the chamber.

Given: $S = 10^{-2}$ m^2 is the surface area of an electrode, $l = 6.2 \times 10^{-2}$ m is the separation between the electrodes, $n = 2 \times 10^{15}$ m^{-3} is the number density of the ions, $b = 3.29 \times 10^{-4}$ m^2/(V·s) is the mobility of the ions, and $t = 1$ s is the time of ionization. From tables, we find the electron charge $e = 1.6 \times 10^{-19}$ C.

Find: the saturation current I and the electric field strength E in the chamber.

Solution. The charge carriers in gases are ions and electrons. At the saturation current, all the charge carriers reach the electrodes. The saturation current can be determined from the formula $I = envS$, where $v = l/t$. This gives

$$I = en\frac{l}{t}S.$$

Using the formula $b = v/E$ for the mobility of charges, we determine the field strength:

$$E = \frac{v}{b} = \frac{l}{bt}.$$

Substituting in the numerical values, we obtain

$$I = 1.6 \times 10^{-19}\ \text{C} \times 2 \times 10^{-15}\ \text{m}^{-3} \frac{6.2 \times 10^{-2}\ \text{m}}{1\ \text{s}}\ 10^{-2}\ \text{m}^2 = 0.2\ \mu\text{A},$$

$$E = \frac{6.2 \times 10^{-2}\ \text{m}}{3.29 \times 10^{-4}\ \text{m}^2/(\text{V}\cdot\text{s}) \times 1\ \text{s}} = 188\ \text{V/m}.$$

Answer. The saturation current is 0.2 μA, and the electric field strength is 188 V/m.

Questions and Problems

13.1. Under the action of an ionizer, a gas has become a conductor. A charged electroscope placed nearby starts to discharge rapidly. Why does the discharge cease after the ionizer is removed?

13.2. Figure 56 shows a dependence of the current through a gas on the applied voltage. What processes correspond to different sections of the graph? Which section describes a self-sustained discharge?

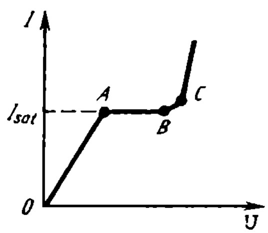


Fig. 56

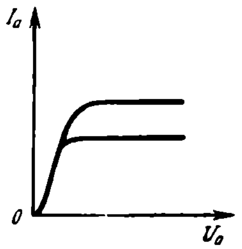


Fig. 57

13.3. What is the difference in the conduction of gases and solutions?

13.4. What should be done to increase the saturation current?

13.5. Is impact ionization possible at a low voltage if a gas is at atmospheric pressure?

13.6. How can you explain that spark discharges occur only intermittently?

13.7. Give examples when a corona discharge is (a) harmful and (b) beneficial.

13.8. What type of discharge is observed in day-light lamps? What particles are the charge carriers in this discharge?

13.9. Why does impact ionization in rarefied gases become more intense with decreasing voltage?

13.10. Why is the surface of the cathode in a vacuum tube coated with a thin layer of metal (say, barium or strontium)?

13.11. Figure 57 shows the anode current versus anode voltage for different temperatures of the filament of a diode. How can the presence of the horizontal lines in the graph be explained? What is the reason behind the increase in the saturation current at higher temperatures?

13.12. How can an electron beam be controlled? How is it controlled in the CRT tube of a television set?

13.13. A gas and a plasma are both electrically neutral as a whole. What is the difference between them?

13.14. Determine the ionization potential for silver atoms if an energy of 6.9×10^{-19} J is required to ionize them.

13.15. How will the velocity of the electrons in a CRT change if the energy of an electron changes from 700 to 1000 eV as a result of a change in the voltage between the anode and the cathode? What is the electron velocity in the two cases?

13.16. For obtaining cathode rays, a voltage of 30 kV is applied to the electrodes of a gas discharge tube. Determine the maximum velocity of the electrons in the cathode beam.

13.17. A potential difference of 300 V is applied between the cathode and the anode of a diode. Determine the velocity of the electrons as they move in the tube if the separation between the cathode and the anode is 10 mm. How long do they move?

13.18. What must the minimum velocity of electrons be to cause the impact ionization of cesium atoms, whose work function is 1.8 eV?

13.19. The ionization energy for a hydrogen atom is 13.5 eV. What must the minimum velocity of an electron be to cause the impact ionization of the hydrogen atom?

13.20. On average, five ion pairs are formed per second

per cm^3 of atmospheric air near the Earth's surface due to radioactivity of soil and cosmic radiation. The separation between two electrodes is 8 cm. Determine the saturation current density between the electrodes for singly charged ions.

13.21. The work function of barium oxide electrons is 1.0 eV. What must the mean free path of the electrons in a parallel-plate capacitor be for the electrons to ionize the barium atoms? The field strength between the plates is 3×10^5 V/m. The field should be treated as uniform.

13.22. The saturation current in an air-filled tube is 2×10^{-10} A at an electric field strength of 30 V/m between the electrodes. The overall mobility of monovalent ions is 3.29×10^{-4} $m^2/(V \cdot s)$, and the area of plane electrodes is 100 cm^2. Determine the number density of the ions.

13.23. Determine the overall mobility of hydrogen ions if the saturation current density is 2.8×10^{-10} A/m^2 at a field strength of 1.2 kV/m and a number density of ions of 10^4 cm^{-3}.

§ 14. ELECTRIC CURRENT IN SEMICONDUCTORS

Basic Concepts and Formulas

Semiconductors are materials having resistivity between that of conductors and insulators; their resistivity decreases with rising temperature and the presence of impurities. Typical semiconductors are elements of group IV of the Periodic Table, whose atoms have four valency electrons in the outer shell (like germanium Ge and silicon Si). At low temperatures, crystals of these elements do not contain free electrons and are good insulators. The covalent bonds in such crystals are ruptured with increasing temperature, illuminance, or due to strong electric fields. Free electrons appear, and intrinsic electron conduction emerges in (n-type) semiconductors. Intrinsic hole conduction of (p-type) semiconductors is due to the displacement of holes.

Impurity conduction in semiconductors is due to the presence of "alien" group V elements (like arsenic or antimony) or group III elements (like boron and aluminium). In the former case, impurity electron conduction is created, and in the latter, impurity hole conduction sets in. Thus, the

introduction of impurities into pure semiconductors can destroy the equilibrium between the p-type and n-type conductions.

When p- and n-type semiconductors are brought in contact, a barrier layer emerges as a result of diffusion through the p-n junction. The application of an external field to a p-n junction may change its conduction and create the conditions for unilateral conduction.

A semiconductor diode is a semiconductor with a p-n junction. Its advantages over vacuum tube diodes include small size, reliability, and efficiency.

Questions and Problems

14.1. Figure 58 shows the temperature dependences of resistance for conductors and semiconductors. Which of them corresponds to semiconductors?

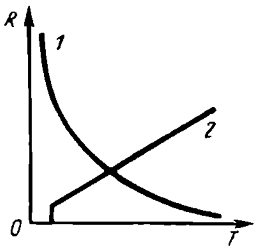


Fig. 58

14.2. What are the mobile charge carriers in a pure semiconductor?

14.3. What is the ratio between the number of holes and the number of free electrons in a pure semiconductor? Is this ratio preserved for impurity conduction of semiconductors?

14.4. What will the type of conduction in germanium with trace impurities of phosphorus or aluminium be?

14.5. How do the conductivities of germanium and silicon vary with lowering temperature?

14.6. By what factor will the current density in a semiconductor change if the velocity of the electrons increases from 0.5 to 0.75 m/s as a result of a temperature increase from 0 to 175°C, while the electron number density increases thereby from 1.3×10^{14} to 2.1×10^{18} m^{-3}?

14.7. The velocity of directional motion of free electrons in a semiconductor is 0.25 m/s for a given temperature. Determine the mobility of the charges and their number density if the current density is 4×10^{-2} A/m^2 for a field strength of 100 V/m.

14.8. What is a thermistor? Why are thermistors called nonlinear resistances?

14.9. What is the difference between a thermistor and a photoresistor?

14.10. What is a transistor? Which regions does the crystal of a transistor contain?

14.11. The thickness of the base in a transistor is very small (1-25 μm). Why?

14.12. What is the ratio between the emitter, base, and collector currents?

14.13. What is the advantage of semiconductor devices over vacuum tubes in radio engineering?

§ 15. ELECTROMAGNETISM

Basic Concepts and Formulas

A magnetic field is a special case of electromagnetic field, being characterized by the action on a moving charged particle of a force proportional to the charge of the particle and its velocity.

The shape of a magnetic field depends on the shape of the current-carrying conductor producing it. For example, the magnetic field formed around a straight current-carrying conductor is graphically represented by magnetic field lines in the form of concentric rings in the plane perpendicular to the direction of the current (Fig. 59). The direction of the magnetic field in this case is given by Ampère's (right-hand screw) rule: rotation of the head of a screw indicates the direction of the magnetic field lines if the motion of the screw body coincides with the direction of the current.

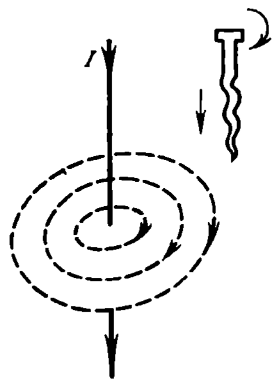


Fig. 59

The magnetic field of a current-carrying coil (solenoid) is similar to the magnetic field of a permanent bar magnet.

The interaction between magnetic fields formed by parallel current-carrying conductors is determined by the

formula

$$F = \mu_m \frac{I_1 I_2 l}{2\pi a},$$

where I_1 and I_2 are the currents in the conductors, l is the length of wire over which the force is acting, a is the separation between the conductors, and μ_m is the absolute permeability of the medium characterizing the dependence of the force of interaction of current-carrying conductors on the properties of the medium:

$$\mu_m = \mu_0 \mu.$$

Here $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic constant, and μ is the relative permeability of the medium (see Table 21).

A magnetic field exerts a force F_A (Ampère's law) on a current-carrying conductor:

$$F_A = BIl \sin \alpha.$$

If the conductor is perpendicular to magnetic field lines ($\alpha = 90°$), we have

$$F_A = BIl.$$

The proportionality factor B is called the magnetic induction and is the force characteristic of the magnetic field. Magnetic induction is a vector quantity. The SI unit of magnetic induction is the tesla (T). At any point in a uniform magnetic field, the magnetic induction has the same magnitude and direction. Therefore, such a field is graphically represented by parallel straight lines with uniform density.

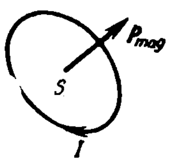


Fig. 60

The magnetic flux is equal to the number of magnetic field lines piercing a surface of area S if the magnetic induction vector coincides with the normal to this surface:

$$\Phi = BS.$$

The SI unit of magnetic flux is the weber (Wb).

The magnetic properties of a current loop are characterized by the magnetic moment P_{mag} (Fig. 60):

$$P_{mag} = IS.$$

A current-carrying loop in a uniform magnetic field of induction B experiences a rotational magnetic moment (torque) M:

$$M = ISB \sin \alpha,$$

where α is the angle between vectors $\mathbf{B}$ and $\mathbf{P}_{mag}$. For $\alpha = 90°$, the torque will have the maximum value

$$M_{max} = ISB = P_{mag}B.$$

A closed current-carrying loop of length l is displaced by a force F_A through a distance b. Consequently, the following work is done:

$$A = F_A b = BIlb.$$

But lb is the change in the area, ΔS, so

$$A = BI\,\Delta S, \quad \text{or} \quad A = I\,\Delta\Phi.$$

The magnetic induction of a straight current-carrying conductor is

$$B = \mu_m \frac{I}{2\pi r},$$

where r is the shortest distance from the current-carrying conductor to the point at which the induction B is being determined.

The magnetic induction of the field produced by a circular current is

$$B = \mu_m \frac{I}{2r}.$$

The magnetic induction of the field in a solenoid is

$$B = \mu_m \frac{I\omega}{l},$$

where ω is the number of turns and l is the length of the solenoid.

The magnetic field inside a long solenoid is uniform. Therefore, the magnetic flux in the solenoid is

$$\Phi = BS = \mu_m \frac{I\omega}{l} S.$$

The magnetic field is also characterized by the magnetic field strength H which is related with the magnetic induction

through the formula

$$B = \mu_m H.$$

The force acting on an electric charge Q moving in a magnetic field is known as the Lorentz force F_L:

$$F_L = BvQ \sin \alpha,$$

where α is the angle between vectors **B** and **v**. The Lorentz force is always perpendicular to the plane containing vectors **B** and **v** and hence does no work. Without changing the magnitude of the velocity of the charge, it only changes its direction and is responsible for centripetal acceleration. If $\alpha = 90°$, we have

$$BvQ = \frac{mv^2}{r},$$

i.e. a particle of mass m and charge Q moves in a circle under the action of force F_L.

Worked Problems

Problem 74. A direct current passes in the same direction through two parallel wires separated by a distance of 30 cm. The distance between the supports to which the wires are fixed is 50 m. The current in the wires is 150 A. Determine the magnitude and direction of the force with which the wires interact.

Given: $I_1 = I_2 = I = 150$ A is the current in each wire, $a = 0.3$ m is the separation between the wires, and $l = 50$ m is the distance between the supports. From tables, we find the magnetic constant $\mu_0 = 4\pi \times 10^{-7}$ H/m and the permeability of air, $\mu = 1$.

Find: the force F between the wires (its magnitude and direction).

Solution. Let us consider the distance between the supports to be the active length of the wires. Noting that $l \gg a$, we can assume that the wires are infinitely long so that the following formula is applicable for the force of interaction:

$$F = \mu_0 \mu \frac{I^2 l}{2\pi a},$$

$$F = \frac{4\pi \times 10^{-7} \text{ H/m} \times 1 (150 \text{ A})^2 \, 50 \text{ m}}{2\pi \times 0.3 \text{ m}} = 0.75 \text{ H}.$$

In order to determine the direction of the force acting between the wires, we shall analyze Fig. 61. The magnetic field between the wires is weaker (the magnetic field lines are directed against one another). In the outer region, the magnetic field lines have the same direction, and the magnetic field is larger. Consequently, wires carrying current in the same direction must attract each other.

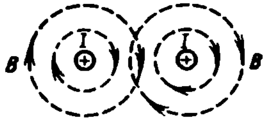


Fig. 61

Answer. The force of attraction between the wires is 0.75 N.

Problem 75. When a 0.5-m long straight conductor carrying a current of 4 A is at right angles to a uniform magnetic field, the field acts on it with a force of 2.8 N. What will the force exerted by the same field on the conductor be if the angle between them is 30°?

Given: $l = 0.5$ m is the length of the conductor, $I = 4$ A is the current in it, $F_1 = 2.8$ N is the force corresponding to 90°, and $\alpha_1 = 90°$ and $\alpha_2 = 30°$ are the angles between the magnetic field lines and the current in the first and the second case.

Find: the force F_2 acting on the conductor.

Solution. Using Ampère's law $F_1 = BIl \sin \alpha_1$, we determine the magnetic induction of the uniform magnetic field:

$$B = \frac{F_1}{Il \sin \alpha_1}, \quad B = \frac{2.8 \text{ H}}{4 \text{ A} \times 0.5 \text{ m} \times 1} = 1.4 \text{ T}.$$

Knowing the magnetic induction, we can determine the force exerted by the magnetic field on the current-carrying conductor when the angle between them is 30°:

$$F_2 = BIl \sin \alpha_2, \quad F_2 = 1.4 \text{ T} \times 4 \text{ A} \times 0.5 \text{ m} \times 0.5 = 1.4 \text{ N}.$$

Answer. The force acting on the conductor in the second case is 1.4 N.

Problem 76. A 0.8-m long straight conductor moves in a magnetic field of induction 5×10^{-2} T. The current in the conductor is 15 A. The conductor is at an angle of 30° with the magnetic induction vector. Find the force acting on the conductor and the work done by the magnetic field to move the conductor 1.8 m.

Given: $B = 5 \times 10^{-2}$ T is the magnetic induction of the field, $l = 0.8$ m is the length of the conductor, $I = 15$ A

is the current in the conductor, $\alpha = 30°$ is the angle between the direction of current I and vector $\mathbf{B}$, and $b = 1.8$ m is the distance by which the conductor is moved.

Find: the force F acting on the conductor and the work A done by the magnetic field to move the conductor.

Solution. A current-carrying conductor in a magnetic field is acted upon by the force

$$F = BIl \sin \alpha,$$

$$F = 5 \times 10^{-2}\ \text{T} \times 15\ \text{A} \times 0.8\ \text{m} \times 0.5 = 0.3\ \text{N}.$$

Knowing the force acting on the conductor and the distance by which it is moved, we can find the work:

$$A = Fb, \quad A = 0.3\ \text{N} \times 1.8\ \text{m} = 0.54\ \text{J}.$$

Answer. The magnetic field acts on the current-carrying conductor with a force of 0.3 N and does a work of 0.54 J to displace it.

Problem 77. A 30-cm long solenoid contains 3000 turns. The diameter of each turn is 11 cm. Determine the magnetic induction of the field inside the solenoid for a current of 1.5 A and the magnetic flux piercing each turn. What will the change in the magnetic induction and in the magnetic flux be if a carbon iron core is inserted into the solenoid?

Given: $l = 0.3$ m is the length of the solenoid, $N = 3000$ is the number of turns in it, $d = 11 \times 10^{-2}$ m is the diameter of a turn, $I = 1.5$ A is the current; from tables, we find the magnetic constant $\mu_0 = 4\pi \times 10^{-7}$ H/m and the permeability of carbon iron, $\mu = 3000$.

Find: the magnetic induction B_1 of the field in the solenoid without a core, the magnetic induction B_2 of the field in the solenoid with the core, and the corresponding magnetic fluxes Φ_1 and Φ_2.

Solution. Assuming that the solenoid is long enough for the magnetic field in it to be uniform and directed along the axis, the magnetic induction can be determined from the formula

$$B = \mu_0 \mu \frac{IN}{l}.$$

In the absence of a core, $\mu = 1$ (for air). Then we have

$$B_1 = 4\pi \times 10^{-7}\ \text{H/m} \frac{1.5\ \text{A} \times 3000}{0.3\ \text{m}} = 1.9 \times 10^{-2}\ \text{T}.$$

In the presence of the core, the magnetic induction increases by a factor of μ:

$$B_2 = 3000 \times 1.9 \times 10^{-2}\ \mathrm{T} = 57\ \mathrm{T}.$$

The magnetic flux can be determined from the formula

$$\Phi = BS \cos \alpha.$$

In the problem under consideration, $\alpha = 0$ and $\cos \alpha = 1$, while $S = \pi d^2/4$. This gives

$$\Phi_1 = B \frac{\pi d^2}{4},$$

$$\Phi_1 = \frac{1.9 \times 10^{-2}\ \mathrm{T} \times 3.14 \times 121 \times 10^{-4}\ \mathrm{m}^2}{4} = 1.8 \times 10^{-4}\ \mathrm{Wb}.$$

In the presence of the core,

$$\Phi_2 = 3000 \times 1.8 \times 10^{-4}\ \mathrm{Wb} = 0.54\ \mathrm{Wb}.$$

Answer. The magnetic field induction in the solenoid without a core is 1.9×10^{-2} T and 57 T with the core. The magnetic fluxes in the solenoid are 1.8×10^{-4} and 0.54 Wb respectively.

Problem 78. A current-carrying conductor is looped into a circle of radius 10 cm. The magnetic moment of the current loop becomes 0.314 A·m². Determine the current in the loop and the maximum torque exerted on it by a uniform magnetic field of induction 5×10^{-3} T.

Given: $r = 0.1$ m is the radius of the loop, $p_{mag} = 0.314$ A·m² is the magnetic moment of the loop, and $B = 5 \times 10^{-3}$ T is the magnetic induction of the field.

Find: the current I in the loop and the maximum torque M_{max}.

Solution. Knowing the magnetic moment of the loop, we can determine the current in it: $p_{mag} = IS$, where $S = \pi r^2$, which gives $p_{mag} = I\pi r^2$, and hence

$$I = \frac{p_{mag}}{\pi r^2}, \quad I = \frac{0.314\ \mathrm{A \cdot m^2}}{3.14 \times 10^{-2}\ \mathrm{m^2}} = 10\ \mathrm{A}.$$

The maximum torque can be determined from the formula

$$M_{max} = p_{mag} B,$$

$$M_{max} = 0.314\ \mathrm{A \cdot m^2} \times 5 \times 10^{-3}\ \mathrm{T} \simeq 1.6 \times 10^{-3}\ \mathrm{N \cdot m}.$$

Answer. The current in the loop is 10 A, and the maximum torque is approximately 1.6×10^{-3} N·m.

Problem 79. A proton that has acquired a velocity while moving across a potential difference of 1 kV enters a uniform magnetic field of induction 0.2 T at right angles to the magnetic field lines. Determine the radius of the circle in which it will move and the period of its revolution.

Given: $U = 1000$ V is the accelerating potential difference, $B = 0.2$ T is the magnetic induction of the field, $\alpha = 90°$ is the angle between vectors **B** and **v**. From tables, we find the mass of the proton $m_p = 1.67 \times 10^{-27}$ kg and its charge $Q = 1.6 \times 10^{-19}$ C.

Find: the radius r of the circle and the period T of revolution of the proton.

Solution. An electric charge (in our case, the proton) moving in a magnetic field is acted upon by the Lorentz force $F_L = BvQ \sin \alpha$, where α is the angle between vectors **B** and v. Since $\alpha = 90°$ and $\sin \alpha = 1$, we obtain $F_L = BvQ$. Since the Lorentz force is always normal to the plane containing vectors **B** and **v**, it does no work, i.e. it does not change the kinetic energy of the moving charge. The force only changes the direction of the velocity. Hence we can write $BvQ = m_p v^2/r$, whence

$$r = \frac{m_p v}{BQ}.$$

In order to find the proton velocity, we can use the energy conservation law: the work done by the electric field U is equal to the kinetic energy acquired by the proton: $QU = m_p v^2/2$, whence

$$v = \sqrt{\frac{2QU}{m_p}}, \quad v = \sqrt{\frac{2 \times 1.6 \times 10^{-19}\ \text{C} \times 10^3\ \text{V}}{1.67 \times 10^{-27}\ \text{kg}}} = 4.4 \times 10^5\ \text{m/s}.$$

We can now determine the radius r of the circle:

$$r = \frac{1.67 \times 10^{-27}\ \text{kg} \times 4.4 \times 10^5\ \text{m/s}}{0.2\ \text{T} \times 1.6 \times 10^{-19}\ \text{C}} \simeq 0.023\ \text{m}.$$

Given the proton velocity and the radius of its orbit, we can find its period:

$$T = \frac{2\pi r}{v},$$

$$T = \frac{2 \times 3.14 \times 0.023\ \text{m}}{4.4 \times 10^5\ \text{m/s}} \simeq 0.033 \times 10^{-5}\ \text{s} \simeq 0.3\ \mu\text{s}.$$

Answer. The proton moves in a circle of radius 0.023 m with period of revolution 0.3 μs.

Questions and Problems

15.1. A looped flexible conductor tends to expand into a circle when a current is passed through it. Why?

15.2. Using a d.c. voltmeter and a magnetic needle on a pivot, one can determine the side on which a generator is located in a two-wire d.c. cable. How?

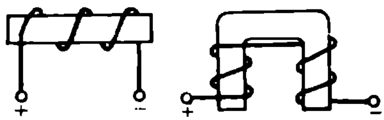


Fig. 62

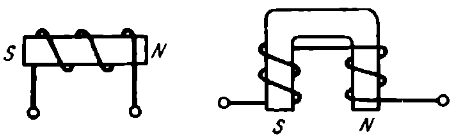


Fig. 63

15.3. Determine the polarity of the electromagnets shown in Fig. 62.

15.4. Determine the direction of the currents in the windings shown in Fig. 63.

15.5. Determine the polarity and the direction of the magnetic field lines for the generator shown in Fig. 64.

15.6. Determine the direction of motion of current-carrying conductors in the magnetic fields shown in Fig. 65.

15.7. What will happen to a permanent bar magnet if the current in the solenoid has the direction indicated in Fig. 66?

15.8. What will the motion of the coil be relative to the solenoid for the direction of the current indicated in Fig. 67?

15.9. What must the polarity of the magnet in Fig. 68 be for the current loop to rotate clockwise? Through what angle will the loop be turned?

15.10. What is the force of the interaction between the wires of a d.c. trolleybus line over 30 m if the separation between the wires is 520 mm and the current in them is 200 A?

15.11. A d.c. transmission line is rated for a current of 150 A. What is the separation between two wires if they

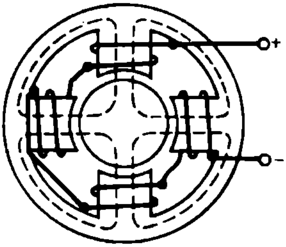


Fig. 64

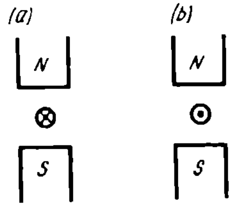


Fig. 65

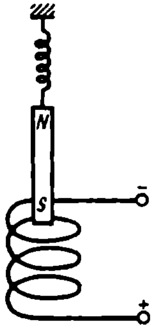


Fig. 66

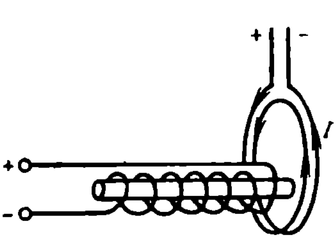


Fig. 67

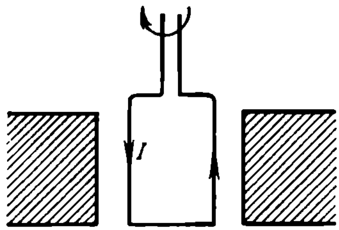


Fig. 68

interact with a force of 2.8×10^{-1} N over a distance of 50 m?

15.12. Two parallel conductors carrying the same current are 25 cm apart. Determine the current in each wire if it experiences a force of 2 mN per metre.

15.13. Two parallel wires are 6 cm apart. The currents in the wires are 15 and 10 A. Over what segment of the wires will a force of 1.4 mN act?

15.14. A 0.5-m long straight conductor carrying a current of 5 A is in a uniform magnetic field of induction 0.16 T.

Determine the forces acting on the conductor when the angles between the direction of current in it and the magnetic induction vector are 90°, 30°, and 0°.

15.15. Determine the magnetic induction of a uniform magnetic field in which a 0.7-m long straight conductor carrying a current of 10 A is acted upon by a force of 42 mN. The angle between the direction of current and the magnetic induction vector is 30°.

15.16. Using a strong horse-shoe magnet, one can determine whether an incandescent lamp is connected to an a.c. or a d.c. source. How can this be done?

15.17. A force of 0.6 N acts on a 1-m long straight conductor placed in a uniform magnetic field at right angles to the magnetic field lines. The current in the conductor is 12 A. What will the force exerted on this conductor be if the angle between the direction of current in it and the magnetic induction vector is 45°? Solve the problem using two different methods.

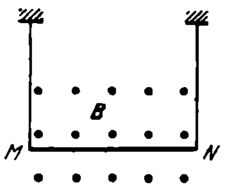


Fig. 69

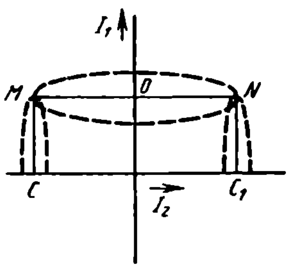


Fig. 70

15.18. A conductor MN made of a material with density ρ and having cross section S is suspended on two weightless unstretchable strings in a uniform magnetic field of induction B (Fig. 69). At what current will the tension in the string be zero? What must the direction of this current be?

15.19. The magnetic field strength 10 cm from a long current-carrying straight conductor is 20 A/m. What force will act on each metre of this wire if it is placed in a uniform magnetic field with induction 2.5 T so that the angle between the direction of the current and the magnetic induction vector is 30°?

15.20. Determine the current that must be passed through

a long straight wire to produce a magnetic field of the same magnitude 1 m away as the magnetic field of the Earth near its surface. The magnetic induction of the magnetic field of the Earth should be taken to be 5.5×10^{-5} T.

15.21. Determine the magnetic field strength and the magnetic induction of the field produced by a straight conductor carrying a current of 7.8 A at a point 4.8 cm away.

15.22. A straight conductor carrying a current of 10 A produces at a certain point a magnetic field of strength 40 A/m. Determine the magnetic induction at this point and the distance between this point and the conductor. K

15.23. Two long current-carrying conductors are arranged at right angles in the same plane (Fig. 70). Determine the resultant magnetic induction of the field at points M and N if $I_1 = 10$ A, $I_2 = 6$ A, $MO = NO = 5$ cm, and $MC = NC_1 = 4$ cm.

15.24. A current-carrying circular conductor produces a magnetic field of strength 25 A/m at the centre. Determine the magnetic induction of the field and the radius of the loop if the current is 3.45 A.

15.25. The induction at the centre of a circular current of radius 4 cm is 1.57×10^{-4} T. Determine the magnetic field strength at the centre of the loop and the current.

15.26. The current passing through a straight long conductor produces a magnetic field of induction 0.8×10^{-4} T 4.4 cm away. The permeability of the medium is 1.1. Determine the current in the conductor and the magnetic field strength 16 cm away.

15.27. Determine the magnetic moment of a circular loop carrying a current of 10 A and having a radius of 6.0 cm.

15.28. The magnetic moment of a current-carrying wire ring 15 cm in diameter is 4.2×10^{-2} A$\cdot$m^2. Determine the current in the ring and the magnetic field strength at its centre.

15.29. A wire ring of radius 5.0 cm carrying a current of 6×10^{-2} A is in a uniform magnetic field of induction 1.2×10^{-2} T. The plane of the ring is parallel to the magnetic field lines. Determine the maximum magnetic moment exerted by the magnetic field on the ring.

15.30. A solenoid without a core is 100 cm long and contains 600 turns. Determine the magnetic induction of the field in the solenoid for a current of 0.4 A.

15.31. The current in a solenoid whose diameter is small in comparison with its length is 6.5 A. The solenoid is 65 cm long and contains 750 turns. Determine the magnetic field strength and the magnetic induction in the solenoid without a core.

15.32. The magnetic induction of a very long solenoid carrying a current of 3.0 A is 2.52×10^{-3} T. The solenoid is wound tightly in one row and has no core. Determine the diameter of the wire of which the solenoid is made.

15.33. Determine the permeability of soft steel in two magnetic fields with strengths 1.5 and 5.0 kA/m respectively (see graph 22). How does the permeability change with increasing magnetic field strength during primary magnetization? Why?

15.34. Determine the magnetic induction in a nickel core and the magnetic flux if the strength of the uniform magnetic field in the core is 2.0×10^3 A/m, the cross-sectional area of the core is 30 cm^2, and the permeability is 200.

15.35. The winding of a solenoid without a core is made of a wire whose diameter is 1.0×10^{-3} m. The turns have a radius of 1.0 cm and are wound tightly. Determine the magnetic flux in the solenoid for a current of 2.0 A.

15.36. A rectangular frame made of a wire carrying a current of 2.0 A is in a uniform magnetic field of induction 0.10 T. The size of the frame is 4×5 cm^2. In a certain position, the magnetic flux through the frame is 0.80×10^{-4} Wb. Determine the maximum magnetic flux in the frame when its plane is perpendicular to the magnetic field lines. Determine the work done in rotating the frame to this position.

15.37. The magnetic induction in the core of an electromagnet is 1.2 T and its cross-sectional area is 0.12 m^2. Determine the magnetic flux through the core.

15.38. A steel rod having a cross-sectional area of 4.5 cm^2 and a permeability of 160 is placed in a uniform magnetic field of strength 7970 A/m so that the magnetic field lines coincide with the normal to the cross section of the rod. Determine the magnetic flux piercing the rod.

15.39. The magnetic field strength in a solenoid with an iron core is 1600 A/m. The cross-sectional area of the core is 10 cm^2. Determine the magnetic induction of the field and

the permeability of iron if the magnetic flux through the core is 2×10^{-4} Wb.

15.40. An electron flies into a uniform magnetic field as shown in Fig. 71. Determine the direction of the force acting on the electron at the initial moment. What will its trajectory be?

15.41. An electron moves in a uniform magnetic field at a velocity of 1.0×10^4 km/s at right angles to the magnetic induction vector. Determine the force acting on the electron for a magnetic field strength of 150 A/m.

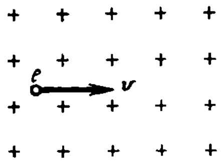


Fig. 71

15.42. An electron flies into a uniform magnetic field with induction 9.1×10^{-5} T. The electron velocity is 1.9×10^7 m/s and at right angles to the magnetic induction vector. Determine the radius of the circle in which the electron will move, the period, and frequency of its revolution.

15.43. The Lorentz force exerted on an electron by crossed electric and magnetic fields is determined by the formula $F_L = eE + evB$. What must the direction and magnitude of the electron velocity be for it to move uniformly in a straight line?

15.44. An electron flies into a uniform magnetic field with induction 2.5×10^{-3} T and moves in a circle of radius 40 cm. The electron velocity vector forms an angle of 90° with the direction of the magnetic field. Determine the kinetic energy of the electron.

15.45. An electron having a velocity of 8.8×10^7 m/s flies into a uniform magnetic field of induction 6.28×10^{-2} T. The angle between the velocity and magnetic induction vectors is 30°. Determine the radius and the lead of the helical trajectory of the electron. Use the charge-to-mass ratio of the electron to the third significant digit. K

15.46. Two identical, singly charged ions fly at different velocities into a uniform magnetic field. What will the periods of their revolution be?

15.47. An electron and a singly charged ion fly at the same velocity into a uniform magnetic field. What will the periods of their revolution be?

§ 16. ELECTROMAGNETIC INDUCTION

Basic Concepts and Formulas

Electromagnetic induction consists in the emergence of an induced emf and induced current in a closed loop if the magnetic flux bounded by this loop varies with time:

$$\mathscr{E}_1 = -\frac{\Delta\Phi}{\Delta t}.$$

Let us suppose that a rectangular frame lying in a plane perpendicular to a magnetic field moves at a velocity v and leaves the magnetic field. Then the magnetic flux piercing the frame varies as

$$\Delta\Phi = -Blv\,\Delta t.$$

Therefore, the induced emf can be written in the form

$$\mathscr{E}_1 = -Blv.$$

If vectors v and **B** form an angle α, we have $\mathscr{E}_1 = -Blv \sin\alpha$.

An emf can also be induced in a stationary loop if the magnetic induction of the field varies over time.

The direction of the current induced in a closed loop can be determined using the right-hand rule or Lenz's law. According to Lenz's law, the direction of the induced current is such that the magnetic field of this current opposes any change in the magnetic field inducing the current.

Self-inductance can be regarded as a special case of electromagnetic induction when a changing magnetic flux is produced by a current as it varies. For example, when a circuit is closed, the self-inductance emf opposes (in accordance with Lenz's law) the increase in the current. When a circuit is disconnected, the self-inductance emf opposes the decrease in the current. For this reason, the induced current is directed against the main current when the circuit is connected and along the main current, when the circuit is disconnected. The self-inductance emf is proportional to the rate of change in the current in the circuit:

$$\mathscr{E}_s = -L\frac{\Delta I}{\Delta t},$$

where L is the inductance of the circuit and is determined by the size and shape of the conductor in it and by the mag-

netic properties of the medium containing the circuit. The SI unit of inductance is the henry (H).

Since it is a component of electromagnetic field, the magnetic field has energy which is given by

$$W = \frac{1}{2}\Phi I = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\Phi^2}{L}.$$

Worked Problems

Problem 80. A conductor AB is 0.6 m long and has a resistance of 0.2 Ω. It can move along a copper busbars CD connected to a current source with an emf of 0.96 V and an internal resistance of 0.1 Ω (Fig. 72). The resistance of the busbars is negligibly small. Determine the current in the conductor if it (a) is at rest, (b) moves at a velocity of 0.5 m/s in a uniform magnetic field of induction 1.6 T directed along the normal to the plane of the figure away from us.

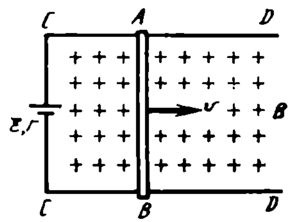


Fig. 72

Given: $l=0.6$ m is the length of the conductor, $R = 0.2$ Ω is its resistance, $\mathscr{E} = 0.96$ V is the emf of the current source, $r = 0.1$ Ω is its internal resistance, $v = 0.5$ m/s is the velocity of the conductor, and $B = 1.6$ T is the magnetic induction of the field.

Find: (a) the current I_1 in the stationary conductor and (b) the current I_2 in the conductor moving in the magnetic field.

Solution. (a) If the conductor is at rest, the current in it is determined from Ohm's law for a circuit:

$$I_1 = \frac{\mathscr{E}}{R+r}, \quad I_1 = \frac{0.96 \text{ V}}{0.2 \ \Omega + 0.1 \ \Omega} = 3.2 \text{ A}.$$

(b) If the conductor moves at a velocity v in a uniform magnetic field, an emf $\mathscr{E}_i$ is induced in it. If the conductor is a part of a closed circuit, then a current is induced in it with a direction determined by the right-hand rule. In the problem under consideration, the induced current is directed

against the current I_1. Ohm's law in this case has the form

$$I_2 = \frac{\mathscr{E} - \mathscr{E}_1}{R + r}.$$

Since $\mathscr{E}_1 = Blv$, we obtain

$$I_2 = \frac{\mathscr{E} - Blv}{R + r}.$$

Here we must note that the magnetic induction vector and the velocity vector are at right angles to each other. Hence,

$$I_2 = \frac{0.96\ \text{V} - 1.6\ \text{T} \times 0.6\ \text{m} \times 0.5\ \text{m/s}}{0.2\ \Omega + 0.1\ \Omega} = 1.6\ \text{A}.$$

Answer. The current is 3.2 A in the stationary conductor, and 1.6 A when the conductor moves in the magnetic field.

Problem 81. A circular coil 10 cm in diameter is placed in a uniform magnetic field with induction 0.12 T so that the magnetic induction vector is normal to the plane of the coil (Fig. 73). Determine the number of turns in the coil if an emf of 0.942 V is induced in it as a result of its rotation through 180° in 0.14 s.

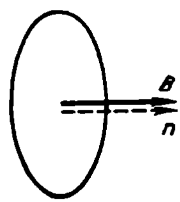


Fig. 73

Given: $B = 0.12$ T is the magnetic induction of the field, $d = 0.1$ m is the diameter of a turn, $\alpha = 180°$ is the angle of rotation of the coil, $\Delta t = 0.14$ s is the time interval, and $\mathscr{E}_1 = 0.942$ V is the induced emf.

Find: the number N of turns in the coil.

Solution. When the coil is rotated, the magnetic flux bounded by the contour of the coil changes, and the emf induced in it is

$$\mathscr{E}_1 = -\frac{\Delta\Phi}{\Delta t} N.$$

The problem statement indicates that before the coil is rotated the angle between its normal and the magnetic induction is $\alpha_0 = 0$, and the magnetic flux through the coil is $\Phi_1 = BS \cos \alpha_0$. After the rotation, the magnetic flux becomes $\Phi_2 = BS \cos \alpha$, where $\alpha = 180°$. The change in the magnetic flux is $\Phi_2 - \Phi_1 = \Delta\Phi$, $\Delta\Phi = BS \cos 180° - BS \cos 0°$, $\cos 180° = -1$ and $\cos 0° = 1$. Therefore, $\Delta\Phi =$

$-BS - BS = -2BS$. Since $S = \pi d^2/4$, we obtain

$$\Delta\Phi = -\frac{B\pi d^2}{2}.$$

Substituting the change in the magnetic flux into the expression for emf, we obtain $\mathcal{E}_1 = \frac{B\pi d^2}{2\Delta t} N$. Then the number of turns is

$$N = \frac{2\mathcal{E}_1 \Delta t}{B\pi d^2}, \quad N = \frac{2 \times 0.942 \text{ V} \times 0.14 \text{ s}}{0.12 \text{ T} \times 3.14 \times 10^{-2} \text{ m}^2} = 70.$$

Answer. The number of turns is 70.

Problem 82. Determine the inductance of a coil in which the self-inductance emf induced by a change in the current from 5 to 10 A in 0.1 s is 10 V. What is the change in the energy of the magnetic field of the coil in this case?

Given: $I_1 = 5$ A is the initial current in the coil, $I_2 = 10$ A is the current after time Δt, $\Delta t = 0.1$ s is the time during which the current changes, and $\mathcal{E}_s = 10$ V is the self-inductance emf.

Find: the inductance L in the coil and the change ΔW in the magnetic field energy of the coil.

Solution. We write the formula for the self-inductance emf $\mathcal{E}_s = -L\frac{\Delta I}{\Delta t}$ and then determine the inductance of the coil:

$$L = \frac{\mathcal{E}_s \Delta t}{\Delta I} = \frac{\mathcal{E}_s \Delta t}{I_2 - I_1}, \quad L = \frac{10 \text{ V} \times 0.1 \text{ s}}{10 \text{ A} - 5 \text{ A}} = 0.2 \text{ H}.$$

The magnetic field energies for currents I_1 and I_2 are

$$W_1 = \frac{LI_1^2}{2}, \quad W_2 = \frac{LI_2^2}{2}.$$

The change in the energy is then

$$\Delta W = W_2 - W_1 = \frac{LI_2^2}{2} - \frac{LI_1^2}{2} = \frac{L}{2}(I_2^2 - I_1^2),$$

$$\Delta W = \frac{0.2 \text{ H}}{2}[(10 \text{ A})^2 - (5 \text{ A})^2] = 7.5 \text{ J}.$$

Answer. The inductance of the coil is 0.2 H, the energy of the magnetic field increases by 7.5 J with current.

Problem 83. A solenoid with a nickel core has 1000 turns on 0.5 m, the cross-sectional area of a turn being 50 cm^2. Determine the magnetic flux in the solenoid and the magnet-

ic field energy if the current in the solenoid is 10 A and the permeability of nickel is 200.

Given: $l = 0.5$ m is the length of the solenoid, $N = 1000$ is the number of turns, $S = 5 \times 10^{-3}$ m^2 is the cross-sectional area of a turn, $I = 10$ A is the current, $\mu = 200$ is the permeability of nickel. From tables, we find the magnetic constant $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Find: The magnetic flux Φ in the solenoid and the energy W of the magnetic field of the solenoid.

Solution. The magnetic flux can be calculated from the formula $\Phi = BS$. Since the magnetic induction of the field of a solenoid with a core is $B = \mu_0\mu \frac{IN}{l}$, we have

$$\Phi = \mu_0\mu \frac{IN}{l} S,$$

$$\Phi = 4\pi \times 10^{-7}\ \text{H/m} \times 200 \frac{10\ \text{A} \times 1000}{0.5\ \text{m}} 5 \times 10^{-3}\ \text{m}^2$$
$$= 2.5 \times 10^{-2}\ \text{Wb}.$$

Given the magnetic flux and the current, we can determine the energy of the magnetic field in the solenoid:

$$W = \frac{1}{2}\Phi I, \quad W = \frac{1}{2} \times 2.5 \times 10^{-2}\ \text{Wb} \times 10\ \text{A} \simeq 1.3 \times 10^{-1}\ \text{J}.$$

Answer. The magnetic flux in the solenoid is 2.5×10^{-2} Wb, and the energy of the magnetic field in the solenoid is 0.13 J.

Questions and Problems

16.1. Will an emf be induced in the conductors moving as shown in Fig. 74?

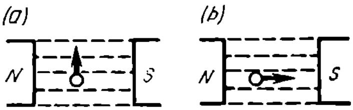


Fig. 74

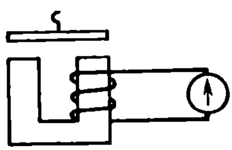


Fig. 75

16.2. When the poles of a horse-shoe magnet are closed with an armature (Fig. 75), the pointer of the galvanometer is deflected. Why?

16.3. Determine the direction of the emf induced in conductors moving in uniform magnetic fields shown in Fig. 76.

16.4. A rectangular frame in a translatory motion (a) enters a uniform magnetic field, (b) moves in it, (c) leaves

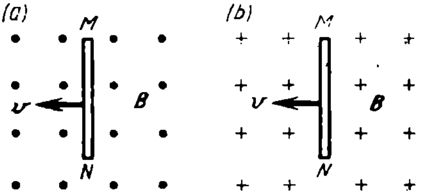


Fig. 76

the field. The normal to the frame plane is directed along the induction line. Is an emf induced in the frame under these conditions? Why?

16.5. A rectangular frame rotates in a uniform magnetic field about an axis parallel to the magnetic field lines. Is an emf induced in this case?

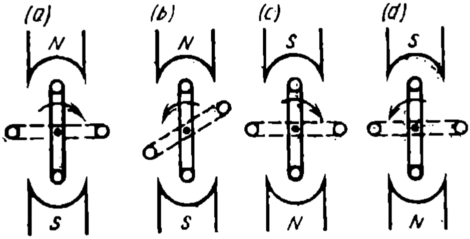


Fig. 77

16.6. Determine the direction of the current induced in a frame (Fig. 77) rotating in a uniform magnetic field in the direction indicated by the arrow.

16.7. Two identical steel rods are brought close to a copper ring suspended in the vertical plane. In one case, the ring is repelled from the rod. Why?

16.8. Determine the direction of the current induced in a solenoid if a bar magnet is moved upwards in it (Fig. 78).

16.9. A bar magnet is caused to approach a copper ring as shown in Fig. 79. Determine the direction of the current induced in the ring.

16.10. A permanent bar magnet is falling through a copper cylinder. Will the motion be free fall?

16.11. A conductor that has been folded in two moves in a uniform magnetic field crossing magnetic field lines. Is an emf induced in the conductor? What would a galvanometer attached to the ends of the moving conductor indicate?

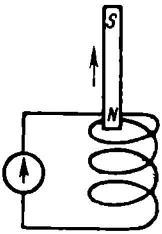


Fig. 78

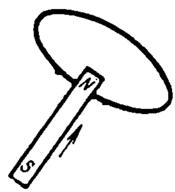


Fig. 79

16.12. What is the value of the emf induced in a circuit if the magnetic flux through it varies by 3.4×10^{-2} Wb/s?

16.13. A conductor of active length 15 cm moves at 10 m/s at right angles to the lines of a uniform magnetic field with induction 2.0 T. What current is induced in the conductor if it is short-circuited? The resistance of the circuit is 0.5 Ω.

16.14. A linear conductor of active length 0.7 m crosses a uniform magnetic field at an angle of 30° and at 10 m/s. Determine the magnetic induction of the field if the emf induced in the conductor is 4.9 V.

16.15. Determine the emf induced at the ends of an aeroplane's wings moving horizontally at 900 km/h if the wing span is 36.5 m and the vertical component of the Earth's magnetic field is 40 A/m.

16.16. An electromagnet produces a magnetic field of strength 4×10^5 A/m in air at its poles. Assuming that the field is uniform, determine the minimum velocity of a conductor of active length 10 cm required for an emf of 1 V to be induced in it if the angle between the vectors of magnetic induction and velocity is 90°.

16.17. A magnetic flux of 30 mWb piercing a closed circuit decreases to zero in 1.5×10^{-2} s. Determine the average emf and current induced in the circuit. The resistance of the circuit is 4 Ω.

16.18. A straight conductor moves at a velocity of 4 m/s in a uniform magnetic field of induction 6 mT. The active length of the conductor is 0.3 m. What is the angle between the velocity and magnetic induction vectors if a potential difference of 3.6×10^{-3} V is induced across the ends of the conductor?

16.19. A solenoid contains 200 turns with a cross-sectional area of 80 cm^2. The magnetic induction in the solenoid increases over 0.1 s from 2 to 6 T. Determine the average emf induced in the solenoid winding.

16.20. What is the change in the magnetic flux in a coil over 0.05 s if the coil contains 1000 turns, and the emf induced in it is 120 V?

16.21. Determine the self-inductance emf in a coil of inductance 0.5 H if the current in it decreases by 0.2 A over 10^{-3} s.

16.22. A voltage of 20 V is applied to a coil of inductance 1 H. How long does it take the current in the coil to reach 30 A?

16.23. The inductance of a circuit is 40 mH. What is the self-inductance emf emerging in the circuit if the current in it has changed by 0.2 A over 0.01 s? What is the change in the magnetic flux in the circuit in this case?

16.24. The inductance of a circuit is 0.05 H. What is the magnetic flux piercing the circuit if the current in it is 8 A?

16.25. Determine the inductance of a coil in which a self-inductance emf of 0.5 V emerges as a result of a decrease of 0.2 A in the current in 0.04 s.

16.26. How can we increase the inductance of a solenoid?

16.27. Why does a strong spark appear when a knife-switch is used to disconnect a circuit containing a coil with a core? How can this effect be eliminated?

16.28. A magnetic flux of 0.14 Wb is coupled with a circuit whose inductance is 0.02 H. Determine the current passing in the circuit.

16.29. At what current in a coil of inductance 40 mH is the magnetic field energy equal to 0.15 J?

16.30. Determine the energy stored in the magnetic field

of a coil of inductance 85 mH if the current passing through the coil is 8 A.

16.31. A charge of 6×10^{-2} C for 0.01 s passes through the cross-section of a coil of inductance 12 mH for a long time. What are the magnetic energy and the magnetic flux in the coil? What will the self-inductance emf emerging at the moment the circuit is disconnected be if the magnetic flux drops to zero in 0.05 s?

16.32. A circular loop of radius 5 cm is placed in a uniform magnetic field of induction 1.2×10^{-2} T so that the normal to the plane of the loop coincides with the direction of the field. The resistance of the loop is 3.1 Ω. How much electricity will pass through the loop if it is turned by an angle of 60°?

16.33. One turn of insulated wire in the form of a planar square frame with side $l = 0.2$ m is placed in a uniform magnetic field perpendicular to the magnetic field lines. Determine the current passing through the turn if the magnetic field starts to decrease at a constant rate of 0.1 T/s. The resistance of the turn is 1 Ω.

Chapter III

Oscillations and Waves

§ 17. MECHANICAL VIBRATIONS AND WAVES. SOUND AND ULTRASOUND

Basic Concepts and Formulas

A vibration (oscillation) is a periodical motion in which a body (particle) passes through its equilibrium position moving alternatively in opposite directions.

The time taken for a vibration to be completed is called the period T, while the quantity reciprocal to the period is known as the vibration frequency ν:

$$\nu = 1/T.$$

The frequency is th number of vibrations that occur per second. The unit of frequency is the hertz (Hz).

The most important type of vibration is the harmonic vibration (harmonic motion) caused by a force proportional to the displacement x. The displacement of a point undergoing a harmonic motion is given by the equation

$$x = A \sin (\omega t + \varphi_0),$$

where A is the amplitude of vibration, $\omega t + \varphi_0 = \varphi$ is the phase, φ_0 is the initial phase, and ω is the circular frequency:

$$\omega = 2\pi\nu, \quad \text{or} \quad \omega = 2\pi/T.$$

The unit of circular frequency is the radian per second (rad/s).

Remark. In the equation for a harmonic motion, A is the maximum displacement (x_{max}) and can be denoted by X. By analogy, the amplitudes of velocity (v_{max}) and acceleration (a_{max}) can be denoted by V and A respectively.

A simple pendulum (a particle suspended on a weightless unstretchable string) has the following period for small vibrations:

$$T = 2\pi \sqrt{l/g}.$$

For a load of mass m vibrating on a spring with spring constant k, the circular frequency and the period are given by

$$\omega = \sqrt{k/m}, \quad T = 2\pi\sqrt{m/k}.$$

A vibrating body has a potential and a kinetic energy. For a load vibrating on a spring, the total energy of the vibrations is given by

$$W = \frac{kA^2}{2}.$$

For an arbitrary displacement x, the energy of the load is

$$W = W_p + W_k = \frac{kx^2}{2} + \frac{mv^2}{2}.$$

If a particle vibrates in a medium, it causes neighbouring particles to vibrate, and a wave propagates in the medium. The velocity v and the wavelength λ of the wave are related thus

$$v = \lambda/T, \quad \text{or} \quad v = \lambda\nu.$$

The velocity of a wave does not depend on the frequency of vibrations and only depends on the properties of the medium. Therefore a transition to a different medium causes the velocity and wavelength to change, while the frequency remains unchanged.

Waves having frequencies from 16 to 20 000 Hz can be perceived by the human ear and are known as acoustic (sound) waves. The velocity of sound is usually denoted by c.

Worked Problems

Problem 84. A boy rocks up and down on a board. The time he takes to move from the upper position to the lower position is 1.5 s. What is the frequency, circular frequency, and period of the vibrations?

Given: $t = 1.5$ s is the time during which the boy moves from the extreme upper to the lower position.

Find: the frequency ν of the vibrations of the boy on the board, the circular frequency ω, and the period T of the vibrations.

Solution. In this case, the period of the vibrations can be defined as the time elapsed between two successive upper-

most (or lowermost) positions of the boy. Thus,

$$T = 2t.$$

The frequency and the circular frequency can be determined from the formulas

$$\nu = 1/T, \quad \omega = 2\pi\nu.$$

Substituting in the numerical values, we obtain

$$T = 2 \times 1.5\ \text{s} = 3\ \text{s}, \quad \nu = \frac{1}{3\ \text{s}} = 0.33\ \text{s}^{-1} = 0.33\ \text{Hz},$$

$$\omega = 2 \times \pi_{\text{rad}} \times 0.33\ \text{s}^{-1} = 2.1\ \text{rad/s}.$$

Answer. The frequency of the vibrations is 0.33 Hz, the circular frequency is 2.1 rad/s, and the period is 3 s.

Problem 85. The coordinates of a point are defined by the equation

$$x = 1.2 \cos \pi\,(2t/3 + 1/4).$$

Determine the amplitude, circular frequency, frequency, period, and the initial phase of the vibrations. Determine the amplitudes of velocity and acceleration. What will the phase be in 0.375 s after the beginning of motion?

Given: $x = 1.2 \cos \pi\,(2t/3 + 1/4)$ is the equation of motion of the vibrating point, $t = 0.375$ s is the time elapsed after the beginning of vibrations.

Find: the amplitude X, the circular frequency ω, the frequency ν, the period T, the initial phase φ_0, the amplitude V of velocity, the amplitude A of acceleration, and the phase φ at the instant t.

Solution. Let us first transform the equation to a form containing a sine instead of a cosine. We know from trigonometry that $\cos \alpha = \sin (\alpha + \pi/2)$. Hence

$$\begin{aligned} x &= 1.2 \cos \pi\,(2t/3 + 1/4) \\ &= 1.2 \sin [\pi\,(2t/3 + 1/4) + \pi/2] \\ &= 1.2 \sin [2\pi t/3 + \pi/4 + \pi/2] \\ &= 1.2 \sin (2\pi t/3 + 3\pi/4). \end{aligned}$$

Comparing this equation with the equation of harmonic motion $x = X \sin (\omega t + \varphi_0)$, we obtain $X = 1.2$ m, $\omega = 2\pi/3$ rad/s $= 2.1$ rad/s, $\varphi_0 = 3\pi/4$ rad $= 2.36$ rad.

The frequency, period, and phase can be determined from the formulas

$$\nu = \omega/2\pi,\ T = 1/\nu,\ \varphi = \omega t + \varphi_0.$$

In order to find the amplitude of the velocity, we must differentiate the equation of motion. We obtain

$$\frac{dx}{dt} = X\omega \cos(\omega t + \varphi_0),\ \text{or}\ v = V\cos(\omega t + \varphi_0).$$

Hence $V = X\omega$.

To find the amplitude of the acceleration, we must differentiate the velocity equation

$$\frac{dv}{dt} = -V\omega \sin(\omega t + \varphi_0),$$

or

$$a = -A \sin(\omega t + \varphi_0).$$

Hence

$$A = -V\omega = -X\omega^2.$$

Substituting in the numerical values, we obtain

$$\nu = \frac{2\pi\ \text{rad/s}}{3 \times 2\pi\ \text{rad}} = 0.33\ \text{Hz},\ T = \frac{1}{\nu} = 3\ \text{s},$$

$$\varphi = \frac{2\pi\ \text{rad/s} \times 0.375\ \text{s}}{3} + \frac{3\pi}{4}\ \text{rad} = 3.14\ \text{rad},$$

$$V = 1.2\ \text{m} \times \frac{2\pi}{3}\ \text{s}^{-1} = 2.5\ \text{m/s},$$

$$A = -1.2\ \text{m} \times \frac{4\pi^2}{9}\ \text{s}^{-2} = -5.27\ \text{m/s}^2.$$

Answer. The amplitude of the vibrations is 1.2 m, the circular frequency is 2.1 rad/s, the frequency is 0.33 Hz, the period is 3 s, the initial phase is 2.36 rad, the velocity amplitude is 2.5 m/s, and the acceleration amplitude is $-5.27\ \text{m/s}^2$. The phase in 0.375 s after the beginning of motion is 3.14 rad.

Problem 86. A load of mass 100 g is fixed on a spring (see Fig. 85) with constant 100 N/m. It is displaced 3 cm from the equilibrium position and receives a velocity of 10 cm/s. What are the potential and kinetic energies of the load at

the initial moment? What is the total energy of the load? Write the equation of its motion.

Given: $m = 100$ g $= 0.1$ kg is the mass of the load, $k = 100$ N/m is the spring constant, $x = 3$ cm $= 0.03$ m is the initial displacement of the load from the equilibrium position, $v = 10$ cm/s $= 0.1$ m/s the initial velocity of the load.

Find: the potential energy E_{p0} of the load at the initial moment, the kinetic energy E_{k0} of the load at the initial moment, the total energy E of the load and the equation of motion.

Solution. The potential and kinetic energies of the load can be determined from the formulas

$$E_{p0} = \frac{kx^2}{2}, \quad E_{k0} = \frac{mv^2}{2}.$$

The total energy of the load is

$$E = E_{p0} + E_{k0}.$$

In order to write the equation of motion, we must find the amplitude A, the circular frequency ω, and the initial phase of vibrations. It should be noted that the initial displacement of the load is not the amplitude since in addition to the initial displacement the load receives a velocity. However, the total energy can be expressed in terms of amplitude: $E = kA^2/2$, whence

$$A = \sqrt{2E/k}.$$

The circular frequency can be found from the formula

$$\omega = \sqrt{k/m}.$$

In order to determine the initial phase, we write the equation for harmonic motion in the general form:

$$x = A \sin(\omega t + \varphi_0).$$

At time $t = 0$, the equation has the form $x = A \sin \varphi_0$, whence

$$\varphi_0 = \arcsin(x/A).$$

Substituting in the numerical values, we obtain

$$E_{p0} = \frac{100\ \text{N/m} \times 9 \times 10^{-4}\ \text{m}^2}{2} = 4.5 \times 10^{-2}\ \text{J},$$

$$E_{k0} = \frac{0.1\ \text{kg} \times 0.01\ \text{m}^2/\text{s}^2}{2} = 5 \times 10^{-4}\ \text{J},$$

$$E = 4.55 \times 10^{-2}\ \text{J},$$

$$A = \sqrt{\frac{2 \times 4.55 \times 10^{-2}\ \text{J}}{100\ \text{N/m}}} = 3.017 \times 10^{-2}\ \text{m},$$

$$\omega = \sqrt{\frac{100\ \text{N/m}}{0.1\ \text{kg}}} = 31.6\ \text{rad/s},$$

$$\varphi_0 = \arcsin \frac{3}{3.017} = \arcsin 0.9945 = 1.4658\ \text{rad}.$$

Answer. The potential and kinetic energies at the initial moment are 4.5×10^{-2} J and 5×10^{-4} J respectively, the total energy is 4.55×10^{-2} J. The equation of motion for the load has the form $x = 3.017 \times 10^{-2} \times \sin(31.6t + 1.4658)$.

Problem 87. What will the change in the period of oscillations of a pendulum be in a lift moving upwards with an acceleration of $0.3g$?

Given: $a = 0.3g$ is the acceleration of the lift.

Find: the period T of oscillations of the pendulum.

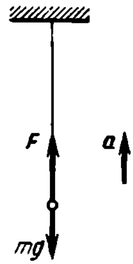


Fig. 80

Solution. Let us go over to a coordinate system fixed to the lift. Then the pendulum will experience the action of an inertial force equal to ma and directed against the acceleration a. To clarify this, let us consider the pendulum at rest. It is acted upon by the tension F of the string and the force of gravity mg. Now let the pendulum move with the acceleration a together with the lift (Fig. 80). We write Newton's second law

$$F - mg = ma, \quad \text{or } F = m(g + a).$$

Thus, the weight of the load as if increases by the quantity ma which is known as the inertial force. We must then replace g in the formula for the period of oscillations by $g + a$

$$T = 2\pi \sqrt{l/(g + a)}.$$

In the absence of acceleration, we have

$$T_0 = 2\pi \sqrt{l/g}.$$

Dividing these two equalities termwise, we obtain

$$\frac{T}{T_0} = \sqrt{\frac{g}{g+a}}, \quad \text{or} \quad T = T_0 \sqrt{\frac{g}{g+a}}.$$

Substituting in the numerical values, we obtain

$$T = T_0 \sqrt{\frac{g}{g+0.3g}} = \frac{T_0}{\sqrt{1.3}} = \frac{T_0}{1.14} = 0.877T_0.$$

Answer. The period of oscillations of the pendulum will decrease by a factor of 1.14.

Problem 88. Using vector diagrams, determine the amplitude and phase of a vibration which is the resultant of three harmonic vibrations $x_1 = \sin \omega t$, $x_2 = 2\sin(\omega t + \pi/2)$, and $x_3 = 2.5 \sin(\omega t + \pi)$. Write its equation.

Given: $x_1 = \sin \omega t$, $x_2 = 2\sin(\omega t + \pi/2)$, and $x_3 = 2.5 \sin(\omega t + \pi)$ are the component vibrations.

Find: the amplitude A and the initial phase φ_0. Write the equation of the resultant vibration.

Solution. From the given equations of harmonic motions, we can determine the amplitudes and the initial phases:

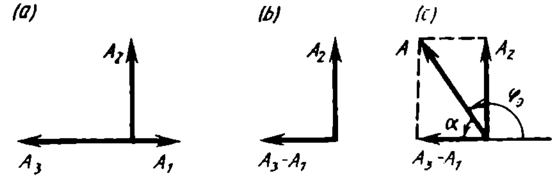


Fig. 81

$A_1 = 1$ m, $A_2 = 2$ m, and $A_3 = 2.5$ m, $\varphi_{01} = 0$, $\varphi_{02} = \pi/2$ rad, and $\varphi_{03} = \pi$ rad.

Using vector diagrams, we can sum vibrations of the same frequency. In the case under consideration, this condition is satisfied.

For each vibration, a vector is plotted from the origin, the length of the vector being equal to the amplitude, and the slope equal to the initial phase. The composition of these three vectors yields the amplitude and phase of the required vibration. Figure 81*a* shows vectors with lengths A_1,

A_2, and A_3 and slopes 0, $\pi/2$, and π. Let us first compose the vectors lying on the same straight line (Fig. 81*b*), and then find the required vector using the parallelogram rule (Fig. 81*c*).

Thus we get

$$A_3 - A_1 = 1.5 \text{ m},$$

$$A = \sqrt{A_2^2 + (A_3 - A_1)^2}, \quad A = \sqrt{4 \text{ m}^2 + 2.25 \text{ m}^2} = 2.5 \text{ m},$$

$$\sin\alpha = \frac{2}{2.5} = 0.8, \quad \alpha = 53°8',$$

$$\varphi = 180° - \alpha = 126°52' = 0.705\pi.$$

Answer. The resultant vibration has an amplitude of 2.5 m and an initial phase of 0.705 rad, and is described by the equation $x = 2.5 \sin(\omega t + 0.705\pi)$.

Questions and Problems

17.1. What is the period of the vibrations of a particle if it completes a single vibration in 5 s?

17.2. The vibration frequency of a particle is 2 Hz. How many vibrations does it complete in 1 s?

17.3. A particle performs 60 complete vibrations in 2 min. What are the period and vibration frequency?

17.4. A load suspended on a spring vibrates at a frequency of 0.4 Hz. Determine the circular frequency and the period of vibrations.

17.5. A ball is dropped on the floor. 0.5 s after the impact, the ball reaches the upper point of its trajectory. Assuming that the impact of the ball against the floor is perfectly elastic, determine the period and frequency of the motion.

17.6. Will the mode of the vibrations of a tuning fork change if it is immersed in water?

17.7. An areometer is immersed in a liquid and then released. In 1/8 s, it attains its equilibrium position. What are the areometer's frequency and period of vibrations?

17.8. Figure 82 shows three pairs of oscillating pendulums. What are their phases relative to each other?

17.9. The disc of a record player turns at 33 revolutions per minute. Determine the circular frequency of the disc.

17.10. The angular velocity of the drive wheel of an automobile is 30 rad/s. What is the frequency and the period of motion of the engine piston on direct gear?

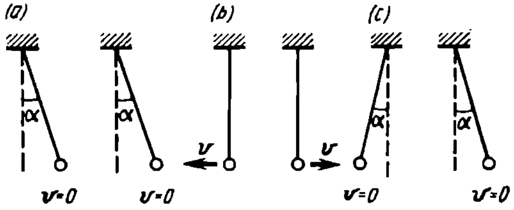


Fig. 82

17.11. A merry-go-round turns through 90° in 0.5 s. Determine its frequency, period, and circular frequency.

17.12. The motion of a body is described by the equation

$$x = 4.25 \sin (0.3t + 0.75).$$

What is the amplitude, circular frequency, initial phase, and the period of oscillations? What is the phase at the instant $t = 0.5$ s?

17.13. The equation of a harmonic motion has the form

$$x = 0.02 \sin \frac{\pi}{2} t.$$

Determine the displacement of the body from the equilibrium position at $t_1 = 0$, $t_2 = T/4$, $t_3 = T/2$, and $t_4 = 7T/12$.

17.14. Using the conditions of Problem 17.13, determine the phase of vibrations for $t_1 = T$, $t_2 = T/2$, and $t_3 = T/4$.

17.15. The position of a vibrating particle is determined by the equation

$$x = 0.05 \sin \omega t.$$

Determine the displacement of a particle if the vibration phase is $\pi/4$.

17.16. The equation of a harmonically vibrating particle has the form

$$x = 1.2 \sin \pi \left(\frac{2t}{T} + \frac{1}{4}\right).$$

Determine the displacement of the particle at the instants $t_1 = 0$, $t_2 = 7T/24$, and $t_3 = 11T/24$.

17.17. Using the conditions of Problem 17.16, determine the coordinate of the particle at $t_1 = 17T/24$, $t_2 = 2T$, $t_3 = 3T/8$. Plot the graph $x = f(t)$.

17.18. The position of a vibrating body is described by the formula $x = 8 \sin \omega t$. Determine the displacement of the body if the vibration phase is 30°.

17.19. Determine the amplitude of the vibrations if the displacement of a particle at a phase of 45° is 10 cm.

17.20. Using the graph of the vibrational motion of a particle, shown in Fig. 83, determine the amplitude, fre-

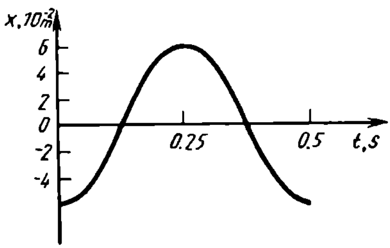


Fig. 83

quency, period, and the initial phase of the vibrations. Write the equation of motion.

17.21. The law of vibration of a particle is represented graphically in Fig. 83. Determine the displacement of the particle at $t = 5T/12$.

17.22. Given the equations of harmonic motion of two bodies

$$\text{(a)}\ x_1 = 2 \sin 0.5\pi\,(2t + 1),$$
$$x_2 = 3 \sin(\pi t + 1.5\pi),$$
$$\text{(b)}\ x_1 = \sin(\omega t + \pi/2),$$
$$x_2 = \cos \omega t.$$

Indicate the phase difference in their vibrations.

17.23. Using the equations of vibrations of two bodies, determine their phase difference:

$$\text{(a)}\ x_1 = 0.1 \sin \pi \frac{3t+5}{2}, \quad x_2 = 0.4 \cos 1.5\pi \left(t + \frac{1}{3}\right),$$

$$\text{(b)}\ x_1 = 4 \sin 1/4\,(\omega t + 7\pi), \quad x_2 = 0.02 \cos 1/3\,(0.75\omega t - 3\pi).$$

17.24. Write the equation of a harmonic motion if
(a) $\omega = \pi$, $x_1 = \sqrt{3}$ for $t_1 = 0$, and $x_2 = 1$ for $t_2 = T/4$.
(b) $x_1 = 0$ for $t_1 = 1$, $x_2 = \sqrt{3}/2$ for $t_2 = 2$, and $x_3 = -3/2$ for $t_3 = 3$.

17.25. Write the equation of a harmonic motion if

(a) $A = 9$ cm, $\nu = 20$ Hz,
(b) $A = 5$ m, $\nu = 0.5$ Hz, φ_0 corresponds to $T/8$,
(c) $A = 1$ m, $T = 6$ s, $\varphi_0 = -\pi/4$.

17.26. A body floating on the surface of a liquid is immersed to a depth of 10 cm and then released. It starts to vibrate at a frequency of 2 Hz/s. How long does it take the body to move 5 cm from its equilibrium position? Determine the time required for the body to traverse a distance of 5 cm from the point of maximum deviation. What distance will the body cover in 10 s? The vibrations should be assumed to be undamped.

17.27. A pendulum consists of a ball suspended on a string. What forces cause it to oscillate? What is the direction of the restoring force? At what point is it at a maximum? At a minimum?

17.28. A ball suspended on a string performs small oscillations about the equilibrium position. How does the velocity of the ball vary during one period? Can the motion of the ball be defined as uniformly variable? Why?

17.29. Two balls of the same size, one aluminium and one lead, are suspended on strings of equal length. The balls are deflected through the same small angle and released. Will the periods of their oscillations be the same?

17.30. Will the two balls in Problem 17.29 come to a halt at the same time? If not, which stops oscillating first?

17.31. At what position does a pendulum have the maximum and minimum acceleration and velocity?

17.32. Determine the restoring force acting on a simple pendulum of mass 10 g for a deflection angle of 45°.

17.33. Determine the mass of a pendulum if the driving force is equal to 1 N at a deflection angle of 30°.

17.34. Through what angle is a copper ball of diameter 2 cm deflected if the restoring force acting on it is 0.183 N?

17.35. Why does a pendulum clock keep the correct time only at a certain latitude?

17.36. The pendulum of a clock consists of a metal rod and a load that can be fixed at any point along the rod. How will the pace of the clock change with decreasing room temperature? How can the correct pace be restored?

17.37. What will the change in the period of oscillations of a pendulum be if its length is increased fourfold?

17.38. How should the length of the pendulum of a clock be changed for it to keep correct time on the Moon?

17.39. Determine the oscillation periods of a 1-m long pendulum in Moscow ($g = 9.816$ m/s^2) at the North pole ($g = 9.832$ m/s^2), on the equator ($g = 9.78$ m/s^2), and on the Moon ($g = 1.63$ m/s^2).

17.40. The pendulum mounted in the St. Isaac's Cathedral in Leningrad is 98 m long and completes 181.5 oscillations per hour. Using these data, determine the free fall acceleration in Leningrad.

17.41. What will the speed of a pendulum clock be in (a) a uniformly ascending lift? (b) a lift falling freely to the ground? What effect will the phase of the pendulum's oscillations at the moment the lift starts falling have?

17.42. A ball of mass m is suspended on a weightless unstretchable string. The period of its oscillations is T_0. In addition to the force of gravity, a force F acts on the ball along the vertical. What must the direction and magnitude of this force (in comparison with the force of gravity) be for the oscillation period to be (a) $2T_0$, (b) $0.8T_0$?

17.43. What must the force F be (see Problem 17.42) to stop the pendulum's oscillations?

17.44. A horizontal force F acts on a ball (see Problem 17.42). What must the magnitude of this force be (compared to the force of gravity) for the oscillation period to be (a) $T = 0.946T_0$, (b) $T = 0.8T_0$? What will be the equilibrium position? Draw a diagram.

17.45. A pendulum consists of a metal ball of mass 10 g and a silk thread. A charge of 10^{-6} C is supplied to the ball, and a horizontal electric field with strength 2×10^4 V/m is applied. How will the mode of the pendulum's oscillations change? What will the ratio of the oscillation periods be in the two cases?

17.46. A seconds pendulum (viz. a pendulum with an oscillation period of 1 s) is in a train moving with accelera-

tion $a = 0.458g$. Determine the period of oscillations of the pendulum.

17.47. A pendulum the period of whose simple oscillations is 0.5 s is in a lift descending with acceleration $a = 0.19g$. What is the frequency of its oscillations?

17.48. A load is fixed on a spring as shown in Fig. 84. When it is set vibrating along the vertical, the load is acted

Fig. 84

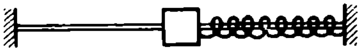


Fig. 85

upon by the elastic force of the spring and the force of gravity. Will the load's oscillations be harmonic? What effect does the force of gravity mg have?

17.49. One end of a spring with a constant of 50 N/m is fixed, and a 1-kg load is suspended from the other end. Determine the vibration frequency of such a pendulum.

17.50. The period of vibrations of a spring pendulum is 0.25 s. What is the spring constant if the mass of the load is 200 g?

17.51. A body of mass 0.5 kg fixed to a spring stretches it by 1 cm at rest. When it is displaced by 3 cm downwards and released, it starts vibrating harmonically. Determine the amplitude, circular frequency, period, and initial phase. Write the equation of motion.

17.52. A body of mass 800 g is fixed to a spring with constant 40 N/m and vibrates as shown in Fig. 85. The amplitude of vibrations is 2 cm. What is the energy of vibrations? Determine the maximum velocity and acceleration.

17.53. The motion of a body whose mass is 2 kg is described by the equation

$$x = 0.8 \sin\left(\pi t + \frac{\pi}{2}\right).$$

Determine the energy of the vibrating body. How does the energy depend on the initial phase?

17.54. A load of mass 1 kg is suspended from a spring with constant 1000 N/m and placed in a rocket that is

launched with acceleration $a = 2g$. When the engine is switched off, the load starts vibrating. What is the energy of the vibrating load? What is its velocity when the load is 0.5 cm away from its equilibrium position? What are its kinetic and potential energies at this point? Write the equation of motion, assuming that the free fall acceleration g does not change during the flight.

17.55. The equation of motion of a particle has the form

$$x = 6 \cos 0.2 \sin \pi \frac{t-0.2}{2} \cos \left(\pi \frac{t}{2} - 0.1\pi\right).$$

Write the equation in a more convenient form and determine from it the amplitude, frequency, period, initial phase, and circular frequency of the vibrations.

17.56. A watch with a spring and a pendulum clock are mounted on a rocket being launched vertically upwards flying with an acceleration $a_1 = g$ for 30 s, after which its engines are switched off. When it is 980 m from the surface of the Earth, the rocket begins slowing down and lands softly. What is the difference between the clock readings?

17.57. Plot the graphs of harmonic motions described by the equations (1) $x = 2 \sin \omega t$, (2) $x = 3 \sin \omega t/2$, and (3) $x = 2 \sin (\omega t + \pi)$.

17.58. Figure 86 shows the graphs of five vibrational motions. Determine the amplitudes of vibrations *II* and *V*. What is the amplitude of the resultant vibration? Compose vibrations *II* and *V* graphically.

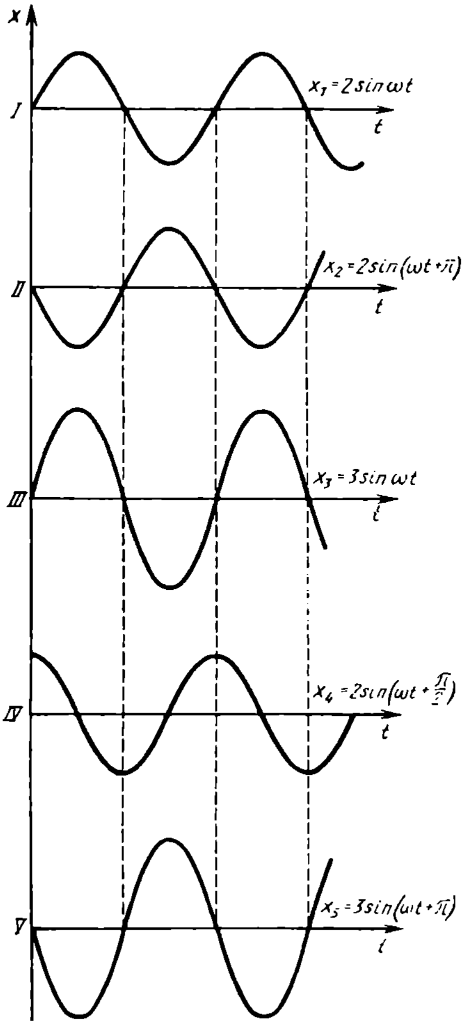
17.59. Compose vibrations *I* and *II* and *I* and *V* (Fig. 86) analytically and graphically. Determine the amplitude of the resultant vibrations.

17.60. Using vector diagrams determine the amplitude of the resultant vibrations (see Fig. 86) for (a) *I* and *IV*, (b) *II*, *IV*, and *V*, and (c) *I*, *III*, and *IV*.

17.61. Using vector diagram, determine the phase shift in the composition of vibrations *I*, *III*, and *IV* (see Fig. 86).

17.62. Using vector diagrams, determine the amplitude and phase of the vibration obtained by subtracting *V* from *IV* (see Fig. 86).

17.63. Describe the motion of a particle participating in a wave motion.



x
I
$x_1 = 2\sin\omega t$
t
II
$x_2 = 2\sin(\omega t + \pi)$
t
III
$x_3 = 3\sin\omega t$
t
IV
$x_4 = 2\sin(\omega t + \frac{\pi}{2})$
t
V
$x_5 = 3\sin(\omega t + \pi)$
t

Fig. 86

17.64. Figure 87 shows the shape of a transverse wave. The wave propagates to the right. Use arrows to indicate the directions of the individual particle velocities.

17.65. Determine the directions in which the waves in Figs. 88*a* and *b* propagate. The arrows indicate the directions of the velocities of the particles.

17.66. On what does the propagation velocity of a transverse wave at the boundary between two liquid media depend?

17.67. A wave propagates over the surface of water at a velocity of 3 m/s. What will the velocities of the particles be on the crest of the wave?

17.68. In southern countries, surfing (riding a board on the crests of waves) is very popular by the sea. At what velocity

Fig. 87

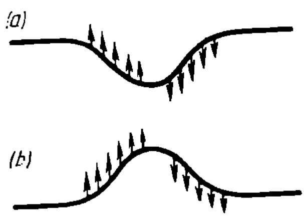


Fig. 88

does a wave carry a surfer if the wavelength is 25 m and the wave particles vibrate with a period of 1.5 s?

17.69. In which medium is the velocity of sound higher: air or iron? Can sound propagate in vacuum?

17.70. While opening a door, we sometimes hear a creak. What is the origin of this sound? What is the role of the door?

17.71. Which properties of a medium determine the velocity of sound? Explain by taking air and water as examples.

17.72. What does the velocity of sound in a given medium, say, air depend on?

17.73. Why do people press an ear against the rails to know whether a train is coming?

17.74. A meteorite strikes the surface of the Moon. How long will it take for sensitive instruments on the Earth to detect the sound of the explosion?

17.75. If a vibrating tuning fork is held against a table, the sound becomes much louder. Why?

17.76. A vibrating tuning fork is first held in the hand and then its end is brought in contact with a table. In which case does the sound cease sooner and why?

17.77. The human ear has its greatest sensitivity in the frequency range 1.5-3 kHz. Determine the wavelengths corresponding to this range if the velocity of sound is 340 m/s.

17.78. Sound propagates in air at 330 m/s for a vibration frequency of 0.5 kHz. Determine the shortest distance in the propagation direction between two points vibrating in phase.

17.79. If sound waves propagate in a medium at 340 m/s and a frequency of 500 Hz, what is the phase difference of two particles of the medium 17 cm apart? The particles lie on the line along which the wave propagates.

17.80. If we blow over the open end of the cap of a fountain pen, a whistling sound is produced. Explain its origin.

17.81. The cap in Problem 17.80 is 5.5 cm long. What is the frequency of the emerging sound?

17.82. The frequency of a tuning fork's vibrations is 1.38 kHz. How long is the vibrating part of the tuning fork? The velocity of sound is 332 m/s.

17.83. How will the frequency of the sound produced by a tuning fork change with decreasing temperature?

17.84. The length of an acoustic wave of the same frequency in air is a factor of 4.25 smaller than in water and a factor of 10.7 smaller than in brick. Determine the velocity of sound in water and in brick assuming that the velocity of sound in air is 340 m/s.

17.85. What is the change in frequency, period of vibrations, and wavelength of sound when it passes from air to steel? The velocity of sound in steel is 5000 m/s.

17.86. Determine the depth of a sea if the response from an echo sounder is received in 1.6 s, and the velocity of sound in water is 1500 m/s.

17.87. Ultrasound is used to find defects in large bodies. At what depth is a defect in an aluminium component detected if the first reflected signal is received after 8×10^{-6} s, and the second, after 2×10^{-5} s? What is the height of the component? The velocity of sound in aluminium is 510 m/s.

17.88. A person heard a clap of thunder 9 s after he saw the

flash of lightning. At what distance from the discharge was he?

17.89. A thunderstorm takes place 4.5 km away from an observer. What is the time interval between seeing the lightning and hearing the thunder?

17.90. A ship surveying the bottom of the ocean moves at 36 km/h. What is the percentage error in the measured depth due to the motion of the ship? What is the error for a depth of 3000 m?

§ 18. ALTERNATING CURRENT

Basic Concepts and Formulas

An alternating current is one that periodically changes its direction in a circuit so that the current averaged over a period T is zero.

Alternating current is produced by vibrational motion of charge carriers, viz. electrons.

When a frame is rotated in a uniform magnetic field, an emf is induced across its ends:

$$e = \mathscr{E}_{\max} \sin \omega t,$$

where e is the instantaneous emf, $\mathscr{E}_{\max}$ is the maximum (amplitude) emf, and ω is the angular velocity of rotation of the frame and the circular frequency of alternating emf. The period and frequency are

$$T = \frac{2\pi}{\omega}, \quad \nu = \frac{1}{T}.$$

The resistance R in which heat is liberated due to the passage of a current through it is known as ohmic resistance. If the frame is connected to a load with an ohmic resistance, an alternating electric current will pass through it, and an alternating voltage will appear across the terminals:

$$i = I_{\max} \sin \omega t, \; u = U_{\max} \sin \omega t.$$

Here $u = iR$.

An alternating current (a.c.) is equivalent to a direct current of the same power. The effective values of alternating voltage and current are

$$U = \frac{U_{\max}}{\sqrt{2}}, \quad I = \frac{I_{\max}}{\sqrt{2}}.$$

An a.c. circuit containing a coil of inductance L has an inductive reactance

$$X_L = \omega L.$$

An a.c. circuit including a capacitor of capacitance C has a capacitive reactance:

$$X_C = \frac{1}{\omega C}.$$

The reactance of a circuit containing a capacitor, a resistor, and an inductance coil in series is given by

$$X = X_L - X_C,$$

while the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

The relation between Z, R, and X is presented graphically by a triangle (Fig. 89), i.e. obeys the rules of geometric composition. The angle φ in the figure is the phase difference between current and voltage.

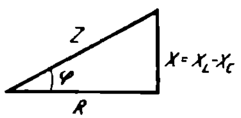


Fig. 89

Using the amplitude and effective values, we can write Ohm's law for an alternating current, i.e.

$$U_{\max} = I_{\max} Z, \quad U = IZ.$$

In the presence of a reactance, the current in the circuit is not in phase with the voltage. If we assume that the initial phase of the current is zero, the voltage across the inductive reactance leads the current by $\pi/2$, while the voltage across the capacitive reactance lags behind the current by $\pi/2$. Therefore, only in an ohmic resistance do the current and voltage oscillate in phase. Hence, alternating currents and voltages cannot be added algebraically, as was the case for direct current. The composition is carried out using vector diagrams in which the emf, voltage, and current are depicted by vectors emerging from the origin and forming with the abscissa axis an angle equal to the initial phase.

The average power liberated in an a.c. circuit is

$$\overline{P} = \frac{I_{\max} U_{\max}}{2} \cos\varphi = IU \cos\varphi.$$

In electrical engineering, the concepts of active, reactive, and total power are introduced. The active power P is asso-

ciated with the conversion of electrical energy into heat and is given by

$$P = I^2R.$$

The unit of active power is the watt (W).

The reactive power Q is given by

$$Q = I^2X_L \text{ or } Q = I^2X_C.$$

The unit of reactive power is the var (volt-ampere reactive).

The total power

$$S = I^2Z$$

is connected with P and Q through the power triangle (Fig. 90). The unit of total power is volt-ampere (VA).

The active power is the average power liberated in the circuit. The reactive power is associated with energy transmission from the generator to a circuit and back (it is stored in inductance coils and capacitors). The reactive power averaged over a period is zero.

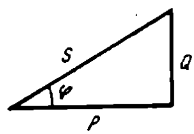


Fig. 90

The active, reactive, and total power can be expressed in terms of the effective current and voltage in the circuit:

$$P = IU \cos\varphi, \; Q = IU \sin\varphi, \; S = IU.$$

Here $\cos\varphi$ is the power factor. It can be found as the ratio of the resistance to the impedance or of the active power to the total power:

$$\cos\varphi = \frac{R}{Z} = \frac{P}{S}.$$

Worked Problems

Problem 89. A resistor with resistance 20 Ω, a coil with inductance 0.0398 H, and a capacitor with capacitance 159 μF are connected in series to a generator. Determine the voltage across the circuit components and in the entire circuit. What is the phase difference between the voltage and the current? Determine the impedance of the circuit. What will happen if the capacitive and inductive reactances are equal? The frequency of the alternating current is 100 Hz and the current is 2 A. Plot the vector diagram.

Given: $R = 20\ \Omega$ is the resistance, $L = 0.0398\ \text{H} = 3.98 \times 10^{-2}$ H is the inductance of the coil, $C = 159\ \mu\text{F} = 1.59 \times 10^{-4}$ F is the capacitance of the capacitor, $I = 2$ A is the effective value of the current, and $\nu = 100$ Hz is the frequency of the alternating current.

Find: the voltages U_R, U_L, U_C, and U across the resistor, the coil, the capacitor, and in the entire circuit respectively, the phase difference φ between the voltage and the current, and the impedance Z of the circuit.

Solution. The reactances of the coil and of the capacitor can be determined from the formulas $X_L = \omega L$ and $X_C = 1/\omega C$.

The impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}.$$

When the components of the circuit are connected in series, the same current passes through it, while the total voltage is the sum of the voltages across the individual components. The voltage across the resistor is

$$U_R = IR,$$

and is in phase with the current. The voltage across the coil is

$$U_L = I\omega L,$$

and leads the current by $\pi/2$. The voltage across the capacitor is

$$U_C = I/(\omega C),$$

and lags behind the current by $\pi/2$.

The total voltage can be found by constructing a vector diagram. We choose the origin and draw a horizontal vector, viz. the current axis relative to which the voltage vectors will be plotted (Fig. 91*a*).

The vector of voltage U_R across the resistor is plotted (from zero) along the current axis since its phase angle is zero.

We now plot the vector of voltage U_L across the inductance coil. Positive angles are measured counterclockwise, so the voltage vector will be plotted upwards at right angles to the current axis.

The vector of the voltage U_C across the capacitor is plotted downwards from zero at right angles to the current axis.

The geometric sum of the three vectors will give the total voltage in the circuit. First we compose the vectors directed along the vertical, and as a result obtain a voltage triangle (Fig. 91*b*) from which we can easily find the voltage and the phase difference:

$$U_X = U_L - U_C,$$

$$U = \sqrt{U_R^2 + U_X^2} = I\sqrt{R^2 + (\omega L - 1/\omega C)^2},$$

$$\tan\varphi = \frac{\omega L - 1/\omega C}{R} = \frac{U_X}{U_R}.$$

If $\omega L = 1/\omega C$, the reactance is zero. This phenomenon is observed at the circular frequency $\omega = \omega_0$ determined

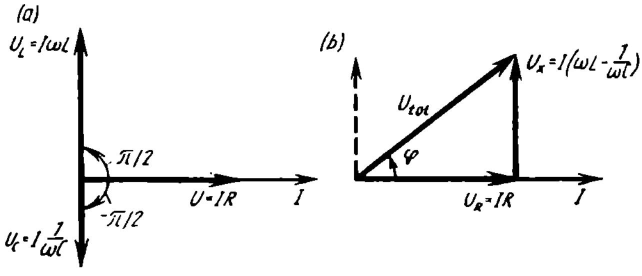


Fig. 91

from the formula $\omega_0^2 = 1/(LC)$. Then the impedance of the circuit attains its minimum value and is equal to the resistance: $Z = R$, while the current attains its maximum value. This phenomenon is known as voltage resonance, and ω_0 is the resonance circular frequency.

Substituting in the numerical values, we obtain

$$U_R = 2\ \mathrm{A} \times 20\ \Omega = 40\ \mathrm{V},$$

$$U_L = 2\ \mathrm{A} \times 2\pi \times 100\ \mathrm{s^{-1}} \times 0.0398\ \mathrm{H} = 50\ \mathrm{V},$$

$$U_C = 2\ \mathrm{A}/(2\pi \times 100\ \mathrm{s^{-1}} \times 159 \times 10^{-4}\ \mathrm{F}) = 20\ \mathrm{V},$$

$$U = \sqrt{(40\ \mathrm{V})^2 + (30\ \mathrm{V})^2} = 50\ \mathrm{V},$$

$$X_L - X_C = 2\pi \times 100\ \mathrm{s^{-1}} \times 0.398\ \mathrm{H} - (2\pi \times 100\ \mathrm{s^{-1}} \times 1.59 \times 10^{-4}\ \mathrm{F})^{-1} = 15\ \Omega,$$

$$\tan\varphi = 15\ \Omega/20\ \Omega = 0.75, \quad \varphi = 36°52',$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \frac{U}{I} = \frac{50\ \mathrm{V}}{2\ \mathrm{A}} = 25\ \Omega.$$

Answer. The voltages across the resistor, the inductance coil, and the capacitor are 40, 50, and 20 V respectively, the total voltage is 50 V, the phase difference is 36°52′, the im-

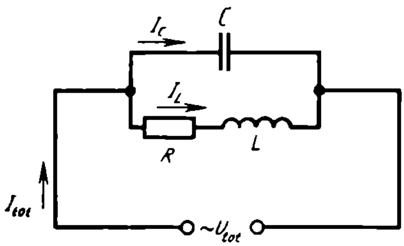


Fig. 92

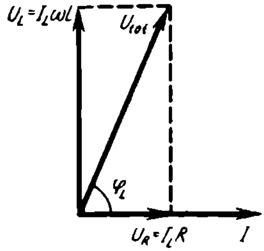


Fig. 93

pedance is 25 Ω. When the inductive and capacitive reactances are equal, resonance is observed.

Problem 90. A generator supplies an a.c. voltage of 80 V at a frequency of 20 Hz and is connected to an inductance coil and a capacitor of 750 μF connected in parallel. The resistance of the coil is 1 Ω and the inductance is 0.1 H (Fig. 92).

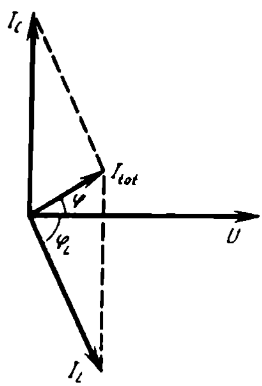


Fig. 94

Determine the currents in the branches and the total current in the circuit. What is the resonance condition for the circuit? Determine the impedance at resonance if $\omega^2L^2 \gg R^2$.

Given: $U = 80$ V is the voltage in the circuit, $\nu=20$ Hz is the frequency, $C = 750\ \mu F = 7.5\times10^{-4}$ F is the capacitance of the capacitor, $L = 0.1$ H and $R = 1\ \Omega$ are the inductance and the resistance of the coil.

Find: I_C, I_L, and I, the currents in the branches and in the entire circuit and the impedance Z at resonance.

Solution. Let us first consider the branch containing the coil. We plot the vector diagram (Fig. 93) in the same way as in Problem 89.

Using Fig. 93, we can find U and φ_L:

$$U = I_L\sqrt{R^2+\omega^2L^2}, \quad \tan\varphi_L = \frac{\omega L}{R}.$$

For the parallel connection (see Fig. 92), the voltage U is the same in each branch, while the current is equal to the sum of the currents in the parallel branches. The currents in the branches have different phases, and hence they should be composed geometrically, using vector diagrams. We plot the voltage axis (Fig. 94). The current in the branch with inductance has the value

$$I_L = \frac{U}{\sqrt{R^2+\omega^2L^2}},$$

and lags behind the voltage by φ_L.

The current through the capacitor is

$$I_C = U\omega C,$$

and leads the voltage by $\pi/2$. The composition of these vectors results in a vector I at an angle φ to the voltage axis.

The current I can be determined by cosine law:

$$I^2 = I_L^2 + I_C^2 - 2I_LI_C\cos\alpha.$$

We know from trigonometry that

$$\sin\varphi_L = \frac{\tan\varphi_L}{\sqrt{1+\tan^2\varphi_L}} = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}},$$

since $\tan\varphi_L = \omega L/R$ (see Fig. 93); we have

$$\cos\varphi_L = \frac{1}{\sqrt{1+\tan^2\varphi_L}} = \frac{R}{\sqrt{R^2+\omega^2L^2}}.$$

Since $\alpha = \pi/2 - \varphi_L$, $\cos\alpha = \sin\varphi_L$. Therefore,

$$I^2 = I_L^2 + I_C^2 - 2I_LI_C\frac{\omega L}{\sqrt{R^2+\omega^2L^2}}.$$

Angle φ can be determined from the equality of the projections of I and I_L on the voltage axis:

$$I\cos\varphi = I_L\cos\varphi_L, \quad \cos\varphi = \frac{I_L}{I}\cos\varphi_L.$$

It is often more convenient to compose the vectors geometrically in a different way. Only one is plotted from

the origin. The next vector is plotted from the end of the preceding one. The resultant vector then connects the origin of the first vector to the end of the last one. For our problem, such a construction yields a current triangle (Fig. 95), the calculations remaining the same.

If one of the parameters ω, C, or L of the circuit is varied, the total current and the phase angle φ will change. For

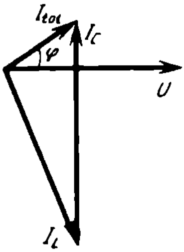


Fig. 95

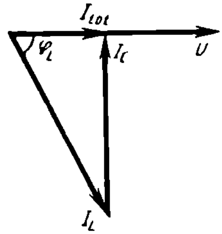


Fig. 96

a certain relation between ω, C, and L, the phase difference vanishes (Fig. 96). Then the current will have the minimum value, and hence the impedance of the circuit, which becomes equal to the resistance, increases. This phenomenon is known as current resonance. The resonance condition can be determined from Fig. 96:

$$I_C = I_L \sin \varphi_L.$$

Substituting I_C, I_L, and $\sin \varphi_L$ into this equation, we obtain

$$U\omega C = \frac{U}{\sqrt{R^2+\omega^2L^2}} \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}.$$

The inductive reactance of inductance coils is usually much higher than their resistance ($\omega L \gg R$), the more so $\omega^2L^2 \gg R^2$. Neglecting R^2 in the last equality and simplifying, we obtain the condition for current resonance:

$$\omega^2 = \omega_0^2 = \frac{1}{LC}.$$

Let us calculate the impedance of the circuit. It follows from Fig. 96 that

$$I = I_L \cos \varphi_L = \frac{U}{\sqrt{R^2+\omega_0^2L^2}} \frac{R}{\sqrt{R^2+\omega_0^2L^2}}.$$

Since $\omega^2 L^2 \gg R^2$, we obtain

$$I = \frac{UR}{\omega_0^2 L^2} = \frac{URC}{L}, \text{ whence } Z = \frac{L}{RC}.$$

Substituting in the numerical values, we obtain

$$I_L = \frac{80\ \text{V}}{\sqrt{(1\ \Omega)^2 + (2\pi \times 20\ \text{s}^{-1} \times 0.1\ \text{H})^2}} = 6.35\ \text{A},$$

$$I_C = 80\ \text{V} \times 2\pi \times 20\ \text{s}^{-1} \times 7.5 \times 10^{-4}\ \text{F} = 7.54\ \text{A},$$

$$\cos\alpha = \frac{2\pi \times 20\ \text{s}^{-1} \times 0.1\ \text{H}}{\sqrt{(1\ \Omega)^2 + (2\pi \times 20\ \text{s}^{-1} \times 0.1\ \text{H})^2}} = 0.997,$$

$$I = \sqrt{(6.35\ \text{A})^2 + (7.54\ \text{A})^2 - 2 \times 6.35\ \text{A} \times 7.54\ \text{A} \times 0.997}$$
$$= 1.28\ \text{A}.$$

The resonance frequency is

$$\nu_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1\ \text{H} \times 7.5 \times 10^{-4}\ \text{F}}} = 18.4\ \text{Hz}.$$

The impedance is

$$Z = \frac{0.1\ \text{H}}{1\ \Omega \times 7.5 \times 10^{-4}\ \text{F}} = 133\ \Omega.$$

Answer. The current through the coil and the capacitor is 6.35 A and 7.54 A respectively, the total current is 1.28 A. Resonance sets in at a frequency of 18.4 Hz, and the impedance of the circuit is then 133 Ω.

Problem 91. An electric bulb rated for 240 V and power of 200 W, a coil with inductance 0.15 H, and a capacitor are connected in series to a lighting circuit at a voltage of 220 V. What is the capacitance of the capacitor if the active power liberated in the circuit is equal to half the total power? Determine the current, the voltages across the components of the circuit, the active, reactive and total powers, and the power factor.

Given: U = 220 V is the voltage in the circuit, ν = 50 Hz is the frequency of the alternating current, U_b = 240 V and P_b = 200 W are the rated voltage and power of the bulb, L = 0.15 H is the inductance of the coil, and $P = 0.5S$ is the relation between the active and total power.

Find: the capacitance C of the capacitor, the current I in the circuit, the voltages U_R, U_L, and U_C across the bulb, coil, and capacitor respectively, the active power P, reactive power Q, total power S, and the power factor $\cos\varphi$.

Solution. The resistance of the bulb can be determined from the formula

$$P_b = U_b^2/R, \quad \text{whence} \quad R = U_b^2/P_b = \frac{(240\ \text{V})^2}{200\ \text{W}} = 288\ \Omega.$$

The power factor can be determined from the relation $P = 0.5S$:

$$\cos\varphi = \frac{P}{S} = \frac{0.5S}{S} = 0.5.$$

In order to determine the capacitance of the capacitor, we must find the inductive and capacitive reactances:

$$X_L = 2\pi\nu L, \quad X_L = 2\pi \times 50\ \text{Hz} \times 0.15\ \text{H} = 47.1\ \Omega,$$

$$\frac{R}{Z} = \frac{P}{S}, \quad R = 0.5Z = 0.5\sqrt{R^2 + (X_C - X_L)^2}.$$

Squaring the last equality, we obtain

$$4R^2 = R^2 + (X_C - X_L)^2, \quad 3R^2 = (X_C - X_L)^2.$$

Hence

$$X_C = R\sqrt{3} + X_L, \quad X_C = 288\ \Omega\sqrt{3} + 47.1\ \Omega = 546\ \Omega,$$

$$C = \frac{1}{2\pi \times 50\ \text{Hz} \times 546\ \Omega} = 5.83 \times 10^{-6}\ \text{F} = 5.83\ \mu\text{F}.$$

The impedance is

$$Z = \sqrt{(288\ \Omega)^2 + (546\ \Omega - 47.1\ \Omega)^2} = 576\ \Omega.$$

Let us now find the current in the circuit

$$I = \frac{U}{Z}, \quad I = \frac{220\ \text{V}}{576\ \Omega} = 0.38\ \text{A}.$$

The voltage across all the components can be found as follows:

$$U_R = IR, \ U_C = IX_C, \quad U_L = IX_L,$$

$$U_R = 0.38\ \text{A} \times 288\ \Omega = 109.4\ \text{V},$$

$$U_C = 0.38\ \text{A} \times 546\ \Omega = 207.5\ \text{V},$$

$$U_L = 0.38\ \text{A} \times 47.1\ \Omega = 17.9\ \text{V}.$$

Let us now calculate the powers:
total power $S = IU$, $S = 0.38\ \text{A} \times 220\ \text{V} = 83.6\ \text{VA}$,
active power $P = 0.5S$, $P = 41.8\ \text{W}$,

reactive power $Q = S\sin\varphi = S\sqrt{1-\cos^2\varphi} = \frac{\sqrt{3}}{2}S$,

$$Q = \frac{\sqrt{3}}{2}\,83.6\ \text{VA} = 72.4\ \text{var}.$$

Answer. The capacitance of the capacitor is 5.83 μF, the current in the circuit is 0.38 A, the voltages across the bulb, the inductance coil, and the capacitor are 109.4, 17.9, and 207.5 V respectively, the active, reactive, and total powers are 41.8 W, 72.4 var, and 83.6 VA respectively, and the power factor is 0.5.

Questions and Problems

18.1. The standard frequency of the alternating current used in the USSR is 50 Hz. Determine the period of the oscillations. How many times per second does the direction of the charge carriers change?

18.2. An ammeter connected to an a.c. circuit indicates 3 A. What is the amplitude of the current?

18.3. The breakdown voltage of a capacitor is 250 V. Can it be connected to a lighting circuit at a voltage of 220 V?

18.4. The emf in an a.c. circuit varies as $e = 250 \times \sin 100\pi t$. Determine the effective value of the emf and the circular frequency.

18.5. The time dependence of an alternating current is described by the equation $i = 90 \sin(314t + \pi/4)$. Determine the effective current and its phase, initial phase, and frequency.

18.6. The instantaneous voltage is $u = 179 \sin \omega t$. Determine the instantaneous voltage at times: 0, 0.0025, 0.005, 5/6, 4/3, 0.015, 0.0175, and 0.02 s. The frequency of the alternating current is 50 Hz.

18.7. The instantaneous current at 1/3 of the period is 2.6 A. What will the current be for a phase of (a) 1.5π (b) $13\pi/6$?

18.8. A wire frame rotates uniformly in a uniform magnetic field about an axis perpendicular to the magnetic field lines. In what position of the frame is the induced emf (a) zero, (b) equal to the amplitude value?

18.9. A rectangular frame of length 10 cm and width 5 cm rotates uniformly in a uniform magnetic field of induction

0.02 T. The speed of rotation is 2865 rph. Determine the amplitude of the emf induced in the frame.

18.10. A rectangular metal frame of length 12 cm and width 4 cm rotates uniformly in a uniform magnetic field. What will the change in the emf induced in the frame be if, without changing the speed of its rotation, it can be deformed so that its length becomes 4 cm and width 12 cm?

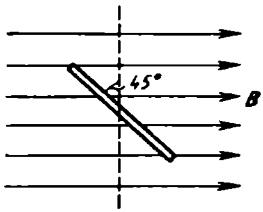


Fig. 97

18.11. A rectangular frame rotates in a uniform magnetic field at a constant speed about an axis perpendicular to the magnetic field lines. The frame length is $l = 14$ cm and its width is $d = 6$ cm. How will the emf induced in the frame change if the frame is deformed such that it takes on the following dimensions: (a) $l = 16$ cm, $d = 4$ cm, (b) $l = 10$ cm, $d = 10$ cm, and (c) $l = 8$ cm, $d = 12$ cm.

18.12. How many turns are wound on a frame having an area of 367 cm^2 and uniformly rotating in a uniform magnetic field with induction 0.115 T if a voltmeter connected to its terminals indicates 90 V? The rotation period of the frame is 2.5×10^{-2} s.

18.13. A frame containing 15 turns of area 200 cm^2 rotates uniformly in a uniform magnetic field at 10 rps. When the frame is at an angle of 30° to the direction of the magnetic field, the emf induced in the frame is 0.75 V. What is the magnetic field's induction?

18.14. A wooden frame having an area of 150 cm^2 and containing ten turns of wire rotates uniformly in a uniform magnetic field with induction 0.05 T. The position of the frame at $t = 0$ is shown in Fig. 97. The frequency of the induced alternating current is 10 Hz. Determine the amplitude, circular frequency, the period, and the initial phase of the induced emf. Write the equation for the instantaneous emf.

18.15. A conductor with inductance 2.55×10^{-3} H is switched from a circuit operating at a frequency of 50 Hz to a circuit operating at a frequency of 300 Hz. Does its impedance change? If it does, in what proportion? What is its impedance in a d.c. circuit equal to?

18.16. A coil with inductance 8.42×10^{-3} H has a resistance of 3 Ω. What is its impedance in an a.c. circuit with a current frequency of 50 Hz and in a d.c. circuit?

18.17. When a coil with inductance 0.3 H is connected to a d.c. circuit at a voltage of 24 V, the current in it is 0.2 A. Determine the impedance of the coil when it is connected to an a.c. circuit of frequency 50 Hz.

18.18. What will the impedance of a 100-pF capacitor be in a d.c. circuit?

18.19. Determine the impedance of a 100-μF capacitor connected to a.c. circuits with current frequencies 50 and 3000 Hz.

18.20. A coil of inductance 0.15 H, a 500-Ω resistor, and a 2-μF capacitor are connected in series to an a.c. circuit operating at a frequency of 400 Hz. Determine the impedance of the circuit.

18.21. Determine the impedance of the circuit connected to the lighting mains if the circuit consists of a 500-μF and a 1000-μF capacitors connected in series, a coil with a resistance of 10 Ω, and inductance 0.08 H.

18.22. A 530-μF capacitor and a potentiometer with a resistance of 5 Ω connected in series are connected to the mains which has a voltage of 220 V and a frequency of 50 Hz. Determine the current passing through the potentiometer.

18.23. A capacitor is connected to the mains which has a voltage of 220 V and frequency of 50 Hz. The amplitude of the current in the circuit is 4.89 A. What is the capacitance of the capacitor?

18.24. A coil of inductance 0.2 H is connected to an a.c. circuit with a frequency of 100 Hz. The current in the coil is 1.01 A. What is the voltage across the coil? What is the amplitude of the voltage?

18.25. A voltage of 80 V is supplied to a variable capacitor and a coil with a core connected in series to it. The current in the circuit was found to be 4 A. Determine the voltage across the circuit elements if halving the capacitance of the capacitor and the inductance of the coil increases the current to 20 A. What is the frequency if the initial inductance is 1.91×10^{-2} H?

18.26. An a.c. circuit contains a resistor. Plot the amplitude of the voltage across the resistor on a vector diagram

based on the current axis. Plot the graphs of the current and voltage.

18.27. Plot the voltage vector diagram if an a.c. circuit contains (a) a capacitor, (b) a solenoid. Plot the graphs for the currents and the voltages.

18.28. Plot the vector diagram for the voltages of the a.c. subcircuits operating at frequency 50 Hz and consisting of the following components connected in series

(a) $C = 1.33 \times 10^3\ \mu F$, $R = 2.4\ \Omega$,

(b) $L = 3.18 \times 10^{-2}$ H, $R = 17.3\ \Omega$.

18.29. Plot the graphs of voltage and current in the components of the circuit in Problem 18.28.

18.30. Plot the vector diagrams of the voltage for the following components when connected in series (a) $L = 0.03$ H and $C = 2 \times 10^{-4}$ F, (b) $L = 4.95 \times 10^{-2}$ H, $C = 398\ \mu F$ and $R = 7.55\ \Omega$. The alternating voltage frequency is 50 Hz.

18.31. Plot the voltage graphs for the components of the a.c. circuit in Problem 18.30.

18.32. Using vector diagrams, prove the formula for the reactance of a subcircuit consisting of a capacitor and an inductance coil.

18.33. Using vector diagrams, prove the formulas for the impedances of series-connected R, C and L in an alternating current.

18.34. An a.c. circuit at voltage 120 V and frequency 50 Hz is connected to series-connected (a) $R = 60\ \Omega$, $L = 0.255$ H, (b) $R = 3.8\ \Omega$ and $C = 2.27 \times 10^{-3}$ F, and (c) $L = 0.0764$ H and $C = 398\ \mu F$. Determine the impedance, current, and voltages on the components of the circuit. Plot the vector diagrams.

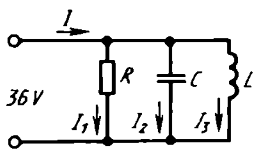


Fig. 98

18.35. An a.c. voltage of 220 V and frequency 50 Hz is applied to series-connected (a) $R = 5\ \Omega$, $L = 0.135$ H, and $C = 75\ \mu F$, (b) $R = 30\ \Omega$, $L = 0.2$ H, and $C = 97\ \mu F$. Determine impedance, current, and voltage across the components of the circuit. Plot the vector diagrams.

18.36. An a.c. generator of voltage 36 V (Fig. 98) is connected in parallel to (a) $R = 3\ \Omega$, $X_L = 4\ \Omega$, (b) $R = 1\ \Omega$ $X_C = 2\ \Omega$, and (c) $X_C = 2\ \Omega$, $X_L = 4\ \Omega$. The frequency of

the alternating current is 50 Hz. Determine the current in the branches, the total current, and cos φ. Plot the vector diagrams.

18.37. Determine the current in the branches and the total current in the circuit (see Problem 18.36) if (a) $R = 6\ \Omega$, $X_L = 3\ \Omega$, and $X_C = 3\ \Omega$, (b) $R = 9\ \Omega$, $X_L = 10\ \Omega$, and $X_C = 4.8\ \Omega$, (c) $R = 4\ \Omega$, $X_L = 2\ \Omega$, and $X_C = 6\ \Omega$. Determine cos φ. Plot the vector diagrams (all the components are connected in parallel).

18.38. A potentiometer and a solenoid with a core are connected to an a.c. circuit of frequency 50 Hz. The phase difference between the total voltage and the current is 30°. The resistance is increased by 100 Ω. What must the change in the inductance of the solenoid be for the phase difference to remain unchanged? What is the voltage in the circuit if the voltage across the solenoid is 60 V?

18.39. A resistor and an inductance coil with a negligibly low resistance are connected in series to an a.c. generator operating at frequency 40 Hz. The current in the curcuit is 3 A and the phase difference between the voltage and the current is 30°. Determine the resistance of the resistor and the inductance of the coil ($U = 60\,V$).

18.40. A potentiometer with resistance 8 Ω is connected in series to a capacitor with capacitance 398 μF and a coil of inductance 0.0383 H. The frequency of the alternating current is 50 Hz. Determine the voltage across the components and in the entire circuit and the phase difference between the current and voltage. The current in the circuit is 4 A. Plot the vector diagram.

18.41. A 0.00636-H coil is connected to a circuit at a frequency 50 Hz. What must the capacitance of a capacitor connected to the circuit be to obtain resonance?

18.42. Why are multipole generators used in hydroelectric plants?

18.43. The rotor of a hydroelectric generator rotates at 150 rpm. How many magnetic pole pairs must the rotor have in order to generate a current of frequency 50 Hz?

18.44. Determine the resonance frequency of the alternating current in the circuit shown in Fig. 99 if the capacitance of the capacitor is 2.5×10^{-5} F and the inductance of the coil is 0.1 H.

18.45. The primary winding of a transformer is connected

to a d.c. circuit. Will the bulb connected to the secondary winding glow?

18.46. The primary of a transformer is connected to an a.c. circuit while the secondary is disconnected. Does the transformer consume energy from the circuit?

18.47. Two neighbouring turns of a transformer's secondary are short circuited. What will happen?

18.48. When a transformer is connected to an a.c. supply with a voltage of 220 V, an emf of 110 V is induced in the secondary winding. What is the transformation ratio?

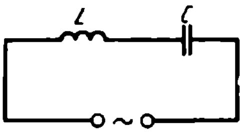


Fig. 99

18.49. The transformation ratio of a transformer is 10. What is the ratio of the numbers of turns in each winding?

18.50. An emf of 600 V is induced in the secondary of a transformer containing 1900 turns. How many turns of the primary winding are connected to a supply with voltage 220 V?

18.51. A transformer is connected to a supply with voltage 120 V. The primary winding contains 300 turns. What must the number of turns in the secondary winding be for a voltage of 6.4 V to be generated across it?

18.52. A voltage of 60 V at a frequency of 50 Hz is applied to a circuit containing a coil with inductance 0.1 H and a 20-μF capacitor. What is the average power developed in the circuit during one period? What are the reactive and total powers?

18.53. An alternating current passes through a series-connected resistor, capacitor, and coil whose resistances and reactances are 2, 3, and 5.67 Ω respectively. Determine the active, reactive, and total power, the power factor ($\cos \alpha$), and the voltage applied to the circuit if the current is 6 A.

18.54. The alternating voltage in a series-connected circuit with a current of 2 A is 90 V. The circuit consists of a capacitor, a coil, and a resistor. The power factor is 0.6. What are the active and the reactive powers? What are the reactances and the resistance of the circuit's components if the reactance of the capacitor is twice that of the coil?

18.55. A source of an a.c. voltage $u = 180 \sin \omega t$ is connected to an unbranched circuit containing a resistance and two reactances, $X_L = 8\ \Omega$ and $X_C = 12\ \Omega$. Deter-

mine the power factor, the active and reactive power if the current in the circuit is 3 A.

18.56. An a.c. voltage of 100 V is applied to a 50-Ω resistor and capacitor with reactance 55 Ω connected in parallel. Determine the total, active and reactive powers, and the power factor.

18.57. A source of a.c. voltage of 60 V and frequency 50 Hz is connected in parallel to a resistance of 15 Ω and an inductance coil with a negligible resistance. The total power in the circuit is 648 VA. What is the inductance of the coil? What are the active and reactive powers?

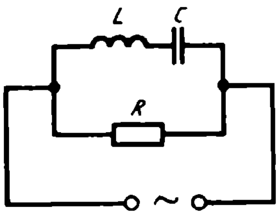


Fig. 100

18.58. An alternating current passes through a coil and a resistor connected in parallel. The voltage across the coil is 120 V, the current in the resistor is 2 A, and total current is 3.5 A. Find the resistance of the resistor and the coil. Determine the active, reactive and total powers, and the power factor.

18.59. A coil, a capacitor, and a resistor are connected as shown in Fig. 100. The voltage of the source is 40 V. Determine the power factor and the total, active, and reactive powers if (a) $X_L = 15\ \Omega$, $X_C = 15\ \Omega$, $R = 10\ \Omega$, (b) $X_L = 8\ \Omega$, $X_C = 18\ \Omega$, and $R = 13.3\ \Omega$.

18.60. A capacitor, a coil, and a resistor are connected in parallel to an a.c. circuit of a voltage 220 V. Their reactances and resistance are 8, 12, and 6 Ω respectively. Determine the power factor, and the active and reactive powers.

18.61. The power in two parallel branches of an a.c. circuit is 200 var and 346 W. The current in the unbranched circuit is 8 A. What are the reactance and the resistance of the branches, the power factor, and the total power? What is the impedance?

§ 19. ELECTROMAGNETIC OSCILLATIONS AND WAVES

Basic Concepts and Formulas

Any varying magnetic field generates an eddy electric field. In turn, an electric field varying over time generates an eddy magnetic field. This continuous transformation of the elec-

tric field's energy into the energy of the magnetic field and back is observed in an oscillatory circuit, viz. a circuit containing a coil and a capacitor (Fig. 101). The resistance of the oscillatory circuit must be low, otherwise the energy of the electromagnetic field will be converted into heat, and the oscillations in the circuit will soon cease. Under these conditions, the period of the natural electromagnetic oscillations is

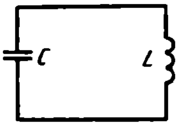


Fig. 101

$$T = 2\pi\sqrt{LC},$$

and the frequency is

$$\nu = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}.$$

An open oscillatory circuit emits waves of the wavelength

$$\lambda = cT, \text{ or } \lambda = c/\nu,$$

where c is the velocity of electromagnetic waves equal to the velocity of light in vacuum.

The velocity of propagation of the electromagnetic waves depends on the properties of the medium, viz.

$$\frac{c}{v} = \sqrt{\mu\varepsilon}, \text{ whence } v = \frac{c}{\sqrt{\mu\varepsilon}},$$

vhere v is the velocity of propagation of the electromagnetic waves in the medium, and μ and ε are the permeability and permittivity of the medium. Since the permeabilities of all dia- and paramagnetic media differ only slightly from unity, we can assume that

$$v = \frac{c}{\sqrt{\varepsilon}}.$$

While using this formula, it should be borne in mind that the ε known from electrostatics cannot always be used since the oscillation frequency affects the permittivity.

Worked Problems

Problem 92. An oscillatory circuit consists of a coil with inductance 20 μH and a capacitor whose capacitance can be varied from 2×10^{-8} to 10^{-8} F. What is the wavelength range of this circuit? Determine its frequency range.

Given: $L = 2 \times 10^{-5}$ H is the inductance of the coil, $C_1 = 2 \times 10^{-8}$ F and $C_2 = 10^{-8}$ F are the maximum and minimum capacitances of the capacitor. From tables, we take the velocity of light in vacuum $c = 3 \times 10^8$ m/s.

Find: the maximum λ_1 and minimum λ_2 wavelengths and the boundary frequencies ν_1 and ν_2 of the oscillatory circuit.

Solution. The wavelengths lie in the interval from $\lambda_1 = cT_1$ for capacitance C_1 to $\lambda_2 = cT_2$ for capacitance C_2. The period of the circuit's oscillations is determined by the formulas

$$T_1 = 2\pi\sqrt{LC_1}, \quad T_2 = 2\pi\sqrt{LC_2},$$

$$T_1 = 2 \times 3.14\sqrt{2 \times 10^{-5}\ \text{H} \times 2 \times 10^{-8}\ \text{F}} = 3.97 \times 10^{-6}\ \text{s},$$

$$T_2 = 2 \times 3.14\sqrt{2 \times 10^{-5}\ \text{H} \times 10^{-8}\ \text{F}} = 2.81 \times 10^{-6}\ \text{s}.$$

Then the wavelengths are

$$\lambda_1 = 3 \times 10^8\ \text{m/s} \times 3.97 \times 10^{-6}\ \text{s} = 1191\ \text{m},$$

$$\lambda_2 = 3 \times 10^8\ \text{m/s} \times 2.81 \times 10^{-6}\ \text{s} = 843\ \text{m}.$$

Given the periods, we can determine the frequencies:

$$\nu_1 = 1/T_1, \qquad \nu_2 = 1/T_2,$$

$$\nu_1 = 252\ \text{kHz}, \ \nu_2 = 356\ \text{kHz}.$$

Answer. The oscillatory circuit can operate in the wavelength range 843-1191 m at frequencies 252-356 kHz.

Problem 93. The maximum voltage across the capacitor of an oscillatory circuit is 120 V. Determine the maximum current if the inductance of the coil is 0.005 H and the capacitance of the capacitor is 10^{-5} F. The resistance of the circuit is assumed to be negligible.

Given: $U_{max} = 120$ V is the maximum voltage across the capacitor, $L = 0.005$ H is the inductance of the coil, and $C = 10^{-5}$ F is the capacitance of the capacitor.

Find: the maximum current I_{max} in the circuit.

Solution. The problem can be solved by two methods.

1st method. At resonance, the reactance of the circuit is minimal, while the current is at its maximum. The resonance frequency is $\omega_0 = 1/\sqrt{LC}$, $I_{max\ C} = U_{max}/X_C$, $X_C = 1/(\omega_0 C) = \sqrt{L/C}$, $I_{max\ C} = U_{max}/\sqrt{L/C} = U_{max}\sqrt{C/L}$. Argueing in the same way, we conclude that the current in

the inductance coil can be calculated from the same formula.

Substituting in the numerical values, we obtain

$$I_{\max C} = 120\ \mathrm{V}\sqrt{\frac{10^{-5}\ \mathrm{F}}{5\times 10^{-3}\ \mathrm{H}}} = 5.37\ \mathrm{A}, \quad I_{\max C} = I_{\max}.$$

2nd method. According to the energy conservation law, the maximum energy of the magnetic field must be equal to the maximum energy of the electric field: $LI^2_{\max}/2 = CU^2_{\max}/2$. Hence

$$I_{\max} = U_{\max}\sqrt{C/L}.$$

Comparing the two solutions, we note that the second method is much simpler.

Answer. In the absence of resistance, the maximum current in the circuit is 5.37 A.

Questions and Problems

19.1. How will the frequency of an oscillatory circuit change if the capacitance is reduced using a variable capacitor?

19.2. Will the insertion of a ferromagnetic core into the coil of the oscillatory circuit affect the period of the electromagnetic oscillations in the circuit?

19.3. A transmitter operates at a frequency of 600 kHz. What wavelength corresponds to this frequency?

19.4. An oscillatory circuit consists of a 200-pF capacitor and a coil of inductance 20 mH. What are the frequency and the period of natural oscillations of the circuit?

19.5. When a motor car enters a tunnel, the sound of its radio is attenuated if it does not vanish altogether. Why?

19.6. The ionosphere reflects radiowaves. How then can communication with spaceships by radio be realized?

19.7. An oscillatory circuit consists of a coil of inductance 2 mH and a capacitor whose capacitance can be varied from 10^{-9} F to 40 pF. What is the wavelength range for this circuit?

19.8. An oscillatory circuit consists of a coil of inductance 0.01 mH and a capacitor of capacitance 10^{-9} F. To what wavelength is this circuit tuned? What frequency corresponds to this wavelength?

19.9. An oscillatory circuit emits electromagnetic waves of length 500 m. Determine the capacitance of the capacitor in the circuit if its inductance is 1.5 mH.

19.10. What is the distance between an aeroplane and a radar if a signal reflected from the plane was received 3×10^{-4} s after it had been emitted?

19.11. How does the power of electromagnetic radiation depend on the oscillation frequency of the emitting circuit?

19.12. Figure 102 shows the circuit diagram of a crystal receiver. What component smooths the current pulses?

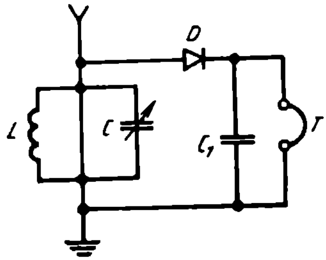


Fig. 102

19.13. Programmes are perceived from a crystal receiver using earphones since the received signals are very weak. How is a signal amplified in modern radios?

19.14. A radar emits electromagnetic waves of length 10 cm at a frequency of 2.25 GHz in a certain medium. What is the propagation velocity of the waves in this medium and what would their wavelength be in vacuum?

19.15. An oscillatory circuit containing a 1.5-μF capacitor should be tuned to a frequency of 1.5 kHz. What inductance is required?

19.16. The time dependence of current in an oscillatory circuit is described by the equation $i = 0.06 \sin 10^6 \pi t$. Determine the frequency of the electromagnetic oscillations and the inductance of the coil if the maximum energy of the magnetic field is 1.8×10^{-4} J.

19.17. An oscillatory circuit tuned to a frequency of 20 MHz contains a coil with inductance 10^{-6} H and a parallel-plate mica capacitor with a plate area of 20 cm^2. Determine the thickness of the mica if its permittivity is 6.

19.18. The maximum current in an oscillatory circuit is 6.28×10^{-3} A and the maximum charge on the capacitor is 10^{-8} C. Determine the period of oscillations in the circuit. To what wavelength is it tuned?

19.19. The maximum potential difference across the capacitor in an oscillatory circuit is 120 V. What will the

maximum current be if the capacitance of the capacitor is 9 μF and the coil has an inductance of 6 mH?

19.20. The time dependence of the current in an oscillatory circuit is given by the equation $i = 0.02 \sin 500\pi t$. The inductance of the circuit is 0.1 H. Determine the period of electromagnetic oscillations, the capacitance of the circuit, and the maximum energies of the electric and magnetic fields.

19.21. As the capacitance of a capacitor decreases by 100 pF, the resonance frequency of the oscillatory circuit increases from 0.2 to 0.25 MHz. What is the inductance of the circuit?

Chapter IV

Optics. Fundamentals of the Special Theory of Relativity

§ 20. GEOMETRICAL OPTICS

Basic Concepts and Formulas

To analyze the phenomena associated with the propagation of visible radiation (light) in a homogeneous medium or through the boundary between two transparent media, we introduce the concept of a light beam.

In a homogeneous medium, light beams propagate in a straight line, which explains the formation of the umbra and penumbra.

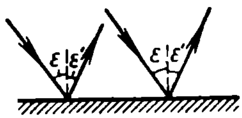


Fig. 103

If a reflecting surface is placed in the path of a light beam, light is reflected in accordance with the following two laws:

1. The incident ray, the reflected ray, and the perpendicular to the reflecting surface at the point of incidence of the ray lie in the same plane.

2. The angle of incidence ε is equal to the angle of reflection ε' (Fig. 103).

If the angles of incidence of two rays on a plane surface are the same, i.e. if the incident rays are parallel, they will remain parallel after reflection. Diverging rays emitted by a source of light remain divergent upon reflection (Fig. 104). In this case, the image formed at the intersection of the continuations of the reflected rays will be virtual and symmetric to the source about the mirror.

If a plane mirror is placed in the path of a converging beam, the image formed by it will be real, the point of intersection of the continuations of the incident rays will be a virtual image of the light source (Fig. 105), and $S'O = SO$.

Light rays incident on a spherical surface are reflected from it according to the same laws. If the mirror (reflecting) surface is the inner surface of the part of a sphere, the mir-

ror is called concave, and a mirror formed on the outer surface of a sphere is called convex.

The straight line passing through the centre of curvature C and point O (pole of a mirror) is known as the optical axis of the mirror (Fig. 106).

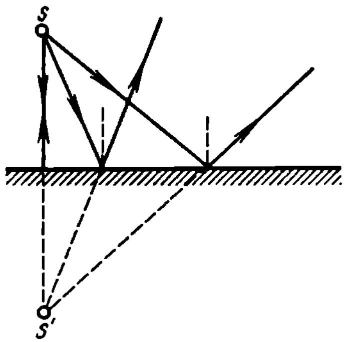


Fig. 104

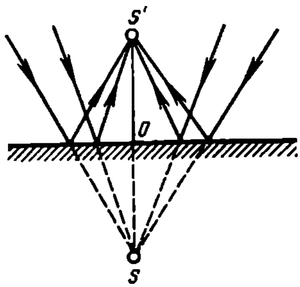


Fig. 105

A beam of rays incident on a concave mirror parallel to the optical axis is reflected such that it intersects the axis at a point F known as the focus. Concave mirrors have real foci, while convex mirrors have virtual foci. The distance FO is called the focal length and is denoted f:

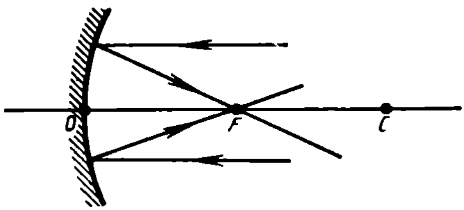


Fig. 106

$$FO = f = R/2,$$

where R is the radius of curvature of the mirror.

The formula for a spherical mirror has the form

$$\pm \frac{1}{f} = \frac{1}{a} + \frac{1}{a'}.$$

Here a and a' are respectively the distances from the object and from its image to the mirror. The minus sign indicates that the mirror is convex, and the plus sign corresponds to a concave mirror.

The linear magnification is given by

$$k = \frac{a'}{a} = \frac{f}{a \mp f}.$$

Here the minus sign in the denominator corresponds to a concave mirror, and the plus sign to a convex mirror.

The image of an object formed by a spherical mirror is constructed by using (1) an incident ray parallel to the optical axis, (2) a ray passing through the focus, and (3) a ray along the radius of the curvature.

Light is refracted when a ray is incident on the interface between two different transparent media, and obeys the following laws:

1. The incident and the refracted rays lie in the same plane as the perpendicular at the point of incidence to the interface between the two media.

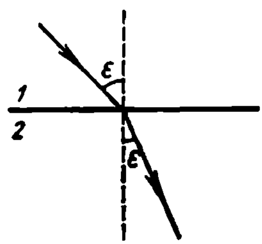


Fig. 107

2. For two given media, the ratio of the sines of the angles formed by the incident and refracted rays with the normal is a constant and called the refractive index of the second medium relative to the first one (Fig. 107):

$$\frac{\sin \varepsilon}{\sin \varepsilon'} = n_{2,1}.$$

In the case under consideration, the second medium is optically denser than the first one. As a ray passes from the first to the second medium, the angle of refraction is always smaller than the angle of incidence. If a ray passes from an optically denser medium to an optically rarer medium, it is possible for the rays not to be refracted but reflected totally, remaining in the same medium. This is called total internal reflection. It occurs when a ray is incident on the interface at an angle larger than a critical angle, defined as the

angle for which the angle of refraction is equal to 90°:

$$\frac{\sin \varepsilon_{cr}}{\sin 90^\circ} = \frac{1}{n}, \quad \sin \varepsilon_{cr} = \frac{1}{n}.$$

The relative refractive index can be expressed in terms of the velocities of light in two media:

$$n_{2,1} = v_1/v_2.$$

The absolute refractive index is

$$n = c/v,$$

where c is the velocity of light in vacuum.

Transparent media bounded by convex or concave surfaces are called lenses. Convex lenses converge rays, while con-

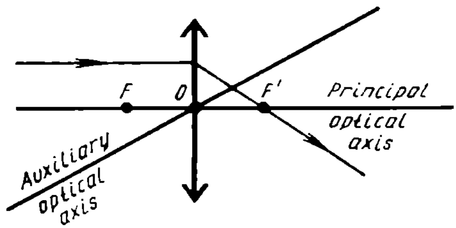


Fig. 108

cave lenses diverge them, if the refractive index of the lens material is larger than the refractive index of the medium surrounding it.

The principal optical axis of a lens is the line passing through the centres of curvature of its two refracting surfaces (Fig. 108). The point O in a lens is called its optical centre. Rays passing through the optical centre are not refracted. Points F and F' are known as the foci of a lens. Any straight line passing through the optical centre of a lens is called an auxiliary optical axis.

In order to construct the image formed by a lens, three rays are normally used: (1) a ray parallel to the principal optical axis, (2) a ray passing through one of the foci, and (3) a ray passing through the optical centre.

As is the case of spherical mirrors, the following formula is applicable to lenses:

$$\pm\frac{1}{f}=\frac{1}{a}+\frac{1}{a'}.$$

The sign rule here is the same as for mirrors.

The unit of optical power $\Phi = 1/f$ is the dioptre (D).

An image formed by a concave lens is always virtual, while a convex lens forms a virtual image when the object lies between the lens and a focus.

Lenses are widely used in optical instruments to magnify images of objects (projectors, magnifiers, and microscopes) or to increase the angle of view of examining objects a long distance from the observer (binocular and telescopes).

Worked Problems

Problem 94. What will happen if a plane mirror is placed in the path of a converging beam?

Solution. Let us examine the path of the rays shown in Fig. 109. Rays *1*, *2*, and *3* are incident on the plane mirror

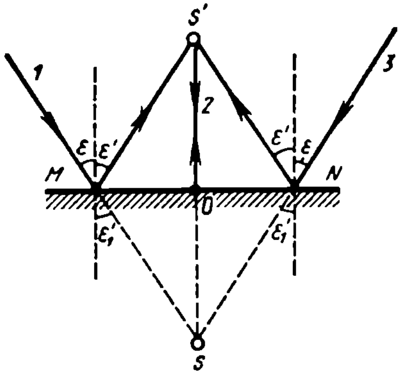


Fig. 109

in a converging beam. After reflection, they will intersect at a point S' which is a real image of point S. At this point the rays would converge in the absence of the mirror. Using

the laws of reflection of light, we can prove the equality of isosceles triangles $MS'N$ and MSN, whence it follows that $S'O = SO$.

Answer. A converging beam of rays will form a real and symmetric image of a virtual bright point.

Problem 95. A concave spherical mirror forms a threefold magnified image of an object. The distance from the object

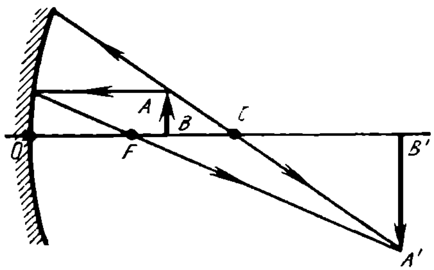


Fig. 110

to the image is 2.6 m. What is the radius of curvature of the mirror?

Given: $\beta = 3$ is the linear magnification of the image, $l = 2.6$ m is the distance from the object to the image.

Find: the radius of curvature R of the mirror.

Solution. We know that $R = 2f$. Consequently, the solution reduces to determining the focal length of the spherical mirror.

The image formed by a concave mirror is magnified in two cases. Let us consider both.

1. The object lies between the focus and the centre of curvature (Fig. 110). In order to construct the image $A'B'$ we shall use two rays, one of which is incident on the mirror parallel to the optical axis and the other is directed along the radius of curvature. After reflection, the rays will intersect at point A'. Dropping the perpendicular from A' onto the optical axis, we obtain point B'. We have a real image of the object with linear magnification $a' = \beta a$.

The problem states that $l = a' - a$, and hence $l = \beta a - a$. Therefore, $a = l/(\beta - 1)$ and $a' = \beta l/(\beta - 1)$. We substitute these values for a and a' into the formula for a

spherical mirror and determine the focal length:

$$\frac{1}{f}=\frac{1}{a}+\frac{1}{a'},\quad \frac{1}{f}=\frac{\beta-1}{l}+\frac{\beta-1}{\beta l},\quad \frac{1}{f}=\frac{\beta^2-1}{\beta l},$$

$$f=\frac{\beta l}{\beta^2-1},\quad f=\frac{3\times 2.6\ \text{m}}{9-1}=0.975\ \text{m}.$$

Consequently, $R = 2 \times 0.975$ m $= 1.95$ m.

2. The object lies between the mirror and the focus (Fig. 111). In order to construct the image, we shall use the same rays as above. It can be seen from the figure that the image

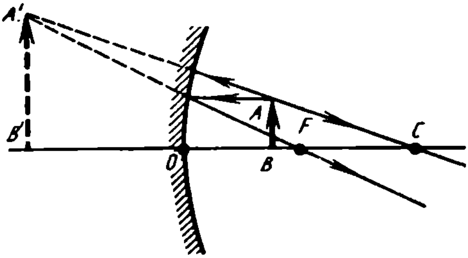


Fig. 111

of point A' is not on the intersection of the rays themselves but on their continuations, i.e. the image is virtual. In the formula for a spherical mirror, the quantity $1/a'$ should be taken with a minus sign. The subsequent reasoning is the same as before, the only difference being that $l = a + a'$. The result will be the same.

Answer. The radius of curvature of the spherical mirror is 1.95 m.

Problem 96. The radius of curvature of a convex spherical mirror is 1.2 m. How far away from the mirror is an object of height 12 cm if the distance between its virtual image and the mirror is 0.35 m? What is the height of the image?

Given: R = 1.2 m is the radius of curvature of the mirror, a' = 0.35 m is the distance between the virtual image and the mirror, and h = 0.12 m is the height of the object.

Find: the distance a between the object and the mirror and the height h' of the image.

Solution. Considering that the focus and the image for a convex mirror are virtual, we put the minus signs in front

of $1/f$ and $1/a'$ in the formula:

$$-\frac{1}{f}=\frac{1}{a}-\frac{1}{a'}, \quad a=\frac{a'f}{f-a'},$$

$$f=\frac{R}{2}=0.6 \text{ m}, \quad a=\frac{0.35 \text{ m}\times 0.6 \text{ m}}{0.6 \text{ m}-0.35 \text{ m}}=0.84 \text{ m}.$$

Let us determine the height of the image:

$$\frac{h'}{h}=\frac{a'}{a}, \quad h'=\frac{ha'}{a},$$

$$h'=\frac{0.12 \text{ m}\times 0.35 \text{ m}}{0.84 \text{ m}}=0.05 \text{ m}, \quad h'=0.05 \text{ m}.$$

Answer. The object is 0.84 m from the mirror, and the image is 5 cm high.

Problem 97. The Sun forms an angle of 60° with the horizon. Determine the length of the umbra at the bottom of an opaque vessel illuminated by sun light. The height of the vessel is 25 cm. What will the change in the length of the umbra be when water is poured into the vessel to a height of 20 cm (Fig. 112)?

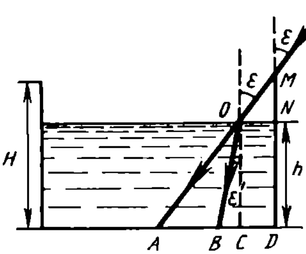


Fig. 112

Given: $H = 25$ cm is the height of the vessel, $\varphi = 60°$ is the altitude of the Sun above the horizon, $h = 20$ cm is the height of the water column. From tables, we take the refractive index for water, $n = 1.33$.

Find: the length l_1 of the umbra at the bottom of the empty vessel and the change Δl of the length of the umbra in the vessel with water.

Solution. We use ε to denote the angle formed by the direction of incident rays with the vertical wall of the vessel. It can be seen from the figure that the angle of incidence ε and the altitude of the Sun φ are related: $\varepsilon + \varphi = \pi/2$, $\varepsilon = 90°—60° = 30°$.

The rays propagate in the empty vessel along a straight line (along AM), and the length l of the umbra can be calcu-

lated from the right-angled triangle AMD:

$$l_1 = AD = H \tan \varepsilon, \tan 30^\circ = 0.577,$$

$$l_1 = 25 \text{ cm} \times 0.577 \simeq 14.4 \text{ cm}.$$

If the vessel contains water, the rays will be refracted at the air-water interface since a light ray bends away from the rarer medium into a denser medium. In this case, the refracted ray is closer to the vertical, and the angle of refraction ε' is smaller than the angle of incidence ε. The length of the umbra in the vessel filled with water is

$$l_2 = BD = BC + CD. \qquad \text{But } CD = ON.$$

In the right-angled triangle OMN, we have $ON = (H - h) \times \tan \varepsilon$, $ON = 5 \text{ cm} \times 0.577 \simeq 2.9$ cm. From the triangle BOC, we find $BC = h \tan \varepsilon'$. In order to determine the angle of refraction, we use the second law of refraction: $\sin \varepsilon / \sin \varepsilon' = n$, $\sin \varepsilon' = \sin \varepsilon / n$, $\sin \varepsilon' = 0.5/1.33 = 0.3759$, and $\varepsilon' \simeq 22^\circ$. From tables, we take $\tan 22^\circ = 0.404$. Consequently, $BC = 20 \text{ cm} \times 0.404 = 8.1$ cm.

The length of the umbra in the vessel with water is

$$l_2 = BD = 8.1 \text{ cm} + 2.9 \text{ cm} = 11 \text{ cm}.$$

When the vessel is filled with water, the length of the umbra decreases by Δl:

$$\Delta l = 14.4 \text{ cm} - 11 \text{ cm} = 3.4 \text{ cm}.$$

Answer. The length of the umbra in the empty vessel is approximately 14.4 cm. The umbra in the vessel with water is 3.4 cm shorter.

Problem 98. A light ray is incident at 45° on a glass slab. The slab is 3 cm thick, and the refractive index of the glass is 1.5. What will the displacement of the ray be as a result of its passage through the slab? At what angle will the ray emerge from the slab?

Given: $\varepsilon = 45^\circ$ is the angle of incidence of the ray, $h = 0.03$ m is the slab thickness, and $n = 1.5$ is the refractive index of the glass.

Find: the displacement δ of the ray and the angle ε_1' at which the ray emerges from the slab.

Solution. Figure 113 shows that the displacement of the ray is the shortest distance between the direction of the

incident ray and the ray emerging from the slab: $\delta = CD$. From the triangle ACD, we have $\delta = AC \sin(\varepsilon - \varepsilon')$, $AC = h/\cos \varepsilon'$. Consequently,

$$\delta = \frac{h \sin(\varepsilon - \varepsilon')}{\cos \varepsilon'}.$$

In order to determine the displacement, we must find the angle ε'. Using the second law of refraction, we obtain $n = \sin \varepsilon / \sin \varepsilon'$, $\sin \varepsilon' = \sin \varepsilon / n$, $\sin \varepsilon = \sqrt{2}/(2 \times 1.5) = 0.47$, and $\varepsilon' \simeq 28°$. This gives

$$\delta = \frac{0.03\ \text{m} \times 0.29}{0.88} \simeq 0.0099\ \text{m} \simeq 9.9\ \text{mm}.$$

Answer. After passing through the slab, the ray remains parallel to the incident ray but is displaced by 9.9 mm; $\varepsilon_1' = \varepsilon$.

Problem 99. An object of height 6 cm is set at right angles to the optical axis of a double-convex lens of optical power 5 D and 25 cm away from the lens. Determine the focal length of the lens, the position of the image, the linear magnification of the lens, and the height of the image formed by it.

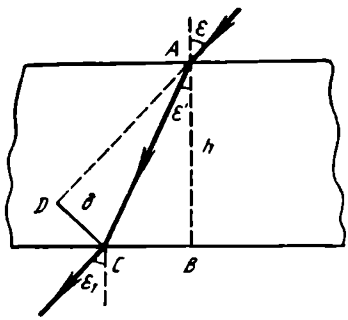


Fig. 113

Given: $h = 0.06$ m is the height of the object, $\Phi = 5$ D is the optical power of the lens, and $a = 0.25$ m is the distance between the object and the lens.

Find: the focal length f of the lens, the distance a' from the image to the lens, the linear magnification β, and the height h' of the image.

Solution. Given the optical power, we can determine the focal length of the lens:

$$f = \frac{1}{\Phi} = \frac{1}{5\ \text{D}} = 0.2\ \text{m}.$$

Let us construct the image of the object (Fig. 114). By choosing an appropriate scale, we can determine the required quantities from the diagram accurately enough.

Using the formula for a thin lens, we calculate a':

$$\frac{1}{f}=\frac{1}{a}+\frac{1}{a'}, \quad a'=\frac{fa}{a-f}, \quad a'=\frac{0.2\,\text{m}\times 0.25\,\text{m}}{0.05\,\text{m}}=1\,\text{m}.$$

The linear magnification β can be determined from the formula

$$\beta=\frac{a'}{a}, \quad \beta=\frac{1\,\text{m}}{0.25\,\text{m}}=4.$$

Consequently, the height of the image will be

$$h'=\beta h, \quad h'=4\times 0.06\,\text{m}=0.24\,\text{m}.$$

Answer. The focal length of the lens is 20 cm, the image is 1 m from the lens, the linear magnification of the lens is 4, and the image is 24 cm high.

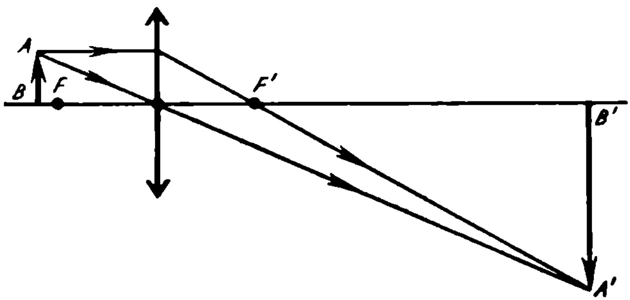


Fig. 114

Problem 100. An object 30 cm high stands vertically 80 cm from a lens with an optical power of -5 D. Determine the position of the image and its height.

Given: $h = 0.3$ m is the height of the object, $a = 0.8$ m is the distance from the object to the lens, and $\Phi = -5$ D is the optical power of the lens.

Find: the distance a' from the lens to the image and the height h' of the image.

Solution. The lens is concave since its optical power is negative. Let us determine its focal length:

$$f=\frac{1}{\Phi}=\frac{1}{-5\,\text{D}}=-0.2\,\text{m}.$$

We construct the image of the object formed by the lens (Fig. 115). The image is virtual. From the formula for a

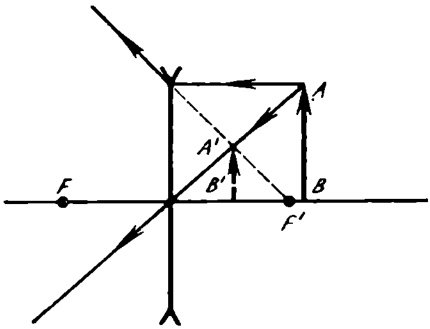


Fig. 115

thin lens, we determine the distance from the lens to the image:

$$\frac{1}{f}=\frac{1}{a}+\frac{1}{a'},\quad a'=\frac{af}{a-f},\quad a'=\frac{0.8\,\mathrm{m}\,(-0.2\,\mathrm{m})}{0.8\,\mathrm{m}+0.2\,\mathrm{m}}=-0.16\,\mathrm{m}.$$

In order to determine the image height, we shall use the formula for magnification:

$$\frac{h'}{h}=\frac{a'}{a},\quad h'=\frac{a'}{a}h,\quad h'=\frac{|0.16\,\mathrm{m}|}{0.8\,\mathrm{m}}\,0.3\,\mathrm{m}=|0.06|\,\mathrm{m}.$$

Answer. The distance from the lens to the image is —16 cm (the minus sign indicates that the image is virtual) and it is 6 cm high.

Problem 101. Given the principal optical axis MN, an object AB, and the image A' of point A (Fig. 116). Determine the positions of the lens and its foci. Is the lens converging or diverging? Construct the image of the object.

Fig. 116

Solution. We draw a ray from A to A'. It intersects the principal optical axis at the optical centre of the lens. We

direct the second ray from point A parallel to the principal optical axis. It is refracted by the lens, to hit A', intersecting the optical axis at a focus F' of the lens. We draw a vertical plane through the focus, which is known as the focal plane. In order to construct the image of the object, we need only construct the image of B. We draw a ray from

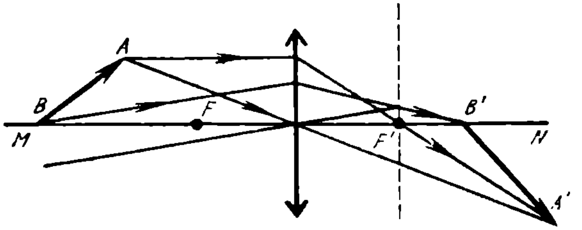


Fig. 117

B towards the lens at an arbitrary angle to the optical axis Parallel to this ray, we draw an auxiliary optical axis, which must intersect the ray at a point in the focal plane. Then point B' is obtained where the ray intersects the principal optical axis (Fig. 117). Connecting points A' and B', we obtain the image of the object.

Problem 102. A microscope has a magnification of 157.5. The focal length of the objective is 0.3 cm. An object is 0.31 cm from the objective. Determine the magnification of the eyepiece and its focal length. What is the length of the microscope drawtube if the image is 26.25 cm from the eyepiece?

Given: $\beta = 157.5$ is the magnification of the microscope, $f_1 = 0.3$ cm is the focal length of the objective, $a_1 = 0.31$ cm is the distance from the object to the objective, and $a_2' = 26.25$ cm is the distance from the eyepiece to the final image.

Find: the magnification β_2 of the eyepiece, the focal length f_2 of the eyepiece, and the length L of the drawtube.

Solution. The magnification of a microscope is determined by the magnifications β_1 and β_2 of the objective and eyepiece: $\beta = \beta_1\beta_2$.

In order to determine β_2, we must know the magnification β_1 of the objective. Let us determine the position of

the image formed by the objective from the formula

$$\frac{1}{f_1}=\frac{1}{a_1}+\frac{1}{a_1'},$$

$$a_1'=\frac{f_1a_1}{a_1-f_1}, \quad a_1'=\frac{0.3\text{ cm}\times 0.31\text{ cm}}{0.31\text{ cm}-0.3\text{ cm}}=9.3\text{ cm}.$$

Remark. To avoid cumbersome numbers, we shall express all quantities in centimetres:

$$\beta_1=\frac{a_1'}{a_1}=\frac{9.3\text{ cm}}{0.31\text{ cm}}=30.$$

We can now find the magnification of the eyepiece:

$$\beta_2=\frac{\beta}{\beta_1}=\frac{157.5}{30}=5.25.$$

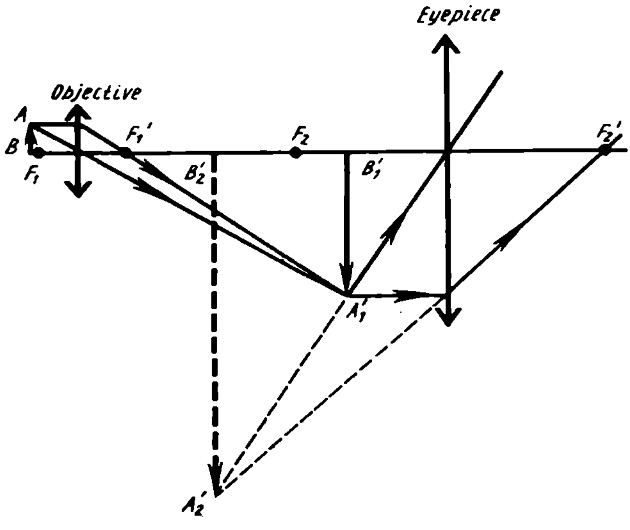


Fig. 118

Let us determine the distance a_2 from the eyepiece to the image $A_1'B_1'$, which is the object relative to the eyepiece (Fig. 118):

$$a_2=\frac{a_2'}{\beta_2}, \quad a_2=\frac{26.25\text{ cm}}{5.25}=5\text{ cm}.$$

The eyepiece is arranged relative to the image $A'_1B'_1$ so that the image is as if magnified. The image $A'_2B'_2$ will thus be virtual, and while determining the focal length f_2 of the eyepiece we must put the minus sign in front of $1/a'_2$:

$$\frac{1}{f_2}=\frac{1}{a_2}-\frac{1}{a'_2},\quad \frac{1}{f_2}=\frac{1}{5\text{ cm}}-\frac{1}{26.25\text{ cm}},\quad f_2=6.2\text{ cm}.$$

The length of the drawtube of the microscope can be determined from the formula

$$L=a'_1+a_2,\ L=9.3\text{ cm}+5\text{ cm}=14.3\text{ cm}.$$

Answer. The magnification of the eyepiece is 5.25, the focal length of the eyepiece is 6.2 cm, and the length of the drawtube is 14.3 cm.

Problem 103. A point is 35 cm along the optical axis from a spherical concave mirror having a focal length 25 cm. At what distance along the optical axis from the concave mirror should a plane mirror be placed for the image it forms to coincide with the point source?

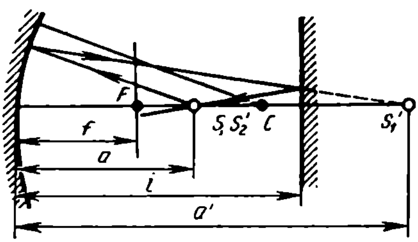


Fig. 119

Given: $f = 0.25$ m is the focal length of the mirror and $a = 0.35$ m is the distance between the point source and the mirror.

Find: the distance l between the plane and concave mirrors.

Solution. Without the plane mirror, the image of point S would be formed at a point S'_1 (Fig. 119). Converging rays are incident on the plane mirror, and hence point S'_1 should be treated as a virtual source of light, its image being real (see Problem 94) and separated from the concave mirror by

a. The distance a' to S_1' can be determined from the formula

$$a' = \frac{af}{a-f}, \quad a' = \frac{0.35\ \text{m} \times 0.25\ \text{m}}{0.35\ \text{m} - 0.25\ \text{m}} = 0.875\ \text{m}.$$

Then the distance between the mirrors is

$$l = a + \frac{a'-a}{2}, \quad l = 0.35\ \text{m} + \frac{0.875\ \text{m} - 0.35\ \text{m}}{2} \simeq 0.61\ \text{m}.$$

Answer. The point source and its image formed by the plane mirror coincide when the distance between the mirrors is approximately 61 cm.

Questions and Problems

Reflection of light. Plane and spherical mirrors

20.1. A light ray is incident on a plane mirror. By what angle will the reflected ray be deflected if the mirror is turned through 20°?

20.2. The angle of incidence of a light ray on a plane mirror is decreased by 15°. How much will the angle between the incident and the reflected ray be reduced?

20.3. Sun rays are incident at an angle of 24° to the horizon. How can they be directed parallel to the horizon using a plane mirror?

20.4. The Sun is 46° above the horizon. What must the angle of incidence of the sun rays on a plane mirror be so that the sun light is reflected (a) vertically downwards, (b) vertically upwards? What will the angle between the reflected ray and the mirror be? Draw the light paths on a diagram.

20.5. A parallel beam propagates horizontally from a projector. How should a plane mirror be arranged so that the image of a slide is formed on the ceiling?

20.6. A mirror hangs vertically on a wall so that its upper edge is level with the person's head. The length of the mirror is 80 cm. What is the minimum height of the person to still see himself full length?

20.7. A plane mirror is at 45° to the horizontal surface of a table on which a book lies. In what plane will the image of the book be formed?

20.8. A beam is incident on a plane mirror (Fig. 120). Show graphically the light path. What type of image will be formed of a lamp and where?

20.9. Two plane mirrors form an angle of 120°. The distance between the two images of a point source formed in

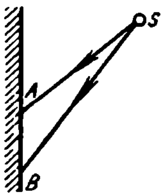


Fig. 120

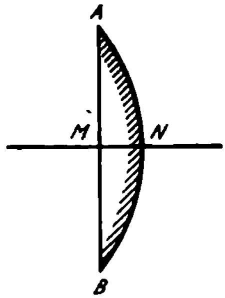


Fig. 121

them is 20 cm. Determine the distance from the light source to the point where the mirrors touch if it lies on the bisector of the angle formed by the mirrors.

20.10. How should two plane mirrors be arranged for a point source of light and its two images to lie at the vertices of an equilateral triangle?

20.11. How many images of an object will be formed by two plane mirrors placed (a) at right angles to each other, (b) at an angle of 60°?

20.12. How many images will be formed by two mirrors lying in parallel planes on both sides of an object?

20.13. How can we see the image of a large building formed by a small plane mirror?

20.14. On a sunny day, the focus of a concave mirror can be determined just by using a ruler. How can this be done?

20.15. Determine the radius of curvature and the focal length of a convex spherical mirror (Fig. 121) if $MN = 3$ cm and $AB = 30$ cm.

20.16. A point source of light is on the axis of a spherical mirror four focal lengths away from it. Construct its image.

20.17. The focal length of a concave spherical mirror is 0.6 m. At what distance from the pole must a point source

be located on the optical axis so that its image is formed at the same point?

20.18. A concave spherical mirror forms a threefold magnified inverted image of an object. What are the principal focal length and the radius of curvature of the surface of the mirror if the distance from the object to the image is 28 cm?

20.19. The focal length of a concave spherical mirror is 25 cm. Where should an object be placed so that its virtual image is formed 1 m from the mirror's pole?

20.20. When an object is 40 cm away from the pole of a concave spherical mirror, the size of its image is equal to that of the object. What is the focal length of the mirror?

20.21. A point source lies on the principal optical axis of a concave spherical mirror, and its virtual image is 40 cm away from the mirror. The radius of curvature of the mirror is 1.2 m. Determine the distance from the source to the pole. What is the magnification?

20.22. A concave spherical mirror forms a real image of an object with magnification 3. When the object is moved a further 5 cm away from the mirror, the magnification of the new real image becomes 2. Determine the radius of curvature of the mirror surface and the initial distance between the object and the mirror.

20.23. A converging beam of solar rays is incident on a concave spherical mirror whose radius of curvature is 0.8 m. Determine the position of the point on the optical axis of the mirror where the reflected rays intersect if the extensions of the incident rays intersect the optical axis 40 cm from the mirror's pole.

20.24. How far from an object in front of a concave spherical mirror will a real image be formed if it is 42 cm from the pole and the radius of curvature of the mirror is 48 cm?

20.25. Why are convex mirrors better than plane ones in motor cars for viewing the road?

20.26. How far from a convex mirror does an object lie if its image is formed 1 m away from the mirror? The focal length of the mirror is 1.5 m.

20.27. When an object is 60 cm from the pole of a concave spherical mirror, its image is formed with a magnification $\beta = -5$. What does the minus sign of the magnification indicate? What is the radius of curvature of the mirror?

20.28. Where will the image of an object be formed by a convex spherical mirror with a 1.2-m radius of curvature and what type of the image will it be if the object is 0.3 m away from the mirror?

20.29. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

20.30. Two spherical mirrors (convex and concave) having the same focal length of 36 cm are arranged so that their

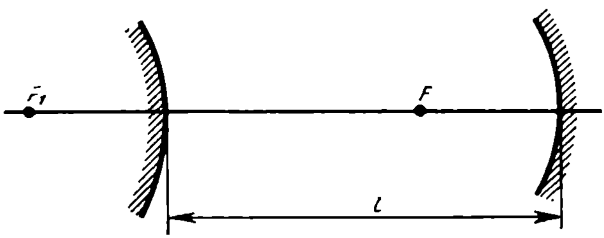


Fig. 122

optical axes coincide (Fig. 122). The separation between the mirrors is 1 m. At what distance from the concave mirror should an object be placed so that its images formed by the concave and convex mirrors are identical?

Refraction of light

20.31. What is the refractive index of flint glass if for a light rays incident at an angle of 63° the angle of refraction is 29°40′?

20.32. A light ray is incident on the surface of water at an angle of 50°. What is the angle of refraction in water?

20.33. At what angle should a light ray be incident on the surface of crown glass so that the angle of refraction is 27°?

20.34. Determine the angle through which a light ray is deflected as it passes from air to water at an angle of incidence of (a) 30°, (b) 45°?

20.35. When a light ray is incident on a quartz plate at an angle of 44°, the angle of refraction is 27°. For what angle of incidence will the ray be refracted at an angle of 30°?

20.36. Why is it difficult to shoot a fish swimming in water?

20.37. A light ray is incident on an air-liquid interface at 45° and is refracted at 30°. What is the refractive index of the liquid? For what angle of incidence will the angle between the reflected and refracted rays be 90°?

20.38. A light ray passes through the interface between two transparent media. Under what condition will the angle of refraction be equal to the angle of incidence?

20.39. Parallel beams of light pass from air to water. The angle of incidence is 55°. Determine the angle of refraction of the rays and the propagation velocity in water.

20.40. Determine the velocity of light in ice if, for an angle of incidence of 61°, the angle of refraction is 42°.

20.41. As a light ray passes from air to a transparent liquid at an angle of incidence of 69°, the angle of refraction is

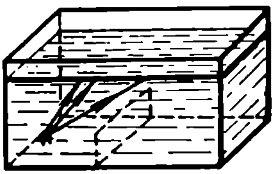


Fig. 123

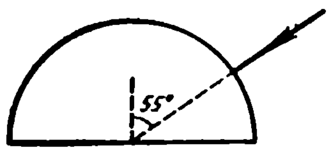


Fig. 124

38°30′. What is the velocity of light in the liquid? What si the refractive index of the liquid?

20.42. Determine the angle of incidence of a light ray on the surface of acetone if the angle between the reflected and refracted rays is 120°.

20.43. The refractive index of rock crystal is 1.54. At what angle must a light ray be incident from air to the crystal so that the reflected ray is perpendicular to the refracted ray?

20.44. A source of visible light at the bottom of an aquarium filled with water emits rays incident on the water-air interface (Fig. 123). Trace the light path afterwards.

20.45. Determine the critical angle of incidence for a light ray passing from diamond to water.

20.46. Determine the refractive index of benzene if the critical angle of incidence of rays on it is 42°.

20.47. For what angle of incidence of a light ray on a diamond-water interface is total reflection observed?

20.48. If a bright metal ball is first blackened with soot in a candle flame and then immersed in water, it appears bright again. Why?

20.49. Can you explain why a test tube immersed at a certain angle in a tumbler of water appears to have a mirror surface for a certain viewing position?

20.50. The direction of an incident light ray is shown in Fig. 124. The semicylinder is made of glass with refractive index 1.8. What is the path of the ray afterwards?

20.51. A bulb is fixed at the bottom of an aquarium divided into two parts by a partition (see Fig. 123). What is the maximum height to which water may be poured if the bottom at the opposite face is to be illuminated and the aquarium is 60 cm long?

20.52. Solar rays are incident on the surface of water at 45°. How long is the shadow of a pole erected at the bottom of a pond if the pole is vertical and completely immersed in water? The length of the pole is 1.2 m.

20.53. Solve Problem 20.52 assuming that 0.2 m of the pole is above the water surface.

20.54. A light ray is incident at 45° on a plate made of crown glass with plane-parallel faces. Determine the thickness of the plate if a ray emerging from it is displaced by 1.5 cm.

20.55. Determine the displacement of a light ray passing through a 2.1-cm thick plate made of crown glass with plane-parallel faces if the angle of incidence is 30°.

20.56. A plate with plane-parallel faces, having refractive index 1.8, rests on a plane mirror. A light ray is incident on the upper face of the plate at 60°. How far from the entry point will the ray emerge after the reflection by the mirror if the plate is 6-cm thick?

20.57. Light rays passing through a plane-parallel plate are displaced. Why do we not notice this through a window? Under what conditions does the displacement become noticeable?

20.58. A light ray is incident on a glass plate with plane-parallel faces at 70°. The plate is 4-cm thick. The refractive

index of glass is 1.5. Determine (1) the displacement of the ray and (2) the path length of the ray in the plate.

20.59. A light ray is incident at right angles on a right-angled glass prism (Fig. 125). What will the path of the ray be afterwards?

20.60. How must two right-angled prisms be arranged in a periscope? Draw the light path in it.

20.61. Two parallel rays *1* and *2* are incident on a glass right-angled prism (Fig. 126). Trace the light path afterwards.

20.62. A light beam is incident along the normal on a lateral face of a prism with an angle of refraction of 30°

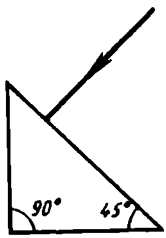


Fig. 125

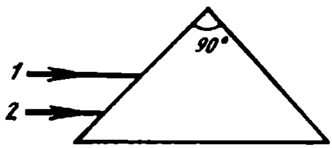


Fig. 126

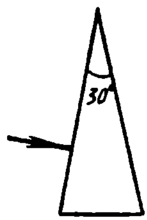


Fig. 127

(Fig. 127). Determine the angular displacement of the beam as a result of its passage through the prism if its refractive index is 1.8.

20.63. A ray incident on the lateral face of a glass prism with an angle of refraction of 30° emerges from it at 30°. The refractive index of glass is 1.5. Determine the angle of incidence of the ray.

20.64. A ray is incident on a lateral face of a glass prism along the normal and emerges from the prism at 25° from the direction of the incident ray. The refractive index of glass is 1.5. What is the angle of refraction of the prism?

20.65. The angle of refraction of a prism is 60°. A light ray emerges from the prism at the same angle as it is incident on it. The refractive index of the prism is 1.5. Determine the angle by which the ray is deflected from its initial direction as a result of its passage through the prism.

20.66. The angle of refraction of a prism is A. A light beam is deflected by an angle δ upon passing through the prism.

Determine the refractive index of the prism if the angle of incidence of the ray is equal to the angle of refraction as it emerges from the prism.

Lenses. Optical instruments

20.67. Determine the optical power of a double-convex lens whose focal length is (a) 12.5 cm, (b) 50 cm.

20.68. Determine the optical powers of concave lenses with focal lengths of —25 cm and —0.4 m.

20.69. Given lenses with optical powers of 4, —5, and —2 D. Determine their focal lengths.

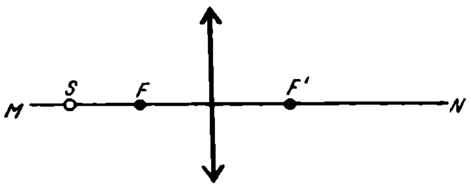


Fig. 128

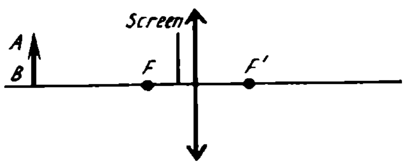


Fig. 129

20.70. In what case will a double-convex lens be diverging?

20.71. Given an optical axis MN, a converging lens, its foci, and a point source S on the optical axis (Fig. 128). Construct the image of the point source.

20.72. An opaque screen is placed between an object and a converging lens (Fig. 129). What will happen to the image? Use a diagram to explain your answer.

20.73. What will the paths of the rays be after refraction in the lenses shown in Figs. 130*a* and *b*?

20.74. Two converging lenses should be placed in the path of parallel rays so that the rays remain parallel after

passing through both lenses. How should the lenses be arranged?

20.75. The focal length of a converging lens is 20 cm. An object is 60 cm from the lens. Where will the image be formed and what kind of image is it?

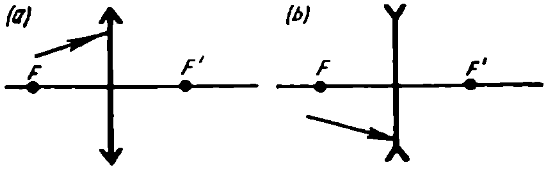


Fig. 130

20.76. How far away will a converging lens with a focal length of 10 cm form the images of objects located 5, 15, 20, and 25 cm from the lens?

20.77. Construct the images of the objects in Problem 20.76 on a 1 : 10 scale and answer the following questions: (1) how does the height of the image change with the distance between the object and the lens? (2) what type of image is it when an object is placed twice the focal length from the lens? (3) where should the object be placed to obtain a virtual image?

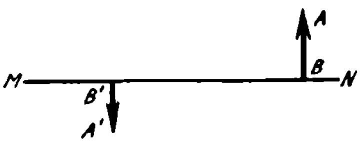


Fig. 131

20.78. When an object is placed 20 cm from a lens, the image is the same size as the object. What are the focal length and optical power of the lens?

20.79. Given an optical axis MN and the positions of an object and its image (Fig. 131), determine graphically the position of the lens (its optical centre O) and its foci. Is it a converging or diverging lens? Is the image real or virtual?

20.80. At what distance from a double-convex lens with focal length 0.42 m is an object placed if its virtual image is formed 56 cm from the object?

20.81. A candle is 2 m from a wall. When a converging lens is placed 40 cm from the candle between the candle and

the wall, a sharp image of the candle is formed on the wall. Determine the optical power of the lens and its magnification.

20.82. A lens placed between a candle and a screen forms a real triply magnified image of the candle on the screen. When the lens is moved away from the candle by 0.8 m without changing the position of the candle, a real image one-third the size of the candle is formed on the screen. Determine the focal length of the lens.

20.83. An object 2 cm high is 15 cm from a double-convex lens whose optical power is 10 D. Determine the height of the image.

20.84. An object is $1.4f$ (where f is the focal length) from a converging lens with an optical power of 2 D. Where is the image formed? What type of image is it?

20.85. An object is placed $0.8f$ from a lens (see Problem 20.84). Where is the image formed? What type of image is it?

20.86. The focal length of a double-concave lens is 12 cm. What is the distance between the lens and an object if its image is 8 cm away from the lens?

20.87. An object is 20 cm from a double-concave lens with a focal length of 30 cm. Where is the image formed (relative to the lens) and how high is it if the object is 6 cm high?

20.88. A virtual image is formed 8 cm from a diverging lens with a focal length of 0.12 m. How far is the object from its image?

20.89. Will the focal length of a glass lens change if it is immersed in a medium with the same optical density as the glass?

20.90. What will the magnification of a converging lens with an optical power of 4 D be if an object is $1.4f$ away from it?

20.91. A virtual image one-fifth of the size of an object is formed 6 cm from a diverging lens. Find the focal length and optical power of the lens and construct the image of the object formed by it.

20.92. The distance from a converging lens to an object is thrice the focal length. Determine the magnification.

20.93. The optical power of a lens is 2.5 D. For a certain position of an object relative to the lens, a real doubly mag-

nified image is formed on a screen. What will the magnification be if the object is moved 0.1 m closer to the lens?

20.94. A converging lens forms a triply magnified real image of an object. In order to obtain a virtual image with the same magnification the lens is moved towards the object by 10 cm. What are the focal length and the optical power of the lens?

20.95. Two identical thin converging lenses brought in contact so that their axes coincide are placed 12.5 cm from an object. What is the optical power of the system and that of a single lens if the real image formed by the system of lenses is four times as large as the object?

20.96. A convex and a concave lens are brought in close contact along their optical axes. The focal length of the convex lens is 10 cm. When the system is placed at 40 cm from an object, a sharp image of the object is formed on a screen on the other side of the system. Determine the optical power of the concave lens if the distance l between the object and the screen is 1.6 m.

20.97. The focal length of a projector's objective is 15 cm. The distance between the slide and the objective is 15.5 cm. What is the linear magnification of the projector?

20.98. Determine the optical power of the projector's objective if it produces a 24-fold magnification when a slide is placed 20.8 cm from the objective.

20.99. The objective of a projector has a focal length of 12.5 cm. How far should the screen be placed away from the objective to obtain a twentyfold magnification?

20.100. The image of a 7×5 cm^2 slide is projected onto a 1.05×0.75 m^2 screen using projector at its maximum magnification. What is the optical power of the objective's lens if the screen is 4 m away?

20.101. What purpose do diaphragms serve in the objectives of cameras?

20.102. The focal length of a camera's objective is 50 mm. The height of the image on the film of a building 80 m away from the camera is 12 mm. What is the actual height of the building?

20.103. The image on the frosted glass of a camera is 13.5 mm high when the object is 8.5 m away and 6 cm when 2 m away. Determine the optical power of the objective.

20.104. The focal length of a microscope's objective is 5 mm and the drawtube is 16 cm long. Determine the magnification of the eyepiece for normal sight if the microscope's total magnification is 200.

20.105. The focal length of a microscope's objective is 0.4 cm. A sample is placed 0.1 mm away from the focus. The magnification of the microscope is 400. Determine the focal length of the eyepiece and the length of the drawtube if the distance of best vision for an observer is 25 cm.

§ 21. PHOTOMETRY

Basic Concepts and Formulas

In photometry, the concept of a point source of visible radiation (light) is employed. A fraction of this radiation corresponding to the wavelength range from 400 to 760 nm acts on the retina of the eye and is perceived as light. A point source is assumed to be considerably smaller than the distance over which its action is analyzed. Such a source is assumed to emit uniformly in all directions. An example of such a source is a star.

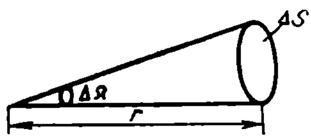


Fig. 132

The luminous flux Φ is the power of visible radiation and is estimated by its action on the retina of a normal eye. The unit of a luminous flux is the lumen (lm).

The luminous intensity (candle power) I of a source is defined as the ratio of the luminous flux propagating from the source in a given direction within a small solid angle to the magnitude of this angle:

$$I = \frac{\Delta\Phi}{\Delta\Omega}.$$

The unit of solid angle is the steradian (sr). A steradian is the solid angle bounded by a conic surface which cuts an area of a sphere's surface ΔS equal to the square of the radius (Fig. 132), the apex of the solid angle being at the centre of the sphere. The solid angle embracing the space around the

point rests on the surface of the sphere and is equal to

$$\Omega = \frac{4\pi R^2}{R^2} = 4\pi \text{ sr}.$$

The total luminous flux emitted by a light source is

$$\Phi_{tot} = I\Omega = 4\pi I.$$

The unit of luminous intensity is the candela (cd).

A luminous flux incident on the surface of a body is partially reflected, reaches the eye retina, and causes the sensation of light—we see the things surrounding us.

The illuminance E is defined as the ratio of the luminous flux Φ incident on a small surface element to its area

$$E = \Phi/S.$$

The unit of illuminance is the lux (lx).

The illuminance of a surface by normal rays varies in inverse proportion to the squared distance from the light source to the surface:

$$E_0 = I/R^2.$$

If rays are incident on a surface at an angle, the illuminance depends on the angle of incidence

$$E = E_0 \cos \varepsilon, \text{ or } E = \frac{I}{R^2} \cos \varepsilon.$$

The luminous intensities of two sources can be compared using a photometer. The two sources are placed on either side of the photometer at distances such that the illuminances at the photometer are equal. By measuring the distances from the light sources to the photometer and given the luminous intensity of one of the sources, we can determine the luminous intensity of the other source:

$$E = \frac{I_1}{R_1^2}, \quad E = \frac{I_2}{R_2^2},$$

$$\frac{I_1}{R_1^2} = \frac{I_2}{R_2^2}, \quad \text{or} \quad \frac{I_1}{I_2} = \frac{R_1^2}{R_2^2}.$$

Worked Problems

Problem 104. Two lamps with luminous intensities of 200 and 300 cd are suspended from 3-m high poles (Fig. 133). The distance between the lamps is 4 m. Determine the illuminance on the ground at points A, B, and C.

Given: $h = 3$ m is the height of the lamps above the ground, $I_1 = 200$ cd and $I_2 = 300$ cd are the luminous intensities of the lamps, and $l = 4$ m is the distance between them.

Find: the illuminances E_A, E_B, and E_C at points A, B, and C.

Solution. At points A, B, and C, the surface of the ground is illuminated by both the lamps. Let us consider each case separately. At point A,

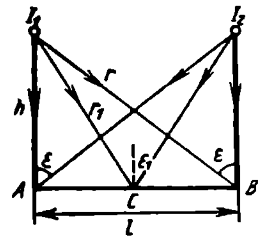


Fig. 133

$$E_A = E_{1A} + E_{2A},$$

where E_{1A} is the illuminance at point A from the first lamp and E_{2A} is that from the second lamp. The light rays from the first lamp are incident along the normal to the surface at point A, and hence

$$E_{1A} = \frac{I_1}{h^2}, \quad E_{1A} = \frac{200\ \text{cd}}{9\ \text{m}^2} = 22.2\ \text{lx}.$$

The illuminance produced at the same point by the second lamp is due to the rays incident at an angle ε (see Fig. 133). Therefore, $E_{2A} = E_0 \cos \varepsilon$, where E_0 is the illuminance of the surface at point A due to rays incident along the normal, $E_0 = I_2/r^2$. Since $r = \sqrt{h^2 + l^2}$, we have $E_0 = \dfrac{I_2}{h^2+l^2}$. It can be seen from the figure that $\cos \varepsilon = \dfrac{h}{r} = \dfrac{h}{\sqrt{h^2+l^2}}$.

The second lamp produces at point A an illuminance of

$$E_{2A} = \frac{I_2}{h^2+l^2} \frac{h}{\sqrt{h^2+l^2}}, \quad E_{2A} = \frac{300\ \text{cd} \times 3\ \text{m}}{25\ \text{m}^2 \times 5\ \text{m}} = 7.2\ \text{lx}.$$

The total illuminance at point A is

$$E_A = 22.2\ \text{lx} + 7.2\ \text{lx} = 29.4\ \text{lx}.$$

The illuminance at the ground at point B is found in the same way:

$$E_B = E_{1B} + E_{2B}.$$

The angle of incidence of the rays from the first lamp remains the same, and

$$E_B = \frac{I_1}{r_1^2}\cos\varepsilon + \frac{I_2}{h^2}, \quad E_B = \frac{200\ \text{cd}}{25\ \text{m}^2}\,0.6 + \frac{300\ \text{cd}}{9\ \text{m}^2} = 38.1\ \text{lx}.$$

The illuminance at point C is

$$E_C = E_{1C} + E_{2C}.$$

Since point C is equidistant from the two lamps, the rays from the lamps form the same angle ε_1 with the normal. Therefore,

$$E_C = \frac{I_1}{r_1^2}\cos\varepsilon_1 + \frac{I_2}{r_1^2}\cos\varepsilon_1, \quad \text{or} \quad E_C = \frac{\cos\varepsilon_1}{r_1^2}(I_1 + I_2).$$

Here $r_1^2 = h^2 + (0.5l)^2 = 13\ \text{m}^2$, and $\cos\varepsilon_1 = \frac{h}{r_1} = \frac{3\ \text{m}}{3.6\ \text{m}} = 0.83$.

Finally, we have

$$E_C = \frac{0.83}{13\ \text{m}^2}(200\ \text{cd} + 300\ \text{cd}) = 31.9\ \text{lx}.$$

Answer. The illuminance of the ground at points A, B, and C is 29.4, 38.1, and 31.9 lx respectively.

Problem 105. Two incandescent lamps with luminous intensities 25 and 225 cd are 1 m apart (Fig. 134). Where should a screen be placed between them for it to be equally illuminated from both sides?

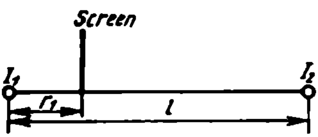


Fig. 134

Given: $I_1 = 25$ cd and $I_2 = 225$ cd are the luminous intensities of the first and second lamps and $l = 1$ m is the separation between them.

Find: the distance from the first lamp to the screen.

Solution. We shall assume that the rays from the first and second lamps are incident on the screen at right angles. Then the illuminances of the screen by the lamps are

$$E_1 = \frac{I_1}{r_1^2}, \quad E_2 = \frac{I_2}{r_2^2}.$$

By hypothesis, $E_1 = E_2$, and hence $\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}$. But $r_2 = l - r_1$. Therefore, $\frac{I_1}{r_1^2} = \frac{I_2}{(l-r_1)^2}$, or $\frac{I_1}{I_2} = \frac{r_1^2}{(l-r_1)^2}$.

We shall solve this equation for r_1:

$$\frac{\sqrt{I_1}}{\sqrt{I_2}}=\frac{r_1}{l-r_1},\ (l-r_1)\sqrt{I_1}=r_1\sqrt{I_2},\ l\sqrt{I_1}=r_1(\sqrt{I_1}+\sqrt{I_2}),$$

whence

$$r_1=\frac{l\sqrt{I_1}}{\sqrt{I_1}+\sqrt{I_2}},\ r_1=\frac{1\ \mathrm{m}\sqrt{25\ \mathrm{cd}}}{\sqrt{25\ \mathrm{cd}}+\sqrt{225\ \mathrm{cd}}}=0.25\ \mathrm{m}.$$

Answer. The screen should be placed 0.25 m from the first lamp.

Problem 106. A 25-cd point source of light is at the centre of a spherical surface of radius 0.5 m. Determine the luminous flux incident on the inner surface of the sphere over an area of 50 cm².

Given: $r = 0.5$ m is the radius of the spherical surface, $I = 25$ cd is the luminous intensity of the source, and $S = 50\ \mathrm{cm}^2 = 5 \times 10^{-3}\ \mathrm{m}^2$ is the area of the surface element.

Find: the luminous flux Φ.

Solution. In order to determine the luminous flux, we must know the area of the surface on which it is incident and its illuminance:

$$\Phi = ES.$$

The illuminance is produced by the rays incident along the normal on the inner surface of the sphere. Therefore, $E = I/r^2$, and hence

$$\Phi=\frac{I}{r^2}S,\ \Phi=25\ \mathrm{cd}\ \frac{5\times10^{-3}}{0.25}\ \mathrm{sr}=0.5\ \mathrm{lm}.$$

Answer. A luminous flux of 0.5 lm is incident on a surface element of area S.

Questions and Problems

21.1. Determine the total luminous flux emitted by a source of luminous intensity 200 cd.

21.2. A luminous flux of 2 lm is uniformly distributed within a solid angle of 0.5 sr. What is the luminous intensity of the point source located at the apex of the angle?

21.3. A 15-cd point source of light is placed at the centre of a hollow sphere of radius 30 cm. Determine the illumi-

nance of the inner surface of the sphere and the total luminous flux emitted by the source.

21.4. Determine the luminous flux passing through a surface of area 20 cm^2 located 5 m from a 100-cd point source of light. The rays are incident normal to the surface.

21.5. The average illuminance in Leningrad during a summer night (when the Sun does not sink deep below the horizon) is 1 lx, while the illuminance during a moonlit night is 0.1 lx. What are the luminous fluxes incident on the Mars Field in Leningrad which covers an area of 0.1 km^2?

21.6. An incandescent lamp (without a shade) of luminous intensity 25 cd is suspended 80 cm above a table. Determine the illuminance on the table.

21.7. The luminous intensity of the lamp in a photographic enlarger is 15 cd. Determine the illuminance on a piece of photographic paper if the enlarger is 30 cm above it and only 15% of the luminous flux is used.

21.8. An incandescent lamp in a room produces an illuminance of 28 lx at one wall and 7 lx on the opposite wall at the same level. What is the ratio of the distances between the lamp and the walls?

21.9. When will the illuminance under a lamp be higher: for a luminous intensity of 120 cd 3 m away or for a luminous intensity of 25 cd 1.2 m away?

21.10. Parallel rays incident at an angle of 25° produce an illuminance of 54 lx. At what angle of incidence will the illuminance of the surface be 45 lx?

21.11. Before a sunset, sun light is incident on the surface of the Earth at an angle of 81°. Compare the illuminances produced on the surface of the Earth and on a vertical wall facing the Sun.

21.12. The maximum illuminance that can be created by sun light on the surface of the Earth is 108 000 lx. How far from the Sun is the planet Mars when the maximum solar illuminance on its surface is 48 000 lx?

21.13. Why does snow melt more quickly on sunlit slopes than on sunlit horizontal areas?

21.14. When parallel rays are incident on an object along the normal to its surface, the illuminance is 70 lx. What will the illuminance of the surface be if the object is turned so that the angle of incidence is 60°?

21.15. An electric bulb of luminous intensity 150 cd is

suspended above a round table of diameter 2 m. Determine the maximum and minimum illuminances on the table if the distance of the cent e of the table from the lamp is 1.5 m.

21.16. A lamp without a shade is suspended 1 m above the centre of a table with diameter 1.2 m. Determine the illuminance at the edge of the table if the total luminous flux from the lamp is 650 lm.

21.17. A bulb without a shade is suspended 1 m above a table. The distance between the bulb and a book lying at the edge of the table is 2 m. What must luminous intensity of the bulb be if the illuminance on the book is 25 lx?

21.18. The light from an electric bulb is incident on a working space at an angle of 45° and produces an illuminance of 141 lx. The luminous intensity of the bulb is 200 cd. How far away is the bulb from the working space? How high above the working space is it suspended?

21.19. Two bulbs with the same luminous intensity of 50 cd are suspended 1 m above a table. The distance between the bulbs is 140 cm. Determine the illuminance on the table under each bulb.

21.20. Two lamps of 200 cd each are fixed to a pole 2 and 3 m from the ground respectively. Determine the illuminance on the ground at 1 m from the foot of the pole.

21.21. A lamp is suspended at the top of a 10-m pole so that the illuminance on the ground 10 m from the base of the pole is 2.5 lx. What is the luminous intensity of the lamp?

21.22. Two lamps of 250 cd each are suspended from a height of 4 m on poles 5 m apart. Determine the illuminance on the ground midway the poles.

21.23. A lamp of 800 cd is suspended 10 m above the ground. Over what area of the ground will the illuminance be at least 1 lx?

21.24. Two incandescent lamps with luminous intensities 300 and 200 cd are suspended at a height of 3 m. The distance between the lamps is 4 m. Determine the illuminance of the point on the ground between the lamps, where rays from the first lamp are incident at 45°.

21.25. A lamp of luminous intensity 32 cd which was suspended 1.2 m above the middle of a table is replaced by another lamp whose luminous intensity is 90 cd. How high

must the second lamp be suspended if the illuminance at the middle of the table is to remain unchanged?

21.26. A lamp suspended from a height of 6 m illuminates a skating rink. How far from the lamp and from the centre of the rink will the illuminance on the surface of ice be a factor of 3.4 less than that at the centre?

21.27. A lamp with luminous intensity 25 cd is placed 15 cm to the left of a photometer. Another lamp is placed 45 cm to the right of the photometer, the illuminance on both sides being the same. Determine the luminous intensity of the second lamp.

21.28. Two lamps with luminous intensities 50 and 200 cd are 2.4 m apart. Where should an opaque screen be placed between the lamps so that it is equally illuminated on both sides?

21.29. The centre of a screen is illuminated by a light source with a luminous intensity I that is placed a distance

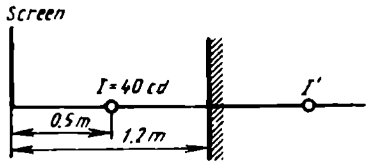


Fig. 135

l from the screen. Will the illuminance change if the luminous intensity and the distance from the light source are both increased n-fold?

21.30. A light source of 40 cd is placed between a screen and a plane mirror 0.5 m from the screen (Fig. 135). The distance between the screen and the mirror is 1.2 m. Determine the illuminance of the screen where the light ray is incident along the normal. Assume that the mirror is perfectly reflecting.

21.31. A lamp used for printing photographs had luminous intensity 50 cd and was placed 1.2 m from the photograph. The exposure time was 3 s. When the lamp burnt out, it was replaced by another lamp of 40 cd placed 1 m from the photograph. Determine the exposure time for the new lamp.

§ 22. PHENOMENA EXPLAINED BY THE WAVE PROPERTIES OF RADIATION. INTERFERENCE. DIFFRACTION

Basic Concepts and Formulas

Optical radiation occupies a small interval in the electromagnetic wave spectrum and includes three regions, viz. the ultraviolet, visible, and infrared wave bands.

Ultraviolet radiation corresponds to wavelengths from about 5 to 400 nm and it can induce marked chemical reactions.

The radiation causing the sensation of light is known as visible radiation, or just light. The lower boundary of the spectrum of visible radiation lies between 380 and 400 nm, and the upper boundary between 760 and 780 nm.

Infrared radiation corresponds to wavelengths from 780 nm (the upper boundary of visible radiation) to 1 mm. It has a well-pronounced thermal effect.

Optical radiation is electromagnetic in nature. The wavelength is given by

$$\lambda = cT, \text{ or } \lambda = c/\nu,$$

where c is the propagation velocity of electromagnetic waves, T and ν being their period and frequency.

As light waves pass from vacuum to a medium, the wavelength changes (the frequency remaining unchanged):

$$n = \frac{c}{v} = \frac{\lambda_1}{\lambda_2},$$

where n is the absolute refractive index and c and v are respectively the propagation velocities of electromagnetic waves in vacuum and in a medium.

Phenomena which confirm the wave nature of visible radiation include interference and diffraction.

The interference of light is the enhancement or attenuation of light as a result of superposition of light waves. In order to observe an interference pattern, the waves must have the same wavelength and a constant phase difference, i.e. they must be coherent.

Using Fresnel's biprism, we can form two virtual images S' and S'' of the same source S, which emit coherent rays (Fig. 136). When monochromatic radiation is incident on the

biprism (i.e. the radiation with a single frequency), bright and dark fringes are formed on the screen. The maximum brightness occurs when the optical path difference Δ is equal to an even number of half-waves:

$$\Delta = 2k\frac{\lambda}{2}, \quad \text{where } k = 1,\ 2,\ \ldots .$$

If, however, one wave lags behind another by a half-wavelength, i.e. the optical path difference is equal to an odd number of half-waves, the maximum attenuation of light is observed:

$$\Delta = (2k+1)\frac{\lambda}{2},$$

where $k = 0,\ 1,\ 2,\ \ldots .$

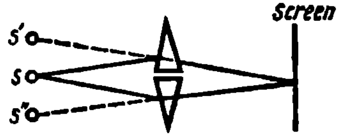


Fig. 136

Interference can be observed in thin films or for light passing through a system formed by a plano-convex lens and a glass plate (Newton's rings).

If we know the radius of curvature R of the lens and measure the radius r_k of one of the dark rings (Fig. 137), we can

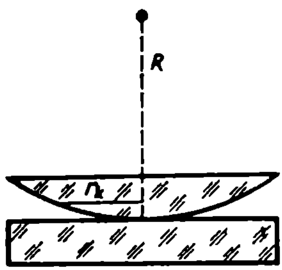


Fig. 137

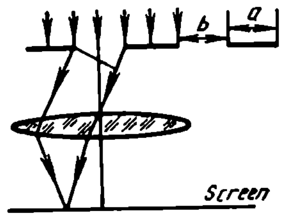


Fig. 138

determine the wavelength of the light illuminating a device that yields Newton's rings:

$$r_k = \sqrt{k\lambda R},$$

where $k = 0,\ 1,\ 2,\ \ldots$ is the number of the dark ring. The formula is valid when air, for which $n = 1$, occupies the space between the plate and the lens.

Diffraction is the bending of light round obstacles, and it occurs when the size of the obstacle is commensurate with the wavelength.

A diffraction grating is formed by alternating transparent and opaque lines. The sum of the width of the line and of a separation between two lines is known as the grating constant (Fig. 138):

$$d = a + b,$$

where d is the grating constant, b is the width of a line, and a is the distance between two lines.

The formula for a diffraction grating is

$$d \sin \varphi = k\lambda,$$

where k is the order of a maximum. The brightest (zero-order) maximum lies on the screen opposite the centre of the grating.

The formula of a diffraction grating shows that one must measure the angle φ to determine the wavelength using a diffraction grating.

When white light is incident on a grating, a spectrum known as the normal spectrum is formed on a screen. It contains all the coloured bands (from red to violet).

Worked Problems

Problem 107. The wavelength of light in glass is 450 nm. Light propagates in glass at a velocity of 1.8×10^5 km/s. Determine the frequency of light, the absolute refractive index of glass, and the wavelength of the light passing from glass into vacuum.

Given: $\lambda_g = 450$ nm $= 4.5 \times 10^{-7}$ m is the wavelength of light in glass, $v = 1.8 \times 10^5$ km/s $= 1.8 \times 10^8$ m/s is the velocity of light in glass. From tables, we take the velocity of light in vacuum, $c = 3 \times 10^8$ m/s.

Find: the frequency ν of light, the absolute refractive index n of glass, and the wavelength λ of light in vacuum.

Solution. From the wavelength and the velocity of light in glass, we can determine its frequency: $\lambda_g = v/\nu$, whence

$$\nu = \frac{v}{\lambda_g}, \quad \nu = \frac{1.8 \times 10^8 \text{ m/s}}{4.5 \times 10^{-7} \text{ m}} = 4.0 \times 10^{14} \text{ Hz} = 400 \text{ THz}.$$

As light passes from one medium to another, its frequency remains unchanged, but its velocity and wavelength change. Consequently, we can determine the wavelength of light in vacuum:

$$\lambda = \frac{c}{\nu}, \quad \lambda = \frac{3 \times 10^8 \text{ m/s}}{4.0 \times 10^{14} \text{ Hz}} = 0.75 \times 10^{-6} \text{ m}.$$

The optical density of a medium is equal to the absolute refractive index:

$$n = \frac{c}{v}, \quad n = \frac{3 \times 10^8 \text{ m/s}}{1.8 \times 10^8 \text{ m/s}} \simeq 1.7.$$

Answer. The frequency of light is 400 THz, the wavelength in vacuum is 750 nm, and the absolute refractive index is about 1.7.

Problem 108. Two coherent light sources emit light of wavelength 550 nm which produces an interference pattern

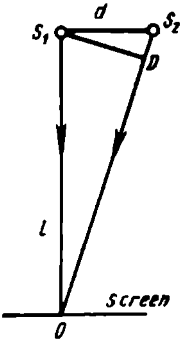


Fig. 139

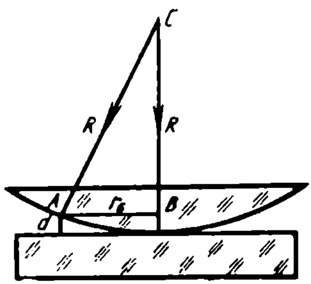


Fig. 140

on a screen (Fig. 139). The sources are 2.2 mm apart and 2.2 m from the screen. Determine whether the interference at point O is constructive or destructive.

Given: $\lambda = 550$ nm $= 5.5 \times 10^{-7}$ m is the wavelength of light, $l = 2.2$ m is the shortest distance from the first source to the screen, and $d = 2.2$ mm $= 2.2 \times 10^{-3}$ m is the separation between the sources.

Find: the path difference Δ of the rays.

Solution. In order to answer the question, we must know the path difference for the rays. The optical path difference

is equal to their geometrical difference (since the rays propagate in the same medium, viz. air):

$$\Delta = S_2D = S_2O - S_1O,\ S_1O = l.$$

From the triangle S_1OS_2, we determine S_2O: $S_2O = \sqrt{l^2+d^2} = l\sqrt{1+(d/l)^2}$. Since d/l is much smaller than l, we can use approximation $\left(\sqrt{1 \pm a^2} = 1 \pm \frac{1}{2}a^2\right)$, whence $S_2O = l\left[1+\frac{1}{2}\left(\frac{d}{l}\right)^2\right]$. Then

$$\Delta = l\left[1+\frac{1}{2}\frac{d^2}{l^2} - 1\right] = \frac{d^2}{2l},$$

$$\Delta = \frac{(2.2\times10^{-3}\,\text{m})^2}{2\times2.2\,\text{m}} = 1.1\times10^{-6}\,\text{m}.$$

There will be constructive interference at O if the path difference contains integral number of waves, i.e. $k = 1, 2, 3, \ldots$:

$$k = \frac{\Delta}{\lambda} = \frac{1.1\times10^{-6}\,\text{m}}{5.5\times10^{-7}\,\text{m}} = 2.$$

Answer. Constructive interference (bright fringe) occurs at O.

Problem 109. A plano-convex lens with a radius of curvature of 12 m is placed on a flat plate as shown in Fig. 140. Monochromatic light is incident along the normal to the plane face of the lens, and dark and bright rings are formed in the reflected light. Determine the wavelength of the monochromatic light if the radius of the sixth dark ring is 7.2×10^{-3} m.

Given: $R = 12$ m is the radius of curvature of the lens, $k = 6$ is the dark ring number, and $r_6 = 7.2\times10^{-3}$ m is the radius of the sixth ring.

Find: the wavelength λ of the monochromatic light.

Solution. The light wave incident on the plane surface of the lens is partially reflected by the convex surface of the lens and partially passes through the air gap d and then reflected from the flat plate, the path difference being increased by $\lambda/2$ by the reflection. Thus, the second wave passes through the gap d twice, and hence the path difference will be $\Delta = 2d + \lambda/2$.

For a dark fringe, we have $\Delta = \frac{\lambda}{2}(2k+1)$, or $2d + \frac{\lambda}{2} = \lambda k + \frac{\lambda}{2}$, whence $d = \frac{\lambda k}{2}$.

From triangle ABC, we obtain $R^2 = (R-d)^2 + r_6^2$, $2Rd - d^2 = r_6^2$. The quantity d^2 can be neglected since d is small. This gives $2Rd = r_6^2$, whence $d = r_6^2/2R$. Comparing the obtained expressions for d, we obtain

$$\frac{\lambda k}{2} = \frac{r_6^2}{2R},$$

whence

$$\lambda = \frac{r_6^2}{kR}, \quad \lambda = \frac{(7.2\times 10^{-3}\,\mathrm{m})^2}{6\times 12\,\mathrm{m}} = 7.2\times 10^{-7}\,\mathrm{m}.$$

Answer. The wavelength of light is 720 nm.

Problem 110. The grating constant of a diffraction grating is 0.016 mm. The red line of the 2nd-order spectrum is 14.2 cm from the middle line. The distance from the grating to a screen is 1.5 m. Determine the wavelength of the red light and the width of the 2nd-order spectrum. The wavelength of violet light is 4×10^{-7} m (Fig. 141).

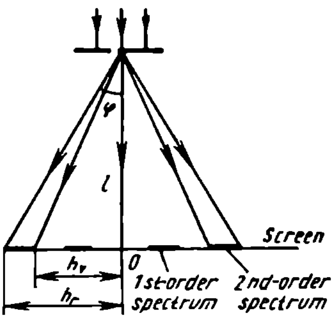


Fig. 141

Given: $d = 0.016$ mm $= 1.6\times 10^{-5}$ m is the grating constant, $h_r = 14.2$ cm $= 1.42\times 10^{-1}$ m is the distance between the red region of the 2nd-order spectrum and the middle line, $k=2$ is the order of the spectrum, $l = 1.5$ m is the distance from the screen to the diffraction grating, and $\lambda_v = 4\times 10^{-7}$ m is the wavelength of violet light.

Find: the wavelength λ_r of the red light and the width h of the 2nd-order spectrum.

Solution. The wavelength of the red light can be determined from the formula for a diffraction grating: $k\lambda_r = d\sin\varphi$. Here the angle φ is very small so that we can

put $\sin\varphi \simeq \tan\varphi$. Then

$$\lambda_r = \frac{d\tan\varphi}{k}, \text{ or } \lambda_r = \frac{dh_r}{kl},$$

$$\lambda_r = \frac{1.6\times10^{-6}\,\mathrm{m}\times1.42\times10^{-1}\,\mathrm{m}}{2\times1.5\,\mathrm{m}} = 7.57\times10^{-7}\,\mathrm{m}.$$

In order to determine the spectral width, we must know the distance from the middle line of the spectrum to the violet region h_v. This can be determined from the formula for a diffraction grating: $h_v = \frac{\lambda_v kl}{d}$, $h_v = \frac{4\times10^{-7}\,\mathrm{m}\times2\times1.5\,\mathrm{m}}{1.6\times10^{-6}\,\mathrm{m}}$ $= 7.5\times10^{-2}$ m.

Thence

$$h = h_r - h_v = 14.2\times10^{-2}\,\mathrm{m} - 7.5\times10^{-2}\,\mathrm{m} = 6.7\times10^{-2}\,\mathrm{m}.$$

Answer. The wavelength of the red light is approximately equal to 760 nm and the width of the 2nd-order spectrum is 6.7 cm.

Questions and Problems

22.1. When would a bright spot be formed on a screen where two beams from a coherent monochromatic light source meet, and when would the spot be dark?

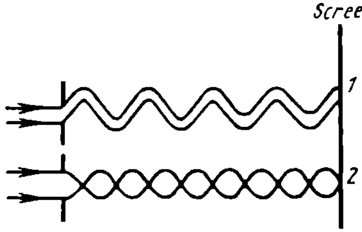


Fig. 142

22.2. Light from two coherent sources is incident at points *1* and *2* on a screen (Fig. 142). Where will constructive and destructive interference occur?

22.3. If crystals of common salt are placed in the flame of a candle and then the flame is observed through a transpa-

rent plate with parallel faces, alternating dark and yellow fringes can be seen against the background of the flame. The same pattern can be observed in reflected light (by placing the plate behind the flame). How can this effect be explained?

22.4. Oil spots on the surface of sunlit water are rainbow coloured. Why? Will the pattern change if the surface is illuminated by monochromatic light?

22.5. Rays from two coherent light sources of wavelength 0.5 μm and with a path difference of 0.5 mm arrive at a certain point in space. Will the interference be constructive or destructive at this point?

22.6. Red light of wavelength 760 nm from two coherent sources is incident on a screen, forming an interference pattern of red and dark fringes. Determine the path difference of the rays if four half-waves fit into it. What type of fringe (red or dark) is formed for such a path difference?

22.7. A soap bubble displays all the colours of the rainbow in sun light. Why?

22.8. Determine the radius of the second dark Newton ring in reflected light if a plano-convex lens with the radius of curvature 8 m and a flat plate (Fig. 140) are illuminated by a monochromatic light with a wavelength of 640 nm.

22.9. An instrument for observing Newton's rings is illuminated by a monochromatic red light. The radius of the third dark ring is found to be 2.8 mm. Determine the wavelength of the red light if the radius of curvature of the plano-convex lens is 4 m.

22.10. The air gap in an instrument for observing Newton's rings (see Fig 140) is filled with water. What will the change in the radii of the interference rings be?

22.11. What is the antireflection coating of objectives based on? Why do objectives have a bluish-violet tinge in reflected light?

22.12. The distance between two coherent monochromatic point light sources is 1.5 cm. The sources are located at 36 m from a screen so that the line connecting them is parallel to the plane of the screen. Determine the wavelength of the light if the separation between adjacent interference fringes is 1.8 mm.

22.13. A plano-convex lens with a radius of curvature of 8 m is put on a flat transparent plate. When the system is

illuminated with green light from thallium (wavelength 536 nm), Newton's rings are formed in the reflected light. Determine the radius of the fifth dark ring.

22.14. The radius of the third dark Newton ring (see Problem 22.13) is found to be 2.8 mm when the system is illuminated by a monochromatic light. Determine the radius of curvature of the plano-convex lens if the wavelength of the monochromatic light is 720 nm.

22.15. If the light of a street lamp is viewed through one's eyelashes, rainbow rings appear around the lamp. Explain this effect.

22.16. What is the difference between a diffraction spectrum and the spectrum obtained with a prism?

22.17. In a practical experiment on the wavelength of light using a diffraction grating, the first diffraction maximum was observed on a screen 30 cm from the central line. The grating constant was 2×10^{-3} mm, and the distance from the screen to the grating was 1.5 m. Using these data, determine the wavelength.

22.18. Light from a gas-discharge tube is incident normally on a diffraction grating whose constant is 2×10^{-3} mm. The orange line in the 1st-order spectrum is seen at an angle of 18°, and the blue line is seen at 14°. Determine the wavelengths of this light.

22.19. Spectra formed by a diffraction grating are projected on a screen 3 m away. Determine the wavelength of a monochromatic light if the distance from the central line to the 1st-order spectrum is 22.8 cm and the grating constant is 0.01 mm.

22.20. Monochromatic light corresponding to a sodium line of wavelength 5.89×10^{-7} m is incident on a diffraction grating. The angle at which this line is seen in the 1st-order spectrum is 17° 18′. Determine the grating constant. How many lines does a centimetre of the grating contain?

22.21. Monochromatic light emitted by a mercury-vapour lamp and having a wavelength of 579 nm is incident on a diffraction grating with constant 2×10^{-5} m and forms a diffraction spectrum on a screen. The distance from the grating to the screen is 1.5 m. How far from the central line will the coloured line in the 1st-order spectrum be?

22.22. How many lines per millimetre will a diffraction grating have for the green line (wavelength 500 nm) in the

3rd-order spectrum to be observed at angle 48° 30′?

22.23. A monochromatic light of wavelength 500 nm is incident normally on a diffraction grating containing 500 lines per millimetre. What is the highest order spectrum observable using this grating?

§ 23. RADIATION AND SPECTRA

Basic Concepts and Formulas

White light is a composite. We can prove this with a trigonal prism. After passing through the prism, a ray of white light splits into the coloured rays of the spectrum.

Dispersion is the dependence of the refractive index on the wavelength of the transmitted light. The refractive index n_r for red light is smallest and for violet light n_v, is largest. This means that red light propagates faster in a transparent medium than violet light: $n_r = c/v_r$, $n_v = c/v_v$. Since $n_r < n_v$, $c/v_r < c/v_v$, or $v_r > v_v$.

The colour of a transparent body depends on the colour of the light rays it transmits. Glass is green if it only transmits the green light from white light.

The colour of an opaque body is determined by the colour of the light it reflects.

All bodies when hot enough produce *emission spectra*:

1. Continuous spectra are emitted by heated liquids or solids. These spectra are the same for all bodies and consist of seven basic coloured bands which continuously merge into each other.

2. Line spectra are obtained from heated gases or vapours. Each chemical element has its own line spectrum which differs from every other spectra in the number of lines, the colour, and the position of the lines against the background of a continuous spectrum.

3. Band spectra are obtained from molecules and consist of a number of bands.

Absorption spectra appear when light passes through a medium which can selectively absorb light and has a lower temperature. The emergence of absorption spectra obeys Kirchhoff's law: a substance mainly absorbs light with the wavelengths which it can emit.

The thermal radiation of a body at a given temperature is determined by its radiant exitance R_e which equals the ratio of the radiant flux emitted from a small surface element to the area of this surface. Bodies can absorb radiation incident on them. If a body completely absorbs the radiant flux incident on it, it is called a blackbody. According to the law on thermal radiation, a blackbody has the maximum radiant exitance.

Stefan-Boltzmann's law. The exitance of a blackbody e_b is proportional to the fourth power of its thermodynamic temperature:

$$R_e = \sigma T^4,$$

where σ is the Stefan-Boltzmann constant.

Wien's displacement law. The product of the wavelength corresponding to the maximum of radiant energy and the thermodynamic temperature is constant for a blackbody:

$$\lambda_{max} T = b,$$

where b is the Wien constant.

Consequently, as the temperature increases, the maximum of the radiant energy is displaced towards shorter waves.

Worked Problems

Problem 111. Assuming that the temperature of the surface of the Sun is approximately 6000 K, determine the wavelength corresponding to the maximum energy, considering that the Sun is a blackbody.

Given: $T = 6000$ K is the temperature of the Sun's surface. From tables, we take the Wien constant: $b = 2.89 \times 10^{-3}$ m·K.

Find: the wavelength λ_{max} corresponding to the maximum energy.

Solution. The wavelength corresponding to the maximum radiant energy in the spectrum of a blackbody at a given temperature can be found from Wien's displacement law $b = \lambda_{max} T$:

$$\lambda_{max} = \frac{b}{T}, \quad \lambda_{max} = \frac{2.89 \times 10^{-3}\ \text{m}\cdot\text{K}}{6000\ \text{K}} = 4.82 \times 10^{-7}\ \text{m}.$$

Answer. The maximum of radiant energy corresponds to the wavelength 482 nm.

Questions and Problems

23.1. Where is the velocity of light higher: in diamond or in water?

23.2. What is the ratio between the velocities of light in vacuum and in water? How long does it take for light to travel 225 km in water?

23.3. One standard metre is as long as 1650763.73 wavelengths of the orange light emitted by krypton-86 atoms in vacuum. What is the frequency of this radiation?

23.4. Determine the wavelengths for the red and violet light at the edges of the visible spectrum if they correspond to frequencies 3.95×10^{14} Hz and 7.5×10^{14} Hz.

23.5. What is the velocity of light in diamond if the frequency 2.73×10^{14} Hz corresponds in it to a wavelength of 450 nm?

23.6. The wavelength of blue light in vacuum is 500 nm. Determine the wavelength corresponding to blue light in water and its frequency.

23.7. Green light passes from air to water, its wavelength decreasing thereby. What colour will be perceived by a diver in the water?

23.8. What will the change in the wavelength of yellow light of frequency 5.3×10^{14} Hz be as a result of its passage from glass to vacuum if its velocity in glass is 1.98×10^8 m/s?

23.9. Determine the velocities of light in a transparent medium for extreme red (800 nm) and violet (400 nm) light if the refractive indices in this medium for these wavelengths are 1.62 and 1.67 respectively. What are the frequencies and wavelengths of these types of light in the transparent medium?

23.10. As light passes from water to vacuum, its wavelength increases by 0.120 μm. Determine the light's wavelengths in vacuum and in water.

23.11. As light passes from vacuum to a transparent medium its wavelength is reduced by a factor of 1.31. What is the medium?

23.12. The word "light" is written in green on a sheet of paper. Through which transparent medium would it be impossible to read the word?

23.13. The star Sirius has a surface temperature of 10^4 K. Determine the wavelength corresponding to the maximum energy emitted by the star.

23.14. If one looks at a bright red object for a while and then at a white wall, a green silhouette of the object will be seen. How can this be explained?

23.15. What method was used to study the chemical composition of the Sun?

23.16. The total energy emitted by the Sun per second is about $E = 4 \times 10^{26}$ J. Assuming that the Sun is a blackbody, determine its surface temperature.

23.17. The spectral analysis of a nebula has revealed that it has a continuous spectrum. What conclusion can be drawn from this?

23.18. The Sun emits about 4×10^{26} J of radiant energy per second. Some data indicate that the radiant exitance of the Sun has remained constant over the last 3×10^9 years (10^{17} s). Determine the mass lost by the Sun per second in the form of radiation and the mass that has been lost over the last 3×10^9 years.

23.19. What is the melting point of tungsten if the wavelength of the radiation corresponding to the radiant energy maximum lies in the red spectral region of tungsten and is equal to 784 nm?

§ 24. PHENOMENA EXPLAINED BY THE QUANTUM PROPERTIES OF RADIATION. PHOTOELECTRIC EFFECT

Basic Concepts and Formulas

In addition to its wave properties, visible radiation also exhibits particle properties. At low frequencies, the wave properties of electromagnetic radiation are more pronounced, while at very high frequencies its particle properties dominate.

According to Planck's theory, radiation is discrete. The units of electromagnetic radiation are known as quanta, and the particles carrying energy quanta are called photons.

The main characteristics of a photon are its energy ε and momentum p:

$$\varepsilon = h\nu = hc/\lambda_0,$$

where h is Planck's constant and λ_0 is the wavelength of the radiation in vacuum.

Since photons, unlike other microparticles, move at the velocity of light, the formula for momentum can be written in the form

$$p = mc.$$

According to Einstein's formula, $m = \varepsilon/c^2$, and hence

$$p = \varepsilon/c = h\nu/c \text{ or } p = h/\lambda_0.$$

The mass of a photon is

$$m = h\nu/c^2.$$

Comparing the formulas for energy, momentum, and mass, we can arrive at the following conclusion: for a monochromatic light, all photons of frequency ν have the same energy, momentum, and mass.

Photons at rest do not exist and therefore the rest mass of a photon is zero.

The photoelectric effect arises during the interaction between light and a substance. Electrons are detached from the surface of a substance at the expense of the radiant energy of photons to which the substance is exposed (the extrinsic photoeffect).

The laws of the photoelectric effect are:

1. The saturation photoelectric current is proportional to the luminous flux incident on the surface of the substance.

2. The maximum kinetic energy of the electrons liberated from an irradiated surface does not depend on the radiation's intensity and is given by

$$\frac{mv_{max}^2}{2} = eU,$$

where v_{max} is the maximum velocity of the electrons, U the minimum voltage at which no photoelectric current is observed, and m and e are the mass and charge of the electron.

Einstein's relation for the photoelectric effect is

$$h\nu = A + \frac{mv_{max}^2}{2},$$

where A is the work function defined as the kinetic energy which must be acquired by an electron in order to leave the surface of the substance.

The maximum wavelength (the minimum frequency) at which a photoeffect can be observed is known as the photoelectric threshold for a given substance.

Equating the kinetic energy of an electron in Einstein's formula to zero, we can determine the wavelength corresponding to the photoelectric thresholds for different materials:

$$\frac{hc}{\lambda_{th}} = A, \quad \lambda_{th} = \frac{hc}{A}.$$

The operation of photoelectric cells and photoresistors is based on the application of intrinsic and extrinsic photoelectric effects. The photoelectric effect is widely used in various fields.

Worked Problems

Problem 112. A surface element of area 2 cm^2 is illuminated for 1 min by a radiation of energy 15 J. Determine the pressure exerted by the radiation if the surface (a) completely absorbs the radiation and (b) completely reflects it.

Given: $W = 15$ J is the radiant energy, $t = 60$ s is the time, $S = 2\ \text{cm}^2 = 2 \times 10^{-4}\ \text{m}^2$ is the area on which the radiation is incident. From tables, we take the velocity of light in vacuum $c = 3 \times 10^8$ m/s.

Find: the pressure p_1 exerted by the radiation when it is completely absorbed and the pressure p_2 when it is completely reflected.

Solution. The pressure arising as a result of the interaction between radiation and the substance can be found from the formula

$$p = \frac{W_0}{c}(1 + \rho),$$

where W_0 is the radiant energy per unit time per unit surface area and ρ is the reflection coefficient.

(a) When the radiation is completely absorbed, $\rho = 0$ and

$$p_1 = \frac{W_0}{c}, \quad W_0 = \frac{W}{St},$$

$$p_1 = \frac{W}{Sct} = \frac{15\ \text{J}}{2 \times 10^{-4}\ \text{m}^2 \times 3 \times 10^8\ \text{m/s} \times 60\ \text{s}} = 4.2 \times 10^{-6}\ \text{Pa}.$$

(b) In the case of total reflection, $\rho = 1$, and

$$p_2 = \frac{2W}{Sct}, \quad p_2 = 8.4 \times 10^{-6}\ \text{Pa}.$$

Answer. The pressure produced by the radiation in the first case is approximately 4.2×10^{-6} Pa, in the second case, it is double this value.

Problem 113. The electron work function for zinc is 3.74 eV. Determine the radiation threshold of photoelectric effect for zinc. Calculate the velocity of the electrons emitted by zinc irradiated by ultraviolet light with a wavelength of 200 nm.

Given: $A = 3.74$ eV $= 3.74 \times 1.6 \times 10^{-19}$ J is the electron work function, $\lambda = 200$ nm $= 2 \times 10^{-7}$ m is the wavelength of incident radiation. From tables, we take the velocity of light in vacuum $c = 3 \times 10^8$ m/s, the Planck constant $h = 6.62 \times 10^{-34}$ J·s, and the mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg.

Find: the radiation threshold λ_{th} of the photoelectric effect for zinc, the maximum electron velocity v.

Solution. The radiation threshold of the photoelectric effect is the maximum wavelength which can cause the photoeffect. In this case the kinetic energy of an electron will be zero, and hence $A = h\nu_{th} = hc/\lambda_{th}$. Thus

$$\lambda_{th} = \frac{hc}{A},$$

$$\lambda_{th} = \frac{6.62 \times 10^{-34}\ \text{J}\cdot\text{s} \times 3 \times 10^8\ \text{m/s}}{3.74 \times 1.6 \times 10^{-19}\ \text{J}} = 3.32 \times 10^{-7}\ \text{m} = 332\ \text{nm}.$$

The energy $h\nu$ of ultraviolet radiation photons incident on the zinc plate is spent on the electron work function A and on the kinetic energy imparted to the electron: $\frac{hc}{\lambda} = A + \frac{m_e v^2}{2}$. From this equation, we can determine the velocity acquired by the electrons in photoeffect:

$$\frac{hc}{\lambda} - A = \frac{m_e v^2}{2}, \quad v = \sqrt{\frac{2(hc - \lambda A)}{m_e \lambda}},$$

$$v = \sqrt{\frac{2\,(6.62\times10^{-34}\,\mathrm{J\cdot s}\times3\times10^{8}\,\mathrm{m/s}-2\times10^{-7}\,\mathrm{m}\times3.74\times1.6\times10^{-19}\,\mathrm{J})}{9.1\times10^{-31}\,\mathrm{kg}\times2\times10^{-7}\,\mathrm{m}}}$$

$$= 9.3\times10^{5}\ \mathrm{m/s}.$$

Answer. The maximum wavelength of the radiation which may cause the photoeffect is 332 nm, the maximum velocity of the knocked out electrons is 9.3×10^5 m/s.

Questions and Problems

24.1. What is the pressure exerted by light on 1 mm^2 of a black surface to which 500 J of radiant energy are transferred per second?

24.2. The solar pressure on the surface of the Earth is 4.7×10^{-4} Pa. Determine the energy of the radiation incident per second on a square metre of the Earth's surface perpendicular to solar rays.

24.3. The number of X-ray photons of frequency 7×10^{19} Hz incident per second on a square metre of a black surface is 2.5×10^{15}. What pressure does this radiation produce?

24.4. The yellow light of sodium vapour has a wavelength of 530 nm. What is the energy of a quantum of this light in joules and electronvolts?

24.5. What is the ratio of the energy of a photon corresponding to the γ-radiation of frequency 3×10^{21} Hz to the energy of an X-ray photon of wavelength 2×10^{-10} m?

24.6. Determine the energy of a quantum corresponding to wavelength 10^{-7} m (in joules and electronvolts).

24.7. How many photons make up 10^{-8} J of radiation at wavelength 2 μm?

24.8. Determine the energy, mass, and momentum of an ultraviolet photon whose wavelength is 360 nm.

24.9. Determine the energy, mass, and momentum of X-ray photons of wavelength 4×10^{-11} m.

24.10. A comet was observed in the sky after sunset. In which direction was its tail pointing?

24.11. The photoelectric threshold for tungsten corresponds to a wavelength of 405 nm. Determine the work function for tungsten.

24.12. The work function for cesium is 1.9 eV. Determine the maximum wavelength of light at which a photoelectric effect is observed.

24.13. Determine the maximum wavelength of light which may cause the extrinsic photoeffect from nickel if the work function for nickel is 4.5 eV.

24.14. The work function for platinum is 6.3 eV. Will photoeffect be observed for a radiation of wavelength 10^{-7} m?

24.15. Light of wavelength 500 nm is incident on the surface of silver. Will the silver become charged or will it remain neutral? If it gets charged, what is the sign of the charge? The photoelectric threshold for silver is 261 nm.

24.16. The maximum wavelength of radiation that can produce a photoelectric effect in platinum is 234 nm. Determine the maximum kinetic energy acquired by the electrons due to radiation of wavelength 200 nm.

24.17. What is the energy of electrons detached from the surface of copper irradiated by light of frequency 6×10^{16} Hz if the work function for copper is 4.5 eV?

24.18. What is the velocity of the electrons knocked out of sodium irradiated by light of wavelength 66 nm? The work function for sodium is 4×10^{-19} J.

24.19. The photoelectric threshold for a certain metal is 690 nm. Determine the work function for this metal and the maximum velocity acquired by its electrons due to radiation of wavelength 190 nm.

24.20.[1] What is the maximum velocity acquired by photoelectrons knocked out of molybdenum by radiation of frequency 3×10^{20} Hz? The work function for molybdenum is 4.27 eV. Is the classical formula applicable in this case?

24.21. If the surface of a metal is successively exposed to radiation of wavelengths 350 and 540 nm, the maximum velocities of the photoelectrons will differ by a factor of two. Determine the metal's work function.

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[1] It would be useful to reconsider this problem after studying the section on special theory of relativity (§ 25).

§ 25. FUNDAMENTALS OF THE SPECIAL THEORY OF RELATIVITY

Basic Concepts and Formulas

The special theory of relativity (STR) is based on two postulates.

1. All physical processes in inertial frames proceed identically and do not depend on the choice of the reference frame.

2. The velocity of electromagnetic waves in vacuum is the same for all inertial frames. It does not depend either on the velocity of the source or on the motion of the observer (receiver of the light signal).

The postulates of the special theory of relativity are in contradiction with the concepts of absolute time and space formed in Newtonian classical mechanics. The STR implies the following concepts.

The *relativity of lengths* (distances): the length l of a rod (body) in a reference frame relative to which it moves will, according to the STR, be less than the length l_0 of the stationary rod:

$$l = l_0 \sqrt{1 - v^2/c^2}.$$

The transverse dimensions of a moving body will be the same in all inertial reference frames.

The *relativity of time intervals* (relativistic time dilation): the time τ measured in a laboratory system in which an observer is at rest and the intrinsic time τ_0 measured using a clock moving together with the reference frame are related as

$$\tau = \frac{\tau_0}{\sqrt{1 - v^2/c^2}} .$$

If the velocity of a moving reference frame is close to the velocity of light, the moving clock will lag behind the stationary clock, i.e. time dilation will occur.

The mass of a body will depend on its velocity: the rest mass m_0 (intrinsic mass) and the mass m of the body moving at a velocity close to that of light are connected through the

following relation:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}.$$

Given that the mass depends on the velocity, we can write the formula for the momentum of the body in the form

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}.$$

The relativistic law for velocity composition: the velocity v_2 of body M relative to a stationary observer in frame K can be determined from the formula

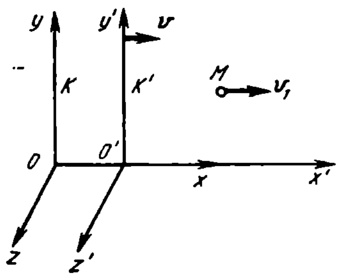


Fig. 143

$$v_2 = \frac{v_1 + v}{1 + v_1 v/c^2},$$

where v_1 is the velocity of the body M relative to the reference frame K', v is the velocity of the reference frame K' relative to the stationary reference frame K (it is necessary that the x-axis be the same for the two reference frames, Fig. 143).

If we assume that $v_1 = v = c$, the resultant velocity cannot exceed the velocity c of light.

The equation relating mass and energy (Einstein's mass-energy relation): for any interconversion of matter into energy, the change in energy is proportional to the change in mass:

$$\Delta E = c^2 \Delta m.$$

Worked Problems

Problem 114. A rocket moves with a velocity equal to 0.6 of the velocity of light in vacuum relative to a stationary observer. What is the change in the length of a 1-m long steel ruler and the density of the steel of which it is made in the rocket (in the direction of motion) with respect to the observer? How much time will elapse according to the clock of the stationary observer if six years have passed according to the clock in the moving rocket?

Given: $v = 0.6c$ is the velocity of the rocket relative to the stationary observer, $l_0 = 1$ m is the proper length of the ruler, $t_0 = 6$ years is the intrinsic time. From tables we take the proper density of steel, $\rho_0 = 7.8 \times 10^3$ kg/m^3.

Find: the length l of the ruler, the density ρ of steel, and the time t.

Solution. The length of the ruler (along the trajectory of motion) for the stationary observer, relative to whom the rocket moves, can be determined from the formula

$$l = l_0\sqrt{1 - v^2/c^2}, \quad l = 1\ \text{m}\sqrt{1 - \frac{0.36c^2}{c^2}} = 0.8\ \text{m}.$$

The density of a substance is expressed in terms of mass and volume: $\rho = m/V$, but $V = lS = l_0S\sqrt{1 - v^2/c^2}$. In this problem, the transverse dimensions do not change, and hence

$$\rho = \frac{m_0}{\sqrt{1-v^2/c^2}\, l_0S\sqrt{1-v^2/c^2}}, \quad \text{but}\ \frac{m_0}{l_0S} = \rho_0.$$

Therefore

$$\rho = \frac{\rho_0}{1 - v^2/c^2}, \quad \rho = \frac{7.8 \times 10^3\ \text{kg/m}^3}{0.64} = 1.2 \times 10^4\ \text{kg/m}^3.$$

The time for the stationary observer will be dilated and is defined by the formula

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}, \quad t = \frac{6\ \text{years}}{0.8} = 7.5\ \text{years}.$$

Answer. The length of the ruler for the stationary observer is 0.8 m, the density of steel is 1.2×10^4 kg/m^3, and the time period is 7.5 years.

Problem 115. The rest energy of a proton is approximately 938 MeV. Determine the rest mass of the proton and the mass and velocity of protons to which a kinetic energy of 70 GeV has been imparted in an accelerator.

Given: $E_0 = 938$ MeV is the rest energy of the proton, $E_k = 70$ GeV is the kinetic energy of a proton acquired in the accelerator. From tables, we take the velocity of light in vacuum $c = 3 \times 10^8$ m/s.

Find: the rest mass m_{p_0} of the proton, the mass m_p of the proton after the acceleration, and the velocity v of the proton as a result of the acceleration.

Solution. We first recalculate the proton's energy in SI units (joules) 1 eV $= 1.6 \times 10^{-19}$ C $\times$ 1 V $= 1.6 \times 10^{-19}$ J, 1 MeV $= 1.6 \times 10^{-13}$ J. Consequently, the rest energy is

$$E_0 = 938 \times 1.6 \times 10^{-13}\ \text{J} = 1.5 \times 10^{-10}\ \text{J}.$$

The kinetic energy of the proton after acceleration is

$$E_k = 7 \times 10^{10} \times 1.6 \times 10^{-19}\ \text{J} = 1.12 \times 10^{-8}\ \text{J}.$$

In order to determine the rest mass of the proton, we shall use Einstein's relation $E_0 = m_{p_0}c^2$:

$$m_{p_0} = \frac{E_0}{c^2}, \quad m_{p_0} = \frac{1.5 \times 10^{-10}\ \text{J}}{9 \times 10^{16}\ \text{m}^2/\text{s}^2} = 1.67 \times 10^{-27}\ \text{kg}.$$

The total energy of the proton can be determined from the formula $E = E_k + E_0 = m_p c^2$. Dividing the two sides of the equality by $E_0 = m_{p_0}c^2$, we obtain $\frac{m_p}{m_{p_0}} = \frac{E_k + E_0}{E_0}$, or $\frac{m_p}{m_{p_0}} = \frac{E_k}{E_0} + 1$, whence

$$m_p = \left(\frac{E_k}{E_0} + 1\right) m_{p_0},$$

$$m_p = \left(\frac{1.12 \times 10^{-8}\ \text{J}}{1.5 \times 10^{-10}\ \text{J}} + 1\right) 1.67 \times 10^{-27}\ \text{kg} = 1.26 \times 10^{-25}\ \text{kg}.$$

Having determined the mass of the proton after the acceleration, we can find its velocity:

$$m_p = \frac{m_{p_0}}{\sqrt{1 - v^2/c^2}}, \quad \sqrt{1 - v^2/c^2} = \frac{m_{p_0}}{m_p}, \quad 1 - \frac{v^2}{c^2} = \frac{m_{p_0}^2}{m_p^2},$$

whence

$$v = c\sqrt{1 - \frac{m_{p_0}^2}{m_p^2}}, \quad v = c\sqrt{1 - \left(\frac{1.67 \times 10^{-27}\ \text{kg}}{1.26 \times 10^{-25}\ \text{kg}}\right)} = 0.99c.$$

Answer. The rest mass of the proton is 1.67×10^{-27} kg (which corresponds to the tabulated value). The mass of the proton after the acceleration is nearly 75 times larger than its rest mass, and the velocity is equal to 0.99 of the velocity of light.

Problem 116. Two rockets move towards each other each at a velocity of $0.8c$ relative to a stationary observer. De-

termine the velocity with which the rockets approach each other according to (a) classical mechanics and (b) the theory of relativity.

Given: $v_1 = v_2 = 0.8c$ are the velocities of rockets relative to the stationary observer on the Earth.

Find: the velocity u_{cl} at which the rockets approach each other according to classical mechanics and u_{rel}, the velocity according to relativity, as well as the difference Δu in these velocities.

Solution. According to classical mechanics, we have

$$u_{cl} = v_1 + v_2, \ u_{cl} = 0.8c + 0.8c = 1.6c.$$

According to relativistic mechanics, we have

$$u_{rel} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}, \quad u_{rel} = \frac{1.6c}{1 + \frac{0.64c^2}{c^2}} = 0.976c.$$

The difference in these velocities is

$$\Delta u = 1.6c - 0.976c = 0.624c.$$

These calculations show that the classical formula for composing velocities cannot be used when bodies move at velocities close to the velocity of light since this contradicts the relativity's postulate that the velocity of light in vacuum cannot be exceeded.

Questions and Problems

25.1. Compare the lengths of two metre rods moving in their longitudinal directions with velocities $0.5c$ and $0.75c$ relative to a stationary observer.

25.2. At what velocity of a body relative to a stationary observer is its length equal to 0.8 of its proper length?

25.3. At what velocity should a body move relative to a stationary observer for its size along the line of motion to contract by a factor of two? Will the size change to an observer moving together with the body?

25.4. A rocket moves at a velocity of $0.866c$ relative to a stationary observer. The width of a plane rectangle arranged along the line of the motion of the body is equal to half its length. How will the rectangle appear to the stationary observer?

25.5. The radius of an electron at rest is 2×10^{-13} cm. What will the contraction in the electron's radius be in the direction of motion at 0.8 of the velocity of light?

25.6. The flight time measured in a rocket moving at a velocity $0.96c$ is one year. How much time will have elapsed for a terrestrial observer?

25.7. How much time will have elapsed on the Earth if six years have passed in a rocket moving at a velocity of 2.4×10^8 m/s relative to the Earth?

25.8. The distance to the star in the Centaurus constellation nearest the Sun is traversed by light in approximately 4.25 years. Express this distance in astronomical units and in kilometres. Determine the flight time (by a clock in a rocket) to this star and back in the rocket if the velocity of the rocket relative to the observer is $0.99c$, and 1 AU $\simeq$ 150 million kilometres.

25.9. What will be the duration of a flight to a star by a rocket flying at a velocity of $0.9c$ for a cosmonaut and for an observer on the Earth if the distance to the star is 40 light years?

25.10. One kilogram of water is heated by 80 K. What is the increase in the mass of water?

25.11. What must the velocity of a particle be at which its kinetic energy is equal to its rest energy?

25.12. What is the velocity of an electron if its mass is four times its rest mass?

25.13. Determine the mass of an electron moving at velocities of 50% and 90% of the velocity of light.

25.14. What must the velocity of a proton be in an accelerator for its mass to be 5% greater?

25.15. A body with a rest mass of 5 kg and density of 7.8×10^3 kg/m^3 is in a rocket moving relative to a terrestrial observer at a velocity of 2.4×10^5 km/s. Determine the relativistic mass and density of the body.

25.16. A rocket moves at a velocity close to that of light relative to a stationary observer. By what factor will the mass and density of bodies in the rocket change for an observer flying in it?

25.17. What must the velocity of a body be for its density to be five times larger?

25.18. Express the rest energy of an electron (positron) in megaelectronvolts. What will the radiant energy emitted

as a result of the annihilation of an electron and a positron be?

25.19. Determine the momentum of an electron moving at 0.6 of the velocity of light.

25.20. Two aeroplanes fly towards each other at velocities 500 and 400 m/s. What is their relative velocity?

25.21. Two particles move towards each other at a velocity $(5/8)c$ each. What will their approach velocity be as calculated using the classical and relativistic formulas?

Chapter V

Physics of Atomic Nucleus

§ 26. STRUCTURE OF ATOMIC NUCLEUS. ATOMIC ENERGY AND ITS APPLICATION

Basic Concepts and Formulas

According to the nuclear model of atoms proposed by Rutherford and developed by Bohr, an atom consists of a positively charged nucleus and electrons revolving about it. The nucleus contains protons and neutrons to which the general term "nucleons" is applied.

A nucleus occupies a very small volume as compared to the atom. If we assume that the atomic nucleus is a sphere, the radii of the nuclei of different elements can be determined by the formula

$$r = 1.4 \times 10^{-15} \sqrt[3]{A},$$

where A is the mass number of an element.

The density of matter in a nucleus is of the order of 1.3×10^{17} kg/m^3. The mean density of nuclear matter is calculated from the formula

$$\rho = \frac{mA}{(4/3)\pi r^3},$$

where m is the mass of a nucleon and r is the radius of the nucleus.

The positive charge of a nucleus is determined by the product of the atomic number Z of the element in the Periodic Table and the elementary charge, and hence depends on the number of protons in the nucleus.

The mass of a nucleus depends on the number of nucleons in it. The unit of mass of an atom, viz. atomic mass unit (amu), is equal to 1/12 of the mass of the carbon atom $^{12}_{6}C$.

The atomic mass unit can be expressed in SI units:

$$1 \text{ amu} = 1.66057 \times 10^{-27} \text{ kg}.$$

In order to determine the mass of an atom, it is usually sufficient to use the mass number of the element:

$$m_a = A \times 1.66057 \times 10^{-27}\ \text{kg}.$$

The nuclei of atoms of chemical elements are denoted by ${}^{A}_{Z}\text{X}$, where X is the symbol of an element, A is the mass number, and Z is the atomic number of the element, which is equal to the number of protons in the nuclei. The particles are designated as follows: ${}^{0}_{-1}e$ is the electron, ${}^{0}_{1}e$ is the positron, ${}^{1}_{0}n$ is the neutron, ${}^{1}_{1}p$ is the proton (the nucleus of a hydrogen atom is ${}^{1}_{1}\text{H}$), and ${}^{4}_{2}\text{He}$ is the α-particle.

The mass of an atomic nucleus is less than the sum of the masses of the nucleons by a quantity known as the mass defect:

$$\Delta m = Zm_p + Nm_n - m_\text{X},$$

where m_X is the mass of the nucleus, m_p is the mass of the proton, m_n the mass of the neutron. The mass m_a of the atom is taken from the Periodic Table. Then

$$m_\text{X} = m_a - Zm_e.$$

Nucleons are confined in a nuclei by nuclear forces which considerably exceed the electrostatic (repulsive) forces. For this reason, when a nucleus is split, a certain amount of energy is required to overcome the nuclear forces. When nucleons combine to form a new nucleus, the energy known as the binding energy is liberated.

The relation between the binding energy and mass defect is given by Einstein's relation

$$\Delta E = \Delta mc^2.$$

A mass defect of 1 amu corresponds to a binding energy of 931.5 MeV.

Radioactivity (viz. the spontaneous disintegration of the nuclei of some isotopes with the emission of α- and β-particles and γ-rays) is a phenomenon that confirms the compound structure of nuclear atoms.

Three principal types of radiation are emitted during radioactive decay (Fig. 144), α-radiation which consists of helium nuclei ${}^{4}_{2}\text{He}$ (the mass number after an α-decay decreases by 4 and the charge by two units), β-radiation which consists of electrons whose velocities are close to the veloc-

ity of light (an electron is produced during β-decay as a result of the transformation of a neutron into a proton, and hence the positive charge of the nucleus increases by one, and the mass number remains unchanged), and γ-radiation which accompanies α- and β-decay. The emission of a γ-quantum does not involve any change in the mass number or charge.

The rules for the nuclei formed during radioactive decays are

$$\text{for an } \alpha\text{-decay,} \quad {}^{A}_{Z}X_1 \to {}^{A-4}_{Z-2}X_2 + {}^{4}_{2}\mathrm{He},$$

$$\text{for a } \beta\text{-decay,} \quad {}^{A}_{Z}X_1 \to {}^{A}_{Z+1}X_2 + {}^{0}_{-1}e.$$

The *law of radioactive decay* states that the number ΔN of nuclei undergoing a radioactive decay during the time interval from t to $t + \Delta t$ is proportional to the number N of nuclei available at time t and to the length of the time interval Δt:

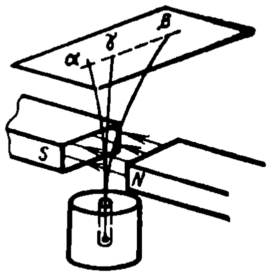


Fig. 144

$$\Delta N = -\lambda N \, \Delta t,$$

where λ is a decay constant which characterizes the rate of radioactive decay for a given species of nuclei. The minus sign indicates that the number of nuclei decreases during the decay process.

In order to characterize the stability of a nucleus, the concept of a half-life is introduced. The half-life $T_{1/2}$ is the time required for half the nuclei capable of decaying to do so:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.$$

Worked Problems

Problem 117. The radioactive waste from nuclear power plants contains the radioactive strontium isotope ${}^{90}_{38}\mathrm{Sr}$ which has a half-life of 28 years. Over what period will the amount of strontium decrease by a factor of four?

Given: ${}^{90}_{38}\mathrm{Sr}$ is the strontium isotope, and $T_{1/2} = 28$ years is its half-life.

Find: the time t during which the amount of strontium decreases by a factor of four.

Solution. The amount of decaying nuclei as a function of the half-life is represented by the curve in Fig. 145. We conclude from this graph that the amount of strontium nuclei will decrease to one quarter of the initial value in 2×28 years = 56 years.

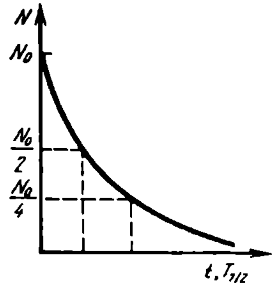


Fig. 145

Problem 118. What fraction of radioactive cesium $^{137}_{55}Cs$ with a half-life of 30 years will decay in a year? Determine the decay constant.

Given: $^{137}_{55}Cs$ is the radioactive cesium isotope, $T_{1/2} = 30$ years is the half-life, and $t = 1$ year is the time.

Find: the fraction $\Delta N/N_0$ of decayed nuclei and the decay constant.

Solution: We use N_0 to denote the initial number of atoms and N to denote the number of atoms remaining after time t. Then we can write $\Delta N = N_0 - N$, where ΔN is the number of atoms that have decayed over the time t.

According to the law for radioactive decay, $N = N_0 e^{(-\lambda t)}$, $e = 2.718...$ being the base of Napierian logarithm.

Assuming that the time t is small in comparison with the half-life T, we can use the approximation

$$\Delta N = \frac{0.693}{T} N_0 t,$$

or

$$\frac{\Delta N}{N_0} = \frac{0.693}{T} t, \quad \frac{\Delta N}{N_0} = \frac{0.693 \times 1 \text{ year}}{30 \text{ years}} = 0.023.$$

The decay constant λ characterizes the rate of the radioactive decay and can be determined from the formula

$$\lambda = \frac{0.693}{T}, \quad \lambda = \frac{0.693}{30 \times 365 \times 24 \times 3600 \text{ s}} = 7.3 \times 10^{-10} \text{ s}^{-1}.$$

Answer. The fraction of cesium nuclei that decays over a year is approximately 2.3%, the decay constant is 7.3×10^{-10} s^{-1}.

Problem 119. Determine the compositions of lithium nuclei, hydrogen isotopes with mass numbers 1 and 2, and uranium isotopes with mass numbers 235 and 238.

Given: ${}^7_3\mathrm{Li}$ is the nucleus of a lithium atom, ${}^1_1\mathrm{H}$ is the nucleus of a hydrogen atom, ${}^2_1\mathrm{H}$ is the nucleus of a heavy hydrogen atom (deuteron), and ${}^{238}_{92}\mathrm{U}$ and ${}^{235}_{92}\mathrm{U}$ are the nuclei of uranium isotopes.

Find: the number Z of protons and the number N of neutrons in the nuclei of the indicated elements.

Solution. The number of protons in a nucleus is determined by the charge number which is equal to the atomic number of the element in the Periodic Table. The charge number is the subscript to the left of the element symbol. The superscript is the mass number A that is equal to the number of nucleons in the nucleus: $A = Z + N$. Therefore, the number of neutrons is

$$N = A - Z.$$

For lithium ${}^7_3\mathrm{Li}$, $Z = 3$ and $N = 7 - 3 = 4$.

For ordinary hydrogen (${}^1_1\mathrm{H}$), $Z = 1$, and for heavy hydrogen (${}^2_1\mathrm{H}$), $Z = 1$, $N = 1$.

For ${}^{238}_{92}\mathrm{U}$, $Z = 92$, $N = 146$.

For ${}^{235}_{92}\mathrm{U}$, $Z = 92$, $N = 143$.

In this problem, hydrogen and uranium are represented by two isotopes. The isotopes of a chemical element have the same number of protons and different number of neutrons.

Problem 120. The nucleus of the magnesium isotope with a mass number 25 is bombarded by protons. The nucleus of which element is formed if the nuclear reaction is accompanied with the emission of α-particles?

Solution. In the Periodic Table, we find the atomic number of magnesium (12) and write the nuclear reaction

$${}^{25}_{12}\mathrm{Mg} + {}^1_1\mathrm{H} \rightarrow {}^A_Z\mathrm{X} + {}^4_2\mathrm{He}.$$

Since charge is conserved, the sum of the charge numbers on the left-hand side must be equal to the sum of the charge numbers on the right-hand side, viz. $12 + 1 = Z + 2$. Hence the charge number Z of the unknown element is 11. The eleventh site in the Periodic Table is occupied by sodium.

Since mass (the mass number) is conserved, the sums of the mass numbers on the left- and right-hand sides must be

equal (here this sum is 26). Consequently, we obtain a sodium isotope with a mass number of 22.

In its final form, the reaction equation will be written as

$$^{25}_{12}\mathrm{Mg} + ^{1}_{1}\mathrm{H} \rightarrow ^{22}_{11}\mathrm{Na} + ^{4}_{2}\mathrm{He}.$$

Problem 121. As a result of bombardment of aluminium by α-particles, a new nucleus and a neutron are formed. Write the nuclear reaction and identify the element whose nucleus is formed.

Solution. We shall write the nuclear reaction. On the left-hand side, we write the initial quantities, while the right-hand side must contain the unknown nucleus and a neutron:

$$^{27}_{13}\mathrm{Al} + ^{4}_{2}\mathrm{He} \rightarrow ^{A}_{Z}\mathrm{X} + ^{1}_{0}n.$$

Equating the charge and mass numbers on the left- and right-hand sides (as in Problem 120), we conclude that the new nucleus is that of phosphorus. In its final form, the reaction is

$$^{27}_{13}\mathrm{Al} + ^{4}_{2}\mathrm{He} \rightarrow ^{30}_{15}\mathrm{P} + ^{1}_{0}n.$$

Problem 122. Determine the mass defect and the binding energy of the nucleus of a nitrogen atom. What is the binding energy per nucleon?

Given: $^{14}_{7}\mathrm{N}$ is the nucleus of the nitrogen atom. From tables, we take the rest mass of a neutron $m_n = 1.00867$ amu, the mass of the nitrogen atom $m_\mathrm{N} = 14.0067$ amu, the mass of a hydrogen atom $m_\mathrm{H} = 1.00797$ amu, the velocity of light in vacuum $c = 2.99792 \times 10^8$ m/s, and 1 amu = 1.66056×10^{-27} kg.

Find: the mass defect Δm, the binding energy ΔE of the nucleus of a nitrogen atom, and the binding energy per nucleon $\Delta E/A$.

Solution. The mass defect is the difference between the sum of the rest masses of the free protons and the neutrons constituting a nucleus on one hand and the mass of the nucleus on the other, i.e.

$$\Delta m = Zm_p + Nm_n - m_\mathrm{X},$$

where m_X is the mass of the nucleus, $m_\mathrm{X} = m_\mathrm{N} - Zm_e$.

The expression for the mass defect can be considerably simplified if instead of the sum of the masses of protons we

take the sum of the masses of hydrogen atoms:

$$\Delta m = Zm_{\text{H}} + Nm_n - m_{\text{N}}.$$

Substituting in the numerical values, we obtain

$$\Delta m = 7 \times 1.00797 \text{ amu} + 7 \times 1.00867 \text{ amu}$$

$$- 14.0067 \text{ amu} \simeq 0.10978 \text{ amu},$$

$$\Delta m = 0.10978 \times 1.66 \times 10^{-27} \text{ kg} \simeq 1.822348 \times 10^{-28} \text{ kg}.$$

The binding energy can be determined using Einstein's relation

$$\Delta E = \Delta mc^2,$$

$$\Delta E = 1.822 \times 10^{-28} \text{ kg } (2.9979 \text{ m/s})^2 = 1.638 \times 10^{-11} \text{ J}.$$

In atomic physics, the binding energy is expressed in megaelectronvolts:

$$\Delta E = \frac{1.638 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV} \simeq 102.4 \text{ MeV}.$$

Considering that 1 amu corresponds to the energy approximately equal to 931.4 MeV, we can determine the binding energy in a simpler way:

$$\Delta E = 0.10978 \times 931.4 \text{ MeV} \simeq 102.25 \text{ MeV}.$$

The results of calculations clearly differ insignificantly.

The binding energy per nucleon is

$$\Delta E/A = 102 \text{ MeV}/14 \simeq 7.3 \text{ MeV/nucleon}.$$

The result is in agreement with the tabulated data.

Answer. The mass defect is 0.1097 amu, the binding energy of the nitrogen nucleus is approximately 102 MeV, and the binding energy per nucleon is 7 MeV.

Problem 123. The thermal power of each reactor installed at the Leningrad nuclear power plant is 3200 MW. The electric power is 1000 MW. The mass of the uranium charge in a reactor is 180 t. Determine the efficiency of the unit, the mass of the uranium-235 consumed by the reactor over a year of continuous operation at the total power. What fraction of the total charge does the mass of the consumed uranium constitute?

Given: $P_t = 3.2 \times 10^9$ W is the thermal power of the reactor, $P_c = 10^9$ W is the electric power of the unit, $t =$

1 year $= 3.15 \times 10^7$ s is the time for which the nuclear fuel consumption is being determined, $m = 1.8 \times 10^5$ kg is the mass of the uranium charge of a reactor. From tables, we take the Avogadro constant $N_A = 6.02 \times 10^{23}$ mol^{-1}, the molar mass of uranium $M = 235 \times 10^{-3}$ kg/mol, and the energy liberated as a result of the fission of a ^{235}U nucleus, $E =$ 200 MeV.

Find: the efficiency η of the power unit, the mass m_1 of uranium consumed and the fraction m_1/m of the consumed uranium.

Solution. The efficiency of a unit is defined as the ratio of the electric power of the unit to the thermal power of a reactor:

$$\eta = \frac{P_e}{P_t} 100\%, \quad \eta = \frac{10^9 \text{ W}}{3.2 \times 10^9 \text{ W}} 100\% = 31.3\%.$$

In order to determine the mass of consumed uranium, we must find the number of ^{235}U atoms entering into the nuclear reaction over the time of operation of the reactor. Since about 200 MeV of energy is liberated during the fission of a ^{235}U nucleus and that 190 MeV are converted into heat, we can write $N = P_t t/E_1$, where $E_1 = 190$ MeV. Given the number of atoms in the reaction and the mass of a uranium atom, $m_U = M/N_A$, we can determine the mass of consumed uranium:

$$m_1 = m_U N, \quad \text{or} \quad m_1 = \frac{M}{N_A} \frac{P_t t}{E_1},$$

$$m_1 = \frac{235 \times 10^{-3} \text{ kg} \cdot \text{mol}^{-1} \times 3.2 \times 10^9 \text{ W} \times 3.15 \times 10^7 \text{ s}}{6.02 \times 10^{23} \text{ mol}^{-1} \times 190 \times 1.6 \times 10^{-13} \text{ J}} = 1294 \text{ kg}.$$

Let us determine the fraction of the total charge consumed over a year:

$$\frac{m_1}{m} = \frac{1294 \text{ kg}}{1.8 \times 10^5 \text{ kg}} = 0.007.$$

Answer. The efficiency of the plant is about 31%, the amount of uranium-235 consumed during a year of operation is about 1300 kg, which is less than 1% of the total.

Questions and Problems

26.1. What is α-radiation? Why is it deflected less in a magnetic field than β-radiation?

26.2. What force deflects α- and β-rays in a magnetic field?

26.3. Which of the three radiations (alpha, beta, and gamma) is not deflected by either magnetic or electric fields?

26.4. What is γ-radiation? What is the difference between this radiation and X-rays?

26.5. How many electrons occupy the electron shell of a neutral atom whose nucleus contains six protons and six neutrons?

26.6. The atomic nucleus of any chemical element consists of protons and neutrons. How can the emission of β-radiation be explained?

26.7. Determine the half-life of radon if 1.75×10^5 out of 10^6 atoms decay per day. What is the decay constant equal to? (Use the approximate formula.)

26.8. The decay constants for bismuth-209 and polonium-210 are $1.6 \times 10^{-6}\ s^{-1}$ and $5.8 \times 10^{-8}\ s^{-1}$ respectively. Determine their half-lives.

26.9. What fraction of the radioactive nuclei of ${}^{14}_{6}C$ decays over 100 years if its half-life is 5570 years?

26.10. It is known that 9.3×10^{18} out of 2.51×10^{19} available atoms of the sodium-24 isotope undergo β-decay. The half-life is 14.8 h. Determine the decay time and constant using the approximate formula (see Problem 118).

26.11. The uranium isotope ${}^{238}_{92}U$ of 1-g mass emits 1.24×10^4 α-particles per second. Determine the half-life and decay constant of the isotope.

26.12. Determine the composition of hydrogen ${}^{3}_{1}H$, helium ${}^{4}_{2}He$, aluminium ${}^{27}_{13}Al$, uranium ${}^{238}_{92}U$, and neptunium ${}^{237}_{93}Np$ nuclei. What can be said about the neutron content of nuclei with increasing atomic number?

26.13. What is the difference between the nuclei of the chlorine isotopes ${}^{35}_{17}Cl$ and ${}^{37}_{17}Cl$? How can you explain the fact that chlorine has a relative atomic mass of 35.5 in the Periodic Table?

26.14. The nucleus of which element contains 14 protons and the same number of neutrons? Which of the first twenty elements in the Periodic Table have nuclei containing equal numbers of protons and neutrons?

26.15. Determine the charges of the lithium, copper, and uranium-238 nuclei in coulombs.

26.16. One gram of radium emits 3.7×10^{10} α-particles

per second. Determine the charge of this radiation in coulombs.

26.17. Determine the radius and nuclear density of helium and uranium-238 atoms.

26.18. Write the reaction for the direct transformation of actinium-227 into francium-223. What type of a radioactive decay is it?

26.19. What will happen to the uranium-237 isotope during β-decay? What will the mass number of the new element be? To which side of the Periodic Table will the nucleus be shifted? Write the reaction equation.

26.20. A beryllium nucleus is formed by the reaction between a lithium nucleus and a deuteron. What particle is liberated in the process? Write the nuclear reaction equation.

26.21. The "age" of the objects discovered during an archeological dig is determined from the isotopes of a certain element the objects contain. Determine the charge and the mass number and identify the element from the following nuclear reaction:

$$^{14}_{7}\mathrm{N} + ^{1}_{0}n \rightarrow ^{A}_{Z}\mathrm{X} + ^{1}_{1}p.$$

26.22. What are the resultant nuclei after the α- and β-decay of xenon?

26.23. Two γ-quanta are formed by the annihilation of an electron and a positron. Assuming that the masses of the electron and the positron are the same, determine the energy of the γ-radiation and its frequency.

26.24. The presence of explosives in the luggage of air passengers can be detected using nuclear physics. An explosive normally contains nitrogen isotopes with mass numbers 14 and 15. As a result of bombardment by neutrons, nitrogen isotopes with mass numbers 15 and 16 are formed. The latter isotope is radioactive and emits γ-quanta that can be detected. Write the equation of the nuclear reaction.

26.25. What must the energy of a γ-quantum be for it to be convertable into an electron-positron pair?

26.26. It is established that a proton is emitted during the bombardment of an aluminium isotope by helium nuclei with the formation of a new nucleus. Write the equation for the nuclear reaction and identify the new nucleus.

26.27. When boron $^{11}_{5}B$ captures a fast proton, three almost identical tracks spreading in different directions are formed in a Wilson cloud chamber where the process takes place. What particles are responsible for these tracks?

26.28. Identify the particle denoted by the question mark in the nuclear reaction $^{7}_{3}Li + ? \to {}^{10}_{5}B + {}^{1}_{0}n$.

26.29. What nuclei and particles are formed as a result of the following reactions:

$$^{239}_{94}Pu + {}^{4}_{2}He \to {}^{A}_{Z}X + {}^{1}_{0}n,$$

$$^{2}_{1}H + \gamma \to {}^{1}_{1}H + ?$$

26.30. The transformation of phosphorus $^{30}_{15}P$ into silicon $^{30}_{14}Si$ is accompanied by the emission of a positron. What changes occur in the nucleus?

26.31. Determine the mass defect of a lithium nucleus in atomic mass units and in kilograms.

26.32. Determine the mass defect of the boron nucleus $^{10}_{5}B$ in atomic mass units and in energy units.

26.33. Determine the mass defect and the binding energy for the uranium-238 nucleus.

26.34. Analyze the following nuclear reactions and determine whether the energy is liberated or evolved:

$$^{4}_{2}He + {}^{4}_{2}He \to {}^{7}_{3}Li + {}^{1}_{1}H,$$

$$^{9}_{4}Be + {}^{2}_{1}H \to {}^{10}_{5}B + {}^{1}_{0}n,$$

$$^{6}_{3}Li + {}^{2}_{1}H \to 2\,{}^{4}_{2}He.$$

26.35. Determine the energy liberated in the nuclear reaction $^{6}_{3}Li + {}^{1}_{1}H \to {}^{4}_{2}He + {}^{3}_{2}He$ (see Table 25).

26.36. In order to obtain 1 GW of electric power, 2×10^{6} t of coal have to be burnt annually, which involves the discharge of 8×10^{3} t of ash and tens of thousands of tons of sulphur dioxide into the atmosphere. How much uranium-235 is required to obtain the same power for the same efficiency?

26.37. The fission of a uranium-235 atom into two fragments is accompanied by the liberation of about 3×10^{-11} J of energy. How much petrol has to be burnt to obtain the same energy as that liberated in a nuclear reaction in which 1 g of uranium is consumed?

26.38. Calculate the energy liberated by the combustion of 1120 t of A-1 grade coal, of 376 t of petroleum, of

5×10^5 m^3 of natural gas and the energy generated by the fission of 260 g of uranium-235.

26.39. The efficiency of the reactors installed at the Kolsk and Rovensk nuclear power plants is 32%. How much uranium 235 is consumed in a nuclear reactor per hour if its electric power is 440 MW?

26.40. How much energy is liberated in the nuclear reactors of the nuclear-powered icebreaker *Lenin* if the daily consumption of uranium-235 is 62 g?

26.41. The thermal reactors installed at atomic power plants (operating on slow neutrons) do not utilize the nuclear fuel very efficiently and cannot ensure the required scale of production of nuclear power. The situation is different for reactors on fast neutrons. Why?

26.42. Which of the particles listed below are stable: a photon, an electron, a neutrino, a proton, a neutron, or a π-meson?

26.43. The annihilation of a proton and an antiproton generates γ-radiation. Calculate the energy of the photons if the masses of the proton and the antiproton are the same and equal to 1.67×10^{-27} kg.

26.44. Controlled nuclear fusion would be a prodigious way of obtaining energy. As much energy is liberated by the fusion of the deuterium contained in one litre of ordinary water as by combustion of 350 l of petrol. Calculate this amount of energy.

26.45. If 5×10^4 kg of hydrogen are fused into 49 644 kg of helium, how much energy is liberated?

Chapter VI

General Remarks on Astronomy

§ 27. FUNDAMENTALS OF ASTRONOMY

Basic Concepts and Formulas

Astronomy facilitates a deeper understanding of the physics of the world and extends dialectical and materialistic ideas about matter and various forms of its existence. The development of cosmonautics has broadened the means for investigating the processes occurring in the Universe and the way they influence the life on our planet.

The celestial sphere is defined as having an arbitrary radius with the centre at the point of observation, onto which

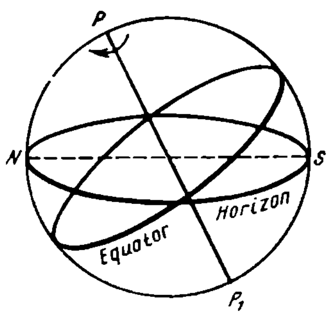


Fig. 146

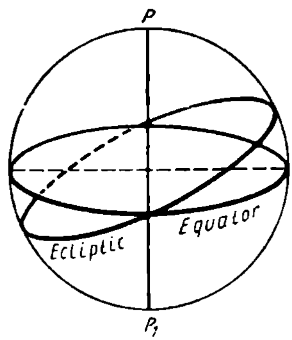


Fig. 147

the celestial bodies are being projected. The celestial axis PP_1 is the axis of the apparent rotation of the celestial sphere. The points where the celestial sphere intersects the celestial axis are known as celestial poles (Fig. 146).

The diurnal rotation of the celestial sphere makes it possible to determine the constellations which set at a given geographical latitude and those which do not set. For

example, the constellation of Ursa Minor does not set at the latitude of Moscow. The brightest star in this constellation, Polaris, is very close to the north celestial pole. Since the geographical latitude of an observation point can be determined from the angular distance from the plane of the horizon to a celestial pole, the geographical latitude in the northern hemisphere can be determined from the altitude of Polaris.

The ecliptic is the apparent annual path of the Sun on the celestial sphere. The ecliptic intersects the celestial equator at points known as vernal equinox and autumnal

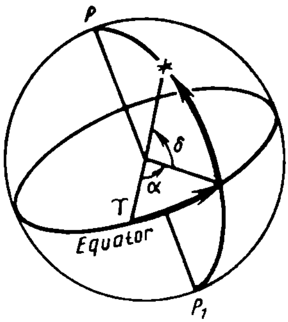


Fig. 148

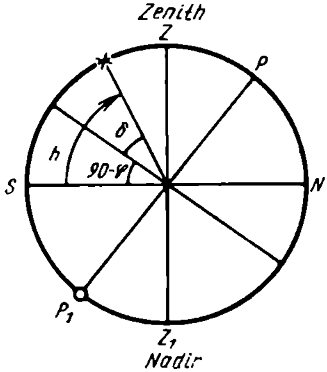


Fig. 149

equinox (Fig. 147). The most remote points from the equator are passed by the Sun on the 22nd of June and on 22nd of December, which are known as summer and winter solstices respectively. The ecliptic passes through twelve constellations of the Zodiac, namely Pisces, Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, and Aquarius.

The position of a star on the celestial sphere is determined by two coordinates: its declination δ and right ascension α (Fig. 148).

The large circle of the celestial sphere passing through the celestial poles and a given star is known as the declination circle, while the angular distance from the celestial equator to the star, measured along the declination circle is known as the declination of the star. Declinations are positive to

the north of the equator and negative to the south of it. The declination of the star is similar to the geographical latitude. The right ascension is measured along the celestial equator from the vernal equinox to the declination circle passing through the given star. This coordinate is similar to the geographical longitude and expressed in units of time.

The passage of a star through the meridian is known as its culmination. Each star passes through the meridian twice a day. The apparent noon is the upper culmination of the centre of the solar disc, while the apparent midnight corresponds to its lower culmination.

The altitude h of a star above the horizon at its upper culmination can be determined from the formula (Fig. 149)

$$h = 90^\circ - \varphi + \delta,$$

where φ is the geographical latitude. In addition to the Sun and planets, the Solar System contains asteroids (small planets), comets, and meteoric dust.

Our star, the Sun, is an ordinary star in the Milky Way (Galaxy). Its diameter is 1390000 km, i.e. 109 times the diameter of the Earth and its mass is 333000 times that of the Earth.

The hot gaseous sphere of the Sun consists of 85% of hydrogen and 13% of helium. Many other elements are present in small amounts. The temperature at the centre of the Sun is about 2×10^7 K and the pressure is of the order of 2×10^7 GPa. Under these conditions, fusion reactions take place, which are the source of the solar energy.

The motion of celestial bodies and their gravitational attraction obey the laws of physics.

Kepler's first law states that all planets revolve around the Sun along elliptical orbits with the Sun as one of the foci.

Kepler's second law: the radius vectors of a planet drawn from the Sun sweep equal areas in equal times (Fig. 150).

Kepler's third law: the squares of the times taken by two planets to describe their orbits are proportional to the cubes of the semimajor axes of the orbits:

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}.$$

Newton refined Kepler's third law for comparing the masses of celestial bodies:

$$\frac{m_1+m_2}{m_3+m_4}\,\frac{T_{12}^2}{T_{34}^2}=\frac{R_{12}^3}{R_{34}^3},$$

where m_1, m_2 and m_3, m_4 are the masses of two pairs of celestial bodies revolving one around the other, and T_{12}, T_{34}, R_{12}, and R_{34} are respectively the periods of revolution and the mean distances between the bodies.

Newton's law of universal gravitation: all bodies in the Universe are attracted to one another with a force propor-

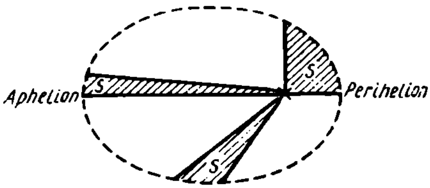


Fig. 150

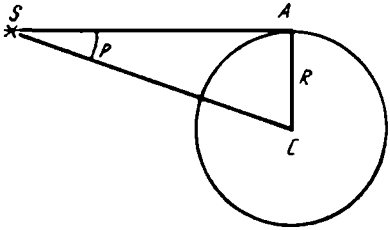


Fig. 151

tional to the product of their masses and inversely proportional to the squared distance between their centres of mass:

$$F=G\frac{m_1m_2}{r^2},$$

where G is the gravitational constant. The distances to celestial bodies are determined by parallax. The horizontal parallax p is the angle at which the radius of the Earth perpendicular to the line of sight is seen from a celestial body (Fig. 151):

$$SC=\frac{B}{\sin p}.$$

The annual parallax of a star is the angle at which the radius of the Earth orbit perpendicular to the line of sight D is seen from the star (Fig. 152):

$$D = \frac{a}{\sin p}.$$

The unit of length in astronomy is the astronomical unit (AU), 1 AU = 149.6×10^6 km is the mean distance between the Sun and the Earth. When discussing stars and galaxies, the units of length are the light year and parsec (pc),

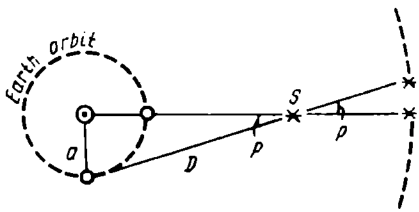


Fig. 152

1 pc = 206265 AU = 30.86×10^{12} km. One light year is the distance traversed by light in one year: 1 light year = 0.3068 pc.

When solving some of the problems, the star chart should be used. To do this, trace the circle on the flyleaf of the book. It should be cut along the line corresponding to the geographical latitude of the site of observation. By applying the circle to the star chart on the front flyleaf, you will obtain the celestial map for the required month and hour of observation. For example, to obtain the star chart at 10 pm, on October 10, it is necessary that 10 pm in the cut circle be matched with the mark for October 10 on the chart. In the slot of the circle you will have all the celestial bodies visible at that moment.

Worked Problems

Problem 124. Determine the ratio of the mass of the Sun to the mass of the Earth if the period of revolution of the Moon round the Earth is 27.2 days and the mean distance from the Earth to the Moon is 384000 km.

Given: $T_{\text{Moon}} = 27.2$ days is the period of revolution of the Moon round the Earth, $a_{\text{Moon}} = 3.84 \times 10^5$ km is the mean distance from the Earth to the Moon, and $T_{\text{Earth}} =$ 365 days is the period of revolution of the Earth round the Sun. From tables, we take the mean distance from the Earth to the Sun $a_{\text{Earth}} = 1.5 \times 10^8$ km.

Find: the ratio between the mass of the Sun and the mass of the Earth, $m_{\text{Sun}}/m_{\text{Earth}}$.

Solution. We shall use the formula for the refined version of Kepler's third law: $\frac{m_{\text{Sun}}+m_{\text{Earth}}}{m_{\text{Earth}}+m_{\text{Moon}}} \cdot \frac{T^2_{\text{Earth}}}{T^2_{\text{Moon}}} = \frac{a^3_{\text{Earth}}}{a^3_{\text{Moon}}}$. Since the mass of the Earth relative to that of the Sun and the mass of the Moon relative to the mass of the Earth are negligibly small, we can rewrite the formula as follows: $\frac{m_{\text{Sun}} T^2_{\text{Earth}}}{m_{\text{Earth}} T^2_{\text{Moon}}} = \frac{a^3_{\text{Earth}}}{a^3_{\text{Moon}}}$. Hence $\frac{m_{\text{Sun}}}{m_{\text{Earth}}} = \frac{a^3_{\text{Earth}}}{a^3_{\text{Moon}}} \times \frac{T^2_{\text{Moon}}}{T^2_{\text{Earth}}}$.

Substituting in the numerical values, we obtain

$$\frac{m_{\text{Sun}}}{m_{\text{Earth}}} = \frac{(1.5 \times 10^8 \text{ km})^3 (27.2 \text{ days})^2}{(3.84 \times 10^5 \text{ km})^3 (365 \text{ days})^2} = 330000.$$

Answer. The ratio between the masses of the Sun and Earth is about 330000.

Problem 125. Determine the mean distance between the Earth and the Moon using the following data: (1) the horizontal parallax of the Moon is $p = 0.57'$, and (2) an electromagnetic signal sent to the Moon from the Earth returns in 2.56 s. What is the mean velocity of motion of the Moon round the Earth if the sidereal month is 27.3 days long?

Given: $p = 0.57'$ is the horizontal parallax of the Moon, $t = 2.56$ s is the time required for the electromagnetic signal to traverse twice the distance from the Earth to the Moon, $T = 27.3$ days $= 27.3 \times 3600 \times 24$ s is the sidereal month. From tables, we take the mean radius of the Earth $R_{\text{Earth}} = 6370$ km and the velocity of light $c =$ 3×10^5 km/s.

Find: the distance d from the Earth to the Moon and the mean velocity $\bar{v}$ of the Moon in its orbit.

Solution. 1. The horizontal parallax of the Moon is defined as the angle at which the radius of the Earth is seen from

the Moon (see Fig. 151). Consequently, $\frac{p}{360^\circ} = \frac{R_{\text{Earth}}}{2\pi d}$. Here it is more convenient to express the parallax in degrees: $p = 0.57' = 0.95^\circ$. Then

$$d = \frac{360^\circ \times 6370 \text{ km}}{0.95^\circ \times 6.28} \simeq 384380 \text{ km}.$$

2. A radio signal sent to the Moon will be reflected by its surface and return to the Earth. We know the time in which the radio signal covers twice the distance from the Earth to the Moon. Therefore,

$$d = \frac{ct}{2}, \quad d = \frac{300000 \text{ km/s} \times 2.56 \text{ s}}{2} \simeq 384000 \text{ km}.$$

In order to determine the mean orbital velocity of the Moon around the Earth we use the formula $\bar{v} = 2\pi d/T$, where T is the sidereal month, i.e. the time taken by the Moon to complete one revolution around the Earth relative to the stars. Substituting the numerical values, we obtain

$$\bar{v} = \frac{6.28 \times 3.84 \times 10^5 \text{ km}}{27.3 \times 24 \times 3600 \text{ s}} \simeq 1.02 \text{ km/s}.$$

Answer. The approximate distance between the Earth and the Moon given by the two methods are (1) 384380 km and (2) 384000 km. The mean orbital velocity of the Moon is 1.02 km/s.

Questions and Problems

27.1. How long does it take light to travel from the Sun to the Earth? The distance from the Sun to the Earth is 149.6×10^6 km, and the velocity of light is 2.998×10^5 km/s.

27.2. Determine the distance to α Centauri (the nearest star to the Solar System) if its light takes 4.25 years to reach the Earth. Express the distance in kilometres and parsecs.

27.3. The mean distance from the Sun to the farthest planet Pluto is 40 AU. How long does it take light to cover this distance?

27.4. It takes 2×10^6 years for light to travel from the nearest to us galaxy in Andromeda. Express this distance in parsecs.

27.5. List the brightest stars, whose magnitudes are ex-

pressed in negative numbers.

27.6. What constellation includes Sirius? What does its negative declination indicate?

27.7. The star Sirius is about 8.4×10^{13} km from the Earth. How long does it take its light to reach the Earth?

27.8. Light takes over 100000 years to traverse the larger diameter of our Galaxy. Determine the approximate size of the Galaxy in parsecs.

27.9. What is the brightest star in the northern celestial hemisphere? What constellation does it belong to?

27.10. Name the points of the horizon lying on the celestial meridian and celestial equator.

27.11. Name the equatorial constellation cut into two unequal parts by another constellation.

27.12. Compile a list of the nonsetting constellations in your locality by observation. Verify the correctness of your observations with star chart.

27.13. Determine the position of the constellation Andromeda on the star chart at the moment of observation and locate it in the sky. The nearest Galaxy to us is a misty spot in the vicinity of this constellation. Observe this galaxy.

27.14. Does the Sun always rise exactly in the east and set exactly in the west?

27.15. Which Zodiac constellations cannot be observed on the north pole?

27.16. Where must an observer be to see the north celestial pole at the zenith?

27.17. The angular distance of the celestial pole from the zenith is 34° 15′ for an observer at Moscow. What is the geographical latitude of Moscow?

27.18. Determine the altitude of Polaris for your locality.

27.19. The geographical latitude of Leningrad is 59° 56′. What is the angular distance between the zenith and the celestial pole for Leningrad?

27.20. Use the star chart to determine the constellation which is located on the northern horizon at midnight on May 15 at the latitude of Moscow.

Determine the constellation at that point and at the same time at your latitude.

27.21. By how many degrees does the Sun move along the ecliptic per day?

27.22. What are the declination and the right ascension

of the Sun on the 22nd of March?

27.23. What are the declination and the right ascension of the autumnal equinox?

27.24. What is the altitude of the Sun at noon in Moscow at the summer solstice?

27.25. What will the angular diameter of the Earth be for a cosmonaut on the Moon if the radius of the Earth is 6370 km and the distance from the Earth to the Moon is 384000 km?

27.26. The angular diameter of the Sun is 32′. Determine the diameter of the Sun.

27.27. The maximum horizontal parallax of Mars is 23″. Calculate the minimum distance between Mars and the Earth.

27.28. The annual parallax of Vega is 0.121″. Calculate the distance from the Earth to Vega in parsecs.

27.29. Calculate the period of revolution of Uranus around the Sun if the mean distance between it and the Sun is 19.19 AU.

27.30. Saturn orbits the Sun once in 29.46 years. Determine the mean distance between it and the Sun.

27.31. The period of revolution of Jupiter round the Sun is 11.86 years. Determine the mean distance from it to the Sun in astronomical units and in kilometres.

27.32. How will the Earth appear to an observer on the Moon at the instant when full Moon is seen from the Earth?

27.33. Venus is sometimes called the morning star and evening star. For what latitudes is this true?

27.34. What is the phase of Venus when it is observed as a morning star?

27.35. The annual parallax of our closest star, α Centauri, is 0.76″. How long would it take for a spacecraft to fly there at a velocity of 17 km/s?

27.36. The distance to Barnard's star is 1.83 pc. What is its annual parallax?

27.37. Why do the seasons not change on Venus?

27.38. Determine the mean density of solar matter. Take the required data from tables.

27.39. Determine the linear velocity of the Vostok-1 spaceship, in which Yuri Gagarin orbited the Earth for the first time, assuming that the orbit was circular and its mean distance from the surface of the Earth was 251 km.

Appendices

1. Basic Physical Constants

Velocity of sound in air under normal conditions	$c = 331.46$ m/s
Velocity of light in vacuum	$c = 2.998 \times 10^{8}$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Free fall acceleration	$g = 9.807$ m/s²
Avogadro constant	$N_A = 6.022 \times 10^{23}$ mol^{-1}
Molar volume of ideal gas under normal conditions	$V_m = 22.4 \times 10^{-3}$ m³/mol
Loschmidt number	$N_L = 2.687 \times 10^{25}$ m^{-3}
Molar gas constant	$R = 8.314$ J/(mol·K)
Boltzmann constant	$k = 1.381 \times 10^{-23}$ J/K
Faraday constant	$F = 9.648 \times 10^{4}$ C/mol
Electric constant	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
Magnetic constant	$\mu_0 = 4\pi \times 10^{-7}$ H/m $= 1.257 \times 10^{6}$ H/m
Hydrogen atom mass	$m_H = 1.673 \times 10^{-27}$ kg
Electron rest mass	$m_e = 9.109 \times 10^{-31}$ kg
Proton rest mass	$m_p = 1.673 \times 10^{-27}$ kg
Neutron rest mass	$m_n = 1.675 \times 10^{-27}$ kg
Elementary charge	$e = 1.602 \times 10^{-19}$ C
Electron charge-to-mass ratio	$e/m_e = 1.759 \times 10^{11}$ C/kg
Ratio of proton and electron masses	$m_p/m_e = 1836.15$
Atomic mass unit	1 amu $= 1.660 \times 10^{-27}$ kg
Planck constant	$h = 6.626 \times 10^{-34}$ J·s
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ W/(m²·K⁴)
Wien constant	$b = 2.897 \times 10^{-3}$ m·K
Rydberg constant (for hydrogen)	$R_\infty = 1.097 \times 10^{7}$ m^{-1}
Triple point for water	$T = 273.16$ K ($t = 0.01$°C)

2. Density of Some Substances, ρ, kg/m³

Solids (at 293 K)

Aluminium	2.7×10^{3}	Cast iron	
Amber	1.1×10^{3}	gray	7.0×10^{3}
Brass	8.5×10^{3}	white	7.5×10^{3}
Brick	1.5×10^{3}		

Concrete	2.2×10^3	Nichrome	8.3×10^3
Constantan	8.9×10^3	Nickel	8.9×10^3
Common salt	2.1×10^3	Nickeline	8.8×10^3
Copper	8.9×10^3	Oak	0.8×10^3
Cork	0.24×10^3	Paraffin	9.0×10^2
Diamond	3.5×10^3	Platinum	21.5×10^3
Ebonite	1.2×10^3	Porcelain	2.3×10^3
Germanium	5.32×10^3	Rubber	0.94×10^3
Gold	19.3×10^3	Silver	10.5×10^3
Graphite	2.1×10^3	Snow	0.2×10^3
Ice (0°C)	0.9×10^3	Tin	7.3×10^3
Iridium	22.4×10^3	Tungsten	1.93×10^3
Iron, steel	7.8×10^3	Uranium	19.0×10^3
Lead	11.4×10^3	Window glass	2.5×10^3
Manganin	8.5×10^3	Zinc	7.1×10^3
Mica	2.8×10^3		

Liquids (at 293 K)

Acetone	0.8×10^3	Nitrobenzene	1.2×10^3
Aniline	1.02×10^3	Oil	
Benzene	0.85×10^3	castor (mineral)	0.92×10^3
Carbon bisulphide	1.26×10^3	transformer	0.89×10^3
Copper sulphate solution (saturated)	1.15×10^3	vegetable	0.93×10^3
		Petrol	0.7×10^3
Ethyl (methyl) alcohol	0.79×10^3	Petroleum	0.9×10^3
		Turpentine	0.87×10^3
Ethyl ether	0.71×10^3	Water	
Glycerine	1.26×10^3	at 277 K	1.0×10^3
Kerosene	0.8×10^3	heavy	1.06×10^3
Mercury (at 273 K)	13.6×10^3	sea	1.03×10^3

Gases
(under normal conditions: $p_0 = 1.013 \times 10^5$ Pa and $T_0 = 273$ K)

Acetylene	1.17	Hydrogen	0.09
Air	1.29	Krypton	3.74
Ammonia	0.77	Methane	0.72
Argon	1.78	Neon	0.9
Butane	0.6	Nitrogen	1.25
Carbon dioxide	1.98	Oxygen	1.43
Chlorine	3.21	Propane	2.01
Helium	0.18	Xenon	5.85

3. Specific Heat Capacity of Some Substances, c, J/(kg·K)

Solids

Aluminium	880	Cast iron	550
Brass	380	Cement	800
Brick	750	Concrete	920

Copper	380	Platinum	125
Glass	840	Sand	970
Gold	125	Silver	250
Ice (snow)	2090	Sulphur	712
Iron, steel, nickel	460	Tin	250
Lead	120	Wood (spruce, pine)	2700
Naphthalene	1300	Zinc	400
Paraffin	3200		

Liquids

Ethyl alcohol	2430	Machine oil	2100
Ethyl ether	2330	Mercury	125
Glycerine	2430	Transformer oil	2093
Iron	830	Turpentine	1760
Kerosene	2140	Water	4187

Gases (at constant pressure)

Air	1000	Hydrogen	14300
Ammonia	2100	Nitrogen	1000
Carbon dioxide	880	Oxygen	920
Helium	5200	Water vapour	2130

4. Specific Heat of Combustion for Some Substances, q, J/kg

Solid fuel

Brown coal	9.3×10^6	Coal	
Charcoal	3.1×10^7	A-1 grade	2.05×10^7
Chocks	1.5×10^7	A-2 grade	3.03×10^7
Coke	3.03×10^7	Donetsk	2.55×10^7
Firewood	8.3×10^6	Ekibastuz	1.63×10^7
Gun powder	3.0×10^6	Peat	1.5×10^7

Liquid fuel

Diesel	4.2×10^7	Kerosene	4.4×10^7
Ethyl alcohol	2.7×10^7	Naphtha	4.33×10^7
Fuel oil	4.0×10^7	Petrol, petroleum	4.6×10^7

Gaseous fuel

(per cubic metre under normal conditions)

Blast-furnace gas	3.7×10^6	Producer gas	5.5×10^6
Coke-oven gas	1.64×10^7	Town gas	2.1×10^7
Natural gas	3.55×10^7		

5. Boiling Point and Specific Latent Heat of Vaporization

Substance	T, K	t, °C	r, J/kg
Acetone	329.2	56.2	5.2×10^5
Air	81	−192	2.1×10^5
Ammonia	239.6	−33.4	1.37×10^6
Ethyl alcohol	351	78	8.57×10^5
Ethyl ether	308	35	3.52×10^5
Freon-12	243.2	−29.8	1.68×10^6
Iron	3023	2750	5.8×10^4
Mercury	630	357	2.85×10^5
Petrol	423	150	3.0×10^5
Turpentine	433	160	2.94×10^5
Water			
heavy	374.43	101.43	2.06×10^6
ordinary	373	100	2.26×10^6

6. Pressure and Density of Saturated Water Vapour at Various Temperatures

t, °C	p, kPa	$\rho \times 10^{-3}$, kg/m³	t, °C	p, kPa	$\rho \times 10^{-3}$, kg/m³
−10	0.260	2.14	16	1.813	13.6
−5	0.401	3.24	17	1.933	14.5
−4	0.437	3.51	18	2.066	15.4
−3	0.476	3.81	19	2.199	16.3
−2	0.517	4.13	20	2.333	17.3
−1	0.563	4.47	21	2.493	18.3
0	0.613	4.80	22	2.639	19.4
1	0.653	5.20	23	2.813	20.6
2	0.706	5.60	24	2.986	21.8
3	0.760	6.00	25	3.173	23.0
4	0.813	6.40	26	3.359	24.4
5	0.880	6.80	27	3.559	25.8
6	0.933	7.30	28	3.786	27.2
7	1.000	7.80	29	3.999	28.7
8	1.066	8.30	30	4.239	30.3
9	1.146	8.80	40	7.371	51.2
10	1.226	9.40	50	12.33	83.0
11	1.306	10.0	60	19.92	130.0
12	1.399	10.7	80	47.33	293
13	1.492	11.4	100	101.3	598
14	1.599	12.1	120	198.5	1123
15	1.706	12.8	160	618.0	3259
			200	1554	7763

7. Young's modulus for Some Substances, E, GPa

Aluminium	70	Copper	130
Brass	110	Iron	200
Brick	28	Lead	17
Cast iron	90	Steel	220
Concrete	20		

8. Boiling Point and Specific Latent Heat of Vaporization for Water under Various Pressures

t, °C	p, MPa	r, MJ/kg	t, °C	p, MPa	r, MJ/kg
10	0.001	2.47	197.4	1.47	1.95
100	0.1	2.26	346	15.7	0.9
151	0.49	2.11	347.15	22.1	0

9. Boiling Point and Critical Parameters for Some Substances

Substance	Boiling point, t, °C	Critical parameters	
		temperature t_{cr}, °C	pressure $p_{cr}\times 10^5$, Pa
Argon	—186	—122.4	48
Ether	35	193.8	35.6
Ethyl alcohol	78	243.1	63
Helium	—269	—267.9	2.25
Hydrogen	—253	—241	12.8
Krypton	—193	—63.62	54.27
Neon	—246	—228.7	26.9
Nitrogen	—196	—147.1	33.5
Oxygen	—183	—118.4	49.7
Water	100	374.15	221.3
Xenon	—108	18.76	57.64

10. Melting Point and Specific Latent Heat of Fusion for Some Solids at Melting Point

Substance	T_m, K	t_m, °C	λ, J/kg
Aluminium	932	659	3.8×10^5
Cast iron			
gray	1423	1150	9.7×10^4
white	1473	1200	1.3×10^5

10. Melting Point and Specific Latent Heat of Fusion for Some Solids at Melting Point (*cont.*)

Substance	T_m, K	t_m, °C	λ, J/kg
Copper	1356	1083	1.8×10^5
Gold	1337	1064	6.6×10^4
Iron	1803	1530	2.7×10^5
Lead	600	327	2.5×10^4
Mercury	234	—39	1.25×10^4
Naphthalene	353	80	1.51×10^5
Silver	1233	960	8.8×10^4
Steel	1673	1400	2.1×10^5
Sulphur	385.8	112.8	5.5×10^4
Tin	505	232	5.8×10^4
Tungsten	3683	3410	2.6×10^4
Water, ice	273	0	3.35×10^5
Water, heavy	276.82	3.82	3.16×10^5
Wood's metal*	341	68	3.2×10^4
Zinc	692	419	1.18×10^5

* Wood's metal with this melting point contains 50% bismuth, 25% lead, 12.5% tin, and 12.5% cadmium.

11. Surface Tension for Some Substances, σ, N/m (at 293 K)

Acetone	0.024	Mercury	0.470
Castor oil	0.033	Milk	0.045
Copper sulphate solution	0.074	Petrol	0.029
Ether	0.017	Soap solution	0.040
Ethyl alcohol	0.022	Turpentine	0.027
Glycerine	0.059	Water	0.072
Kerosene	0.024		

12. Coefficient of Linear Expansion for Some Solids, α, K^{-1}

Aluminium, duralumin	2.3×10^{-5}	Gold	1.4×10^{-5}
Brass	1.9×10^{-5}	Invar*	6×10^{-7}
Bronze	1.8×10^{-5}	Iron, steel	1.2×10^{-5}
Cast iron	1.0×10^{-5}	Lead	2.9×10^{-5}
Concrete, cement	$(1\text{-}1.4) \times 10^{-5}$	Nickel	1.28×10^{-5}
Copper	1.7×10^{-5}	Platinum	9×10^{-6}
Ebonite	7.0×10^{-5}	Platinite	9×10^{-6}
Glass		Tin	2.1×10^{-5}
quartz	6×10^{-7}	Tungsten	4×10^{-6}
window	9×10^{-6}	Zinc	2.9×10^{-5}

* Invar contains 64% iron and 36% nickel.

13. Coefficient of Volume Expansion for Some Liquids, β, K^{-1}

Acetone	1.2×10^{-3}	Sulphuric acid	5.7×10^{-4}
Ethyl alcohol	1.1×10^{-3}	Transformer oil	6.0×10^{-4}
Ethyl ether	1.6×10^{-3}	Water at	
Glycerine	5.0×10^{-4}	5-10°C	5.3×10^{-5}
Kerosene	1.0×10^{-3}	10-20°C	1.5×10^{-4}
Mercury	1.8×10^{-4}	20-40°C	3.02×10^{-4}
Petrol	1.0×10^{-3}	40-60°C	4.58×10^{-4}
Petroleum	1.0×10^{-3}	60-80°C	5.87×10^{-4}
		80-100°C	7.02×10^{-4}

14. Permittivity ε for Some Substances

Air		Paraffin	2.2
at 0.1 MPa	1.0006	Petrol	2.3
at 10 MPa	1.055	Porcelain	4-7
Amber	2.8	Rubber	2-3
Aniline	84	Rutile	130
Ebonite	2.7	Shellac	3.6
Epoxy resin	3.7	Sulphur	3.6-4.3
Glass	5-10	Transformer oil	2.2-2.5
Glycerine	39	Vacuum	1
Hydrogen	1.0003	Water	81
Ice at —18°C	3.2	Water at 0°C	88
Kerosene	2.0	Wax	5.8
Marble	8-9	Waxed paper	2.0
Mica	6-9		

15. Resistivity of Some Materials, ρ, $\Omega \cdot m$

Aluminium	2.7×10^{-8}	Nickel	7.3×10^{-8}
Brass	6.3×10^{-8}	Nickeline	4.2×10^{-7}
Coal	$(4.0\text{-}5.0) \times 10^{-5}$	Nichrome	1.05×10^{-6}
Constantan	4.7×10^{-7}	Osmium	9.5×10^{-8}
Copper	1.68×10^{-8}	Platinum	1.05×10^{-7}
Ferro-aluminium high-resistance alloy	1.1×10^{-6}	Rheotan	4.5×10^{-7}
		Silver	1.58×10^{-8}
		Steel	1.2×10^{-7}
Gold	2.2×10^{-8}	Tin	1.13×10^{-7}
Iron	9.9×10^{-8}	Tungsten	5.3×10^{-8}
Lead	2.07×10^{-7}	Zinc	5.95×10^{-8}
Manganin	3.9×10^{-7}		
Mercury	9.54×10^{-7}		

16. Temperature Resistance Coefficient for Some Substances, α, K^{-1}

Cast iron	0.002	Nickeline	0.0001
Constantan	0.000005	Rheotan	0.0004
Manganin	0.000008	Steel	0.006
Nichrome and ferro-aluminium high-resistance alloy	0.0002	Tungsten	0.0050

17. Electrochemical Equivalent for Some Substances, k, kg/C

Aluminium	9.32×10^{-8}	Lead	1.074×10^{-6}
Calcium	2.077×10^{-7}	Magnesium	1.26×10^{-7}
Chlorine	3.67×10^{-7}	Mercury	2.072×10^{-6}
Chromium (bivalent)	2.79×10^{-7}	Nickel	
Copper		bivalent	3.04×10^{-7}
bivalent	3.29×10^{-7}	trivalent	2.03×10^{-7}
monovalent	6.6×10^{-7}	Oxygen	8.29×10^{-8}
Gold	6.81×10^{-7}	Potassium	4.052×10^{-7}
Hydrogen	1.045×10^{-8}	Silver	1.118×10^{-6}
Iron		Sodium	2.383×10^{-7}
bivalent	2.89×10^{-7}	Zinc	3.388×10^{-7}
trivalent	1.93×10^{-7}		

18. Refractive Index for Some Materials, n

Acetone	1.36	Glycerine	1.47
Air	1.0003	Ice	1.31
Aniline	1.59	Methyl alcohol	1.33
Benzene	1.50	Quartz	1.54
Carbon bisulphide	1.63	Rock salt	1.54
Carbon tetrachloride	1.46	Sugar	1.56
Diamond	2.42	Sylvite	1.49
Ethyl alcohol	1.36	Turpentine	1.51
Glass		Water	1.33
crown	1.5		
flint	1.6-1.8		

19. Mass of Some Isotopes, amu*

Element	Isotope	Mass	Element	Isotope	Mass
Hydrogen	$^{1}_{1}H$	1.00783	Carbon	$^{13}_{6}C$	13.00335
	$^{2}_{1}H$	2.01410	Oxygen	$^{16}_{8}O$	15.99491
	$^{3}_{1}H$	3.01605	Fluorine	$^{19}_{9}F$	18.99843
Helium	$^{3}_{2}He$	3.01603	Aluminium	$^{27}_{13}Al$	26.98153
	$^{4}_{2}He$	4.00260	Phosphorus	$^{30}_{15}P$	29.97867
Lithium	$^{6}_{3}Li$	6.01513	Radon	$^{222}_{86}Rn$	222.01922
	$^{7}_{3}Li$	7.01601	Radium	$^{226}_{88}Ra$	226.02435
Beryllium	$^{8}_{4}Be$	8.00531	Uranium	$^{235}_{92}U$	235.04299
	$^{9}_{4}Be$	9.01219		$^{238}_{92}U$	238.05006
Boron	$^{11}_{5}B$	11.00930	Neptunium	$^{237}_{93}Np$	237.04706
Carbon	$^{12}_{6}C$	12.00000	Plutonium	$^{239}_{94}Pu$	239.05122

* 1 amu is equal to 1/12 of the mass of an atom of $^{12}_{6}C$ isotope.

20. Psychrometric Table

Dry-bulb temperature		Difference between dry-bulb and wet-bulb temperatures											
K	°C	0	1	2	3	4	5	6	7	8	9	10	11
273	0	100	82	63	45	28	11						
274	1	100	83	65	48	32	16						
275	2	100	84	68	51	35	20						
276	3	100	84	69	54	39	24	10					
277	4	100	85	70	56	42	28	14					
278	5	100	86	72	58	45	32	19	6				
279	6	100	86	73	60	47	35	23	10				
280	7	100	87	74	61	49	37	26	14				
281	8	100	87	75	63	51	40	28	18	7			
282	9	100	88	76	64	53	42	31	21	11			
283	10	100	88	76	65	54	44	34	24	14	4		
284	11	100	88	77	66	56	46	36	26	17	8		
285	12	100	89	78	68	57	48	38	29	20	11		
286	13	100	89	79	69	59	49	40	31	23	14	6	
287	14	100	90	79	70	60	51	42	33	25	17	9	
288	15	100	90	80	71	61	52	44	36	27	20	12	5
289	16	100	90	81	71	62	54	45	37	30	22	15	8
290	17	100	90	81	72	64	55	47	39	32	24	17	10
291	18	100	91	82	73	64	56	48	41	34	26	20	13
292	19	100	91	82	74	65	58	50	43	35	29	22	15
293	20	100	91	83	74	66	59	51	44	37	30	24	18
294	21	100	91	83	75	67	60	52	46	39	32	26	20
295	22	100	92	83	76	68	61	54	47	40	34	28	22
296	23	100	92	84	76	69	61	55	48	42	36	30	24
297	24	100	92	84	77	69	62	56	49	43	37	31	26
298	25	100	92	84	77	70	63	57	50	44	38	33	27
299	26	100	92	85	78	71	64	58	51	45	40	34	29
300	27	100	92	85	78	71	65	59	52	47	41	36	30
301	28	100	93	85	78	72	65	59	53	48	42	37	32
302	29	100	93	86	79	72	66	60	54	49	43	38	33
303	30	100	93	86	79	73	67	61	55	50	44	39	34

21. Permeability of Some Materials, μ

Paramagnetics ($\mu > 1$)		*Diamagnetics* ($\mu < 1$)	
Air	1.00000038	Bismuth	0.999824
Aluminium	1.000023	Copper	0.999990
Oxygen	1.0000019	Glass	0.999987
liquid	1.003400	Hydrogen	0.999999937
Tungsten	1.000176	Water	0.999991

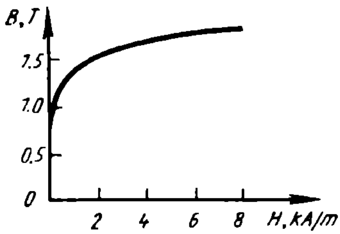
Ferromagnetics

(maximum permeability)

Cast iron	2000
Iron	
carbon	3000
soft	8000
transformer	15000
Permalloy (Ni-Fe alloy for transformer cores)	80000

Remark. The maximum permeabilities of ferromagnetics are given since the parameter depends on the external magnetic field strength.

22. Magnetic Induction as a Function of Magnetic Field Strength for Soft Steel during Primary Magnetization



23. Some Astronomical Quantities (mean values)

Radius of the Earth	6.37×10^6 m
Mass of the Earth	5.98×10^{24} kg
Density of the Earth	5.52×10^3 kg/m^3
Velocity of orbital motion of the Earth	10^5 km/h
Radius of the Sun	6.95×10^8 m
Mass of the Sun	1.98×10^{30} kg
Radius of the Moon	1.74×10^6 m
Mass of the Moon	7.33×10^{22} kg
Distance between the centres of the Earth and the Sun	1.49×10^{11} m
Distance between the centres of the Earth and the Moon	3.84×10^8 m
Period of revolution of the Moon around the Earth	27.3 days
Inclination of the ecliptic to the equator	23.5°

24. Parameters of Some Bright Stars

Star	Magnitude m	Right ascension α	Declination δ
α Tauri (Aldebaran)	1.06	4 h 31 min 54 s	+45°57′
β Orionis (Rigel)	0.34	5 h 12.1 min	−8°15′
α Aurigae (Capella)	0.21	5 h 13 min	+45°57′
α Orionis (Betelgeuse)	0.92	5 h 52.5 min	+7°24′
α Canis Majoris (Sirius)	−1.58	6 h 42.9 min	−16°39′
α Geminorum (Castor)	1.99	7 h 31.4 min	+32°
α Lyrae (Vega)	0.14	18 h 35 min	+38°41′
α Cygni (Deneb)	1.38	20 h 39 min	+45°06′

25. Binding Energy of Atomic Nuclei

Nucleus	E_b, MeV	E_b/A, MeV
${}^{2}_{1}H$	2.2	1.1
${}^{3}_{1}H$	8.5	2.83
${}^{3}_{2}He$	7.7	2.57
${}^{4}_{2}He$	28.3	7.075
${}^{6}_{3}Li$	32.0	5.33
${}^{7}_{3}Li$	39.2	5.60
${}^{9}_{4}Be$	58.2	6.47
${}^{10}_{5}B$	64.7	6.47
${}^{11}_{5}B$	76.2	6.93
${}^{12}_{6}C$	92.2	7.68
${}^{13}_{6}C$	97.1	7.47
${}^{14}_{7}N$	104.7	7.47
${}^{16}_{8}O$	127.6	7.975
${}^{17}_{8}O$	131.8	7.75
${}^{27}_{13}Al$	225.0	8.33
${}^{13}_{14}Si$	255.2	8.51
${}^{30}_{15}P$	250.6	8.35
${}^{222}_{86}Rn$	1708.2	7.69
${}^{226}_{88}Ra$	1731.6	7.66
${}^{235}_{92}U$	1783.8	7.59
${}^{238}_{92}U$	1801.7	7.57
${}^{239}_{94}Pu$	1806.9	7.56

26. Prefixes Used with SI Units

Factor	Prefix	Notation
10^{18}	hexa	H
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

27. Sines and Tangents of Angles from 0 to 90°

Angle		Sine	Tangent
degrees	radians		
0	0	0.0000	0.0000
1	0.0175	0.0175	0.0175
2	0.0349	0.0349	0.0349
3	0.0524	0.0523	0.0524
4	0.0698	0.0698	0.0699
5	0.0873	0.0872	0.0875
6	0.1047	0.1045	0.1051
7	0.1222	0.1219	0.1228
8	0.1396	0.1392	0.1405
9	0.1571	0.1564	0.1584
10	0.1745	0.1736	0.1763
11	0.1920	0.1908	0 1944
12	0.2094	0.2079	0.2126
13	0.2269	0.2250	0.2309
14	0.2443	0.2419	0.2493
15	0.2618	0.2588	0.2679
16	0.2793	0.2756	0.2867
17	0.2967	0.2924	0.3057
18	0.3142	0.3090	0.3249
19	0.3316	0.3256	0.3443
20	0.3491	0.3420	0.3640

27. Sines and Tangents of Angles from 0 to 90° *(cont.)*

Angle		Sine	Tangent
degrees	radians		
21	0.3665	0.3584	0.3839
22	0.3840	0.3746	0.4040
23	0.4014	0.3907	0.4245
24	0.4189	0.4067	0.4452
25	0.4363	0.4226	0.4663
26	0.4538	0.4384	0.4877
27	0.4712	0.4540	0.5095
28	0.4887	0.4695	0.5317
29	0.5061	0.4848	0.5543
30	0.5236	0.5000	0.5774
31	0.5411	0.5150	0.6009
32	0.5585	0.5299	0.6249
33	0.5760	0.5446	0.6494
34	0.5934	0.5592	0.6745
35	0.6109	0.5736	0.7002
36	0.6283	0.5878	0.7265
37	0.6458	0.6018	0.67536
38	0.6632	0.6157	0.7813
39	0.6807	0.6293	0.8098
40	0.6981	0.6428	0.8391
41	0.7156	0.6561	0.8693
42	0.7330	0.6691	0.9004
43	0.7505	0.6820	0.9325
44	0.7679	0.6947	0.9657
45	0.7854	0.7071	1.0000
46	0.8029	0.7193	1.036
47	0.8203	0.7314	1.072
48	0.8378	0.7431	1.111
49	0.8552	0.7574	1.150
50	0.8727	0.7660	1.192
51	0.8901	0.7771	1.235
52	0.9076	0.7880	1.280
53	0.9250	0.7986	1.327
54	0.9425	0.8090	1.376
55	0.9599	0.8192	1.428
56	0.9774	0.9774	1.483
57	0.9948	0.8387	1.540
58	1.0123	0.8480	1.600
59	1.0297	0.8572	1.664
60	1.0472	0.8660	1.732
61	1.0647	0.8746	1.804
62	1.0821	0.8829	1.881
63	1.0996	0.8910	1.963
64	1.1170	0.8988	2.050
65	1.1345	0.9063	2.145
66	1.1519	0.9135	2.246

27. Sines and Tangents of Angles from 0 to 90° (*cont.*)

Angle		Sine	Tangent
degrees	radians		
67	1.1694	0.9205	2.356
68	1.1868	0.9272	2.475
69	1.2043	0.9336	2.605
70	1.2217	0.9397	2.747
71	1.2392	0.9455	2.904
72	1.2566	0.9511	3.078
73	1.2741	0.9563	3.271
74	1.2915	0.9631	3.487
75	1.3090	0.9659	3.732
76	1.3265	0.9703	4.011
77	1.3439	0.9744	4.331
78	1.3614	0.9781	4.705
79	1.3788	0.9816	5.145
80	1.3963	0.9848	5.671
81	1.4137	0.9877	6.314
82	1.4312	0.9903	7.115
83	1.4486	0.9925	8.144
84	1.4661	0.9945	9.514
85	1.4835	0.9962	11.43
86	1.5010	0.9976	14.30
87	1.5184	0.9986	19.08
88	1.5359	0.9994	28.64
89	1.5533	0.9998	57.29
90	1.5708	1.000	∞

28. Basic Formulas in Physics

Formula	Quantities appearing in formulas	Units
	Kinematics	
$\mathbf{r} = \mathbf{r}(t)$	$\mathbf{r}$ is the displacement of a particle	m
	t is the time interval	s
$x = x(t)$, $y = y(t)$	x and y are the coordinates of a particle	m
$s = s(t)$	s is the distance covered by the particle	m
	Uniform motion	
$\mathbf{r} - \mathbf{r}_0 = \mathbf{v}t$	$\mathbf{r} - \mathbf{r}_0$ is the increment of displacement	m
$x - x_0 = v_x t$	x_0 and y_0 are the initial coordinates of a particle	m
$y - y_0 = v_y t$		

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$\lvert \mathbf{r} \rvert = x = s = vt$	s is the path length (the coordinate axis coincides with the direction of displacement)	m
	Uniformly variable motion	
$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$	$\bar{\mathbf{v}}$ is the mean velocity of variable motion	m/s
$\mathbf{v} = \lim\limits_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$	$\mathbf{v}$ is the instantaneous velocity	m/s
	In vector form	
$\mathbf{a} = \text{const}$	$\mathbf{a}$ is the acceleration	m/s^2
$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t}$	$\mathbf{v}_0$ is the initial velocity	m/s
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$	$\mathbf{v}$ is the instantaneous velocity	m/s
	In scalar form	
$v_x = v_{0x} + a_x t$	v_x and v_y are the projections of velocity on the coordinate axes	m/s
$v_y = v_{0y} + a_y t$		
$x - x_0 = v_{0x} + \frac{a_x t^2}{2}$	a_x and a_y are the projections of acceleration on the coordinate axes	m/s^2
$y - y_0 = v_{0y} + \frac{a_y t^2}{2}$		
$v^2 - v_0^2 = 2as$		
	Free fall	
$v = gt$	g is the free fall acceleration	m/s^2
$H = \frac{gt^2}{2}$	H is the height from which the body falls	m
$v^2 = 2gH$	v is the velocity at the end of the free fall	m/s
	Motion of a body thrown upwards	
$v = v_0 - gt$		
$h = v_0 t - \frac{gt^2}{2}$		
$H = \frac{v_0^2}{2g}$	H is the maximum height of ascent	m

28. Basic Formulas in Physics *(cont.)*

Formula	Quantities appearing in formulas	Units
	Curvilinear motion	
$v = 2\pi Rf$	v is the linear velocity	m/s
	R is the radius of rotation	m
	f is the rotational frequency	Hz, s^{-1}
$\omega = 2\pi f$	ω is the angular velocity	rad/s
$\omega = v/R$		
$a_c = v^2/R$	a_c is the centripetal acceleration	m/s^2
$a_c = \omega^2 R$		
	Dynamics	
$\rho = m/V$	ρ is the density	kg/m^3
	m is the mass	kg
	V is the volume	m^3
Newton's second law		
$a = F/m$	F is the (resultant) force	N
$Ft = \Delta mv$	Ft is the impulse of force	N·s
	mv is the momentum	kg·m/s
Conservation of momentum		
$mv = \text{const}$		
Newton's third law		
$m_1a_1 = -m_2a_2$		
$P = mg$	P is the force of gravity	N
Law of universal gravitation		
$F = G\dfrac{m_1m_2}{r^2}$	F is the gravitational force	N
	G is the gravitational constant	$N \cdot m^2/kg^2$
	r is the distance between the centres of mass	m
$F_c = mv^2/R$	F_c is the centripetal force	N
$F_c = m\omega^2 R$		
$F_{el} = kx$	F_{el} is the elastic force	N
	k is the spring constant	N/m
$F_f = \mu N$	F_f is the force of friction	N
	μ is the friction coefficient	
	N is the force of normal pressure	N
	Work, power, energy	
$A = FS\cos\alpha$	A is the mechanical work	J
$A = FS$ for $\alpha = 0$	α is the angle between the direction of force and displacement	deg
$P = A/t$	P is the power	W
$\bar{P} = F\bar{v}$	$\bar{P}$ is the mean power	W
	$\bar{v}$ is the mean velocity	m/s

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$E_k = mv^2/2$	E_k is the kinetic energy	J
$E_p = mgh$	E_p is the potential energy	J
$\eta = \frac{A_{useful}}{A_{spent}} \times 100\%$	η is the efficiency	
	A_{useful} is the useful work	J
	A_{spent} is the spent work	J
	Molecular Physics. Heat	
$n_0 = \frac{N_A}{M}\rho$	N_A is the Avogadro constant	mol^{-1}
	M is the molar mass	kg/mol
$n_0 = N_A/V_m$	n_0 is number density of molecules	m^{-3}
	V_m is the molar volume	m^3/mol
$m_0 = M/N_A$	m_0 is the mass of a molecule	kg
$V_0 = \frac{M}{N_A\rho}$	V_0 is the volume of a molecule	m^3
$r_0 = \sqrt[3]{\frac{3M}{4\pi\rho N_A}}$	r_0 is the radius of a molecule	m
$\bar{\lambda} = \bar{v}/\bar{z}$	$\bar{\lambda}$ is the mean free path of a molecule	m
	$\bar{v}$ is the (arithmetic) mean velocity of molecules	m/s
	$\bar{z}$ is the number of collisions per second	s^{-1}
$\bar{\lambda} = \frac{1}{\sqrt{2}\pi d_{eff}^2 n_0}$	d_{eff} is the effective diameter of a molecule	m
$p = \frac{2}{3} n_0 \frac{m\bar{v}_{rms}^2}{2}$	p is the pressure	Pa
	v_{rms} is the root-mean-square velocity	m/s
$p = n_0 kT$	k is the Boltzmann constant	J/K
	T is the thermodynamic temperature	K
$E_k = \frac{3}{2} kT$	E_k is the kinetic energy of translational motion of molecules	J
Equations of state of an ideal gas		
	R is the molar gas constant	J/(mol·K)
$pV = \frac{m}{M} RT$	$m/M = \nu$ is the amount of substance	mol
$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = \text{const}$	p_1, V_1, and T_1 are the parameters of the first state of a gas	
	p_2, V_2, and T_2 are the parameters of the second state of a gas	

28. Basic Formulas in Physics *(cont.)*

Formula	Quantities appearing in formulas	Units
Dalton's law		
	p is the pressure of a gas mixture	Pa
$p = p'_1 + p'_2 + \ldots$	p'_1 and p'_2 are partial pressures	Pa
$Q = cm(T_2 - T_1)$	Q is the heat required to heat a body or given off as it cools	J
	c is the specific heat	J/(kg·K)
$Q = qm$	Q is the heat given off as fuel burns	J
	q is the specific heat of combustion of fuel	J/kg
	m is the mass of burnt fuel	kg
$\eta = \frac{Q_{useful}}{Q_{spent}} 100\%$	η is the efficiency of the heater	%
	Q_{useful} is the useful heat	J
	Q_{spent} is the spent heat	J
$Q = rm$	Q is the heat required for vaporization or released during condensation	J
	r is the specific latent heat of vaporization	J/kg
	m is the mass of vapour	kg
$B = \frac{\rho_a}{\rho_{sat}} 100\%$	B is the relative humidity of air	%
	ρ_a is the absolute humidity of air	kg/m³
	ρ_{sat} is the saturated vapour density	kg/m³
$B = \frac{p_a}{p_{sat}} 100\%$	p_a is the pressure of vapour contained in air	Pa
	p_{sat} is the saturated vapour pressure	Pa
$\sigma = \frac{F}{l}$	σ is the surface tension	N/m
	F is the force of surface tension	N
	l is the length of the boundary of free surface	m
$\sigma = \frac{A}{\Delta S}$	A is the work of molecular forces	J
	ΔS is the decrease in the area of free surface	m²
$h = \frac{2\sigma}{\rho g r}$	h is the height to which a wetting liquid rises (nonwetting liquid falls) in a capillary	m
$p_L = 2\sigma/r$	r is the radius of a capillary	m
	p_L is the Laplacian pressure for a spherical surface	Pa

28. **Basic Formulas in Physics** (*cont.*)

Formula	Quantities appearing in formulas	Units
$Q = \lambda m$	Q is the heat required for fusion or liberated during crystallization	J
	λ is the specific latent heat of fusion	J/kg
	Deformations	
$\sigma = \frac{F}{S}$	σ is the mechanical stress	Pa
	F is the force of elastic deformation	N
	S is the cross-sectional area of a deformed body	m^2
Hooke's law $\sigma = E\,\frac{\Delta l}{l}$	E is the Young's modulus	Pa
	Δl is the absolute deformation (elongation)	m
	l is the initial length	m
$\varepsilon = \Delta l/l$	ε is the strain	
$E_p = F\Delta l/2$	E_p is the potential energy of elastic deformation	J
	Thermal expansion of bodies	
$\alpha = \frac{l - l_0}{l_0 \Delta T} = \frac{\Delta l}{l_0 \Delta T}$	α is the coefficient of linear expansion	K^{-1}
	l_0 is the length of a body at 273 K	m
$l = l_0 (1 + \alpha \Delta T)$	l is the length of the body at any temperature	m
	ΔT is the change in temperature	K
$S = S_0 (1 + 2\alpha \Delta T)$	S_0 is the surface area of the body at 273 K	m^2
	S is the surface area of the body at any temperature	m^2
$\beta = \frac{V - V_0}{V_0 \Delta T} = \frac{\Delta V}{V_0 \Delta T}$	β is the coefficient of volume expansion	K^{-1}
$V = V_0 (1 + \beta \Delta T)$	V is the volume of the body at any temperature	m^3
	V_0 is the volume of the body at 273 K	m^3
$\rho = \frac{\rho_0}{1 + \beta \Delta T}$	ρ is the density of a substance at any temperature	kg/m^3
	ρ_0 is the density of the substance at 273 K	kg/m^3

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
	Electrostatics	
Charge conservation law	$Q_1, Q_2, \ldots, Q_n$ are electric charges	C
$Q_1 + Q_2 + \ldots + Q_n = \text{const}$	n is the number of electric charges in a closed system	
Coulomb's law		
$F = \frac{1}{4\pi\varepsilon_0} \frac{\lvert Q_1 \rvert \lvert Q_2 \rvert}{\varepsilon r^2}$	F is the force of interaction between point electric charges	N
	Q_1, Q_2 are electric charges	C
	r is the separation between the charges	m
	ε_0 is the electric constant	C/(N·m²)
	ε is the permittivity of the medium	
$E = F/Q_t$	E is the electric field strength	N/C
$E = \frac{Q}{4\pi\varepsilon_0 \varepsilon r^2}$	Q_t is a test charge in the field	C
	Q is the charge producing the field	C
$\varphi = W/Q_t$	φ is the electric field potential	V
$\varphi = \frac{Q}{4\pi\varepsilon_0 \varepsilon r}$	W is the potential energy of the test charge	J
$U = \varphi_1 - \varphi_2$	U is the voltage	V
$A = QU$	A is the work done by the electric field to move a charge from one point in the field to another	J
$E = \frac{U}{d}$	E is the strength of a uniform field	V/m, N/C
	U is the potential difference between two points in the field	V
	d is the distance between the two points along the field line	m
$C = Q/\varphi$	C is the capacitance	F
$C = Q/U$		
$C = \frac{\varepsilon_0 \varepsilon S}{d}$	C is the capacitance of a (parallel-plate) capacitor	F
	S is the area of a plate	m²
	d is the thickness of a dielectric	m
$W = CU^2/2$	W is the energy of the capacitor	J

28. **Basic Formulas in Physics** (*cont.*)

Formula	Quantities appearing in formulas	Units
	Electric current in metals	
$I = Q/t$	I is the electric current	A
$I = en\bar{v}S$	e is the charge of an electron (ion)	C
$I = enuES$	n is the number density of charge carriers	m^{-3}
	$\bar{v}$ is the mean velocity of the directed motion of charge carriers	m/s
$u = \bar{v}/E$	u is the mobility of the charge carriers	$m^2/(V \cdot s)$
$j = \frac{I}{S}$	j is the current density	A/m^2
	S is the cross-sectional area	m^2
$R = \rho \frac{l}{S}$	R is the resistance of the conductor	Ω
	ρ is the resistivity of the conductor material	Ω·m
	l is the conductor length	m
	S is the cross-sectional area of the conductor	m^2
$R_t = R_0 (1 + \alpha \Delta T)$	R_t is the resistance of a conductor at any temperature	Ω
	R_0 is the resistance of the conductor at 273 K	Ω
	α is the temperature resistance coefficient	K^{-1}
Ohm's law for a conductor		
$I = \frac{U}{R}$	U is the voltage across the conductor	V
	R is the resistance of the conductor	Ω
Series connection of conductors		
$R_{eq} = R_1 + R_2 + \ldots + R_n$	R_{eq} is the equivalent resistance of conductors	Ω
$R_{eq} = R_1 n$ for $R_1 = R_2 = \ldots = R_n$	n is the number of the conductors	
	$R_1, R_2, \ldots, R_n$ are the resistances of the conductors	Ω
Parallel connection of conductors		
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$	R_{eq} is the equivalent resistance	Ω

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$R_{eq} = \frac{R_1}{n}$	n is the number of resistors	
for $R_1 = R_2 = \ldots = R_n$	$R_1, R_2, R_3, \ldots, R_n$ are the resistances	Ω
Ohm's law for a closed circuit		
$I = \frac{\mathscr{E}}{R + r}$	$\mathscr{E}$ is the electromotive force of a current source	V
	R is the resistance of the external circuit	Ω
	r is the resistance of the internal circuit	Ω
Ohm's law for a subcircuit containing an emf		
$I = \frac{U - \mathscr{E}}{R}$	$\mathscr{E}$ is the counter emf	V
$A = IUt$	A is the work done by electric current	J
$A = I^2Rt$		
$A = \frac{U^2}{R} t$	t is the time	s
$P = IU$	P is the power of an electric current	W
$P = I^2R$		
$P = U^2/R$		
$P_0 = I\mathscr{E}$	P_0 is the total power developed by a current source	W
$P_l = P_0 - I^2r$	P_l is the power of the load in a closed circuit	W
	Electric current in electrolytes	
Faraday's law		
$m = kQ$ or $m = kIt$	m is the mass of a substance deposited on electrodes	kg
Generalized law		
$m = \frac{1}{F} \frac{M}{n} Q$	k is the electrochemical equivalent	kg/C
	M is the molar mass	kg/mol
	F is the Faraday constant	C/mol
	n is the valency	
	Electromagnetism	
$F = \frac{\mu_0 \mu I_1 I_2 l}{2\pi a}$	F is the force of interaction between parallel current-carrying conductors	N
	I_1 and I_2 are the currents in the conductors	A
	μ is the permeability	
	μ_0 is the magnetic constant	H/m
	a is the separation between the conductors	m

28. Basic Formulas in Physics *(cont.)*

Formula	Quantities appearing in formulas	Units
$F_A = BIl \sin\alpha$	F_A is Ampère's force (acting on a current-carrying conductor)	N
$F_A = BIl$ for $\sin\alpha = 1$	B is the magnetic induction	T
	l is the active length of a conductor	m
	α is the angle between the direction of the current and the magnetic induction vector	deg
$A = IB\Delta S$ $A = I\Delta\Phi$	A is the work done in moving a current-carrying conductor in a magnetic field	J
	ΔS is the change in the area embraced by the current loop	m^2
	$\Delta\Phi$ is the change in the magnetic flux	Wb
$B_{str} = \mu_0\mu \frac{I}{2\pi r}$	B_{str} is the magnetic induction of a straight current	T
	r is the distance from the conductor to the point where the induction is being determined	m
$B_{cir} = \mu_0\mu \frac{1}{2r}$	B_{cir} is the magnetic induction at the centre of a circular current	T
	r is the radius of the circular current	m
$B_{sol} = \mu_0\mu \frac{I\omega}{l}$	B_{sol} is the magnetic induction of the field in a solenoid	T
	ω is the number of turns	
	l is the length of the solenoid	m
$\Phi_{sol} = B_{sol}S$	Φ_{sol} is the magnetic flux in the solenoid	Wb
	S is the cross-sectional area of the solenoid	m^2
$B = \mu_0\mu H$	H is the magnetic field strength	A/m
$F_L = Bve \sin\alpha$	F_L is the Lorentz force (acting on a charged particle in a magnetic field)	N
	e is the charge of a particle	C
	v is the velocity of the particle	m/s
	α is the angle between vectors **B** and **v**	deg
	Electromagnetic induction	
$\mathscr{E} = -\frac{\Delta\Phi}{\Delta t}$	$\mathscr{E}$ is the induced emf	V
	$\Delta\Phi$ is the change in the magnetic flux	Wb
	Δt is the time interval over which the magnetic flux changes	s

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$\mathscr{E}_{sol} = -\omega \frac{\Delta\Phi}{\Delta t}$	$\mathscr{E}_{sol}$ is the emf induced in a solenoid	V
$\mathscr{E}_{str} = Blv \sin\alpha$	$\mathscr{E}_{str}$ is the emf induced in a straight conductor during its motion	V
	l is the active length of the conductor	m
$\mathscr{E}_s = -L\frac{\Delta I}{\Delta t}$	$\mathscr{E}_s$ is the self-inductance emf	V
	L is the inductance of the circuit	H
$W = \frac{LI^2}{2}$	W is the energy of the magnetic field of the circuit	J
	Oscillations and Waves	
$T = \frac{t}{n}$	T is the period of oscillations	s
	n is the number of complete oscillations	
$\nu = 1/T$	ν is the oscillation frequency	Hz
$\omega = 2\pi/T = 2\pi\nu$	ω is the circular frequency	rad/s
$\varphi = \omega t$	φ is the phase of oscillations	rad
$\varphi = \omega t + \varphi_0$	φ_0 is the initial phase	rad
Equation of harmonic	motion	
$x = A\sin(\omega t + \varphi_0)$	x is the displacement	m
	A is the amplitude of vibrations	m
$T = 2\pi\sqrt{\frac{l}{g}}$	T is the period of oscillations of a simple pendulum	s
	l is the length of the pendulum	m
	g is the free fall acceleration	m/s^2
$T = 2\pi\sqrt{m/k}$	T is the period of elastic vibrations	s
$\omega = \sqrt{\frac{k}{m}}$	ω is the circular frequency	rad/s
	k is the spring constant	N/m
	m is the mass of the load	kg
$\lambda = vT$	λ is the wavelength	m
	v is the velocity of propagation of wave	m/s
	Alternating current	
$e = \mathscr{E}_0 \sin(\omega t + \varphi_0)$	e is the instantaneous emf in the circuit	V
	$\mathscr{E}_0$ is the amplitude of the emf	V
$u = U_0 \sin(\omega t + \varphi_0)$	u is the instantaneous voltage	V
	U_0 is the amplitude of the voltage	V

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$i = I_0 \sin(\omega t + \varphi_0)$	i is the instantaneous current	A
	I_0 is the amplitude of the current	A
$I = I_0/\sqrt{2}$	I is the effective current	A
$U = U_0/\sqrt{2}$	U is the effective voltage	V
$\mathscr{E} = \mathscr{E}_0/\sqrt{2}$	$\mathscr{E}$ is the effective emf	V
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$	X_C is the capacitive reactance of the circuit	Ω
	ν is the alternating current frequency	Hz
$X_L = \omega L = 2\pi\nu L$	X_L is the inductive reactance of the circuit	Ω
$Z = \sqrt{R^2+(X_L-X_C)^2}$	Z is the impedance of the circuit with series-connected components	Ω
	R is the resistance of the circuit	Ω
$\cos\varphi = R/Z$	$\cos\varphi$ is the power factor	
$P = UI\cos\varphi$	P is the active power	W
	I and U are the effective current and voltage	A, V
$Q = UI\sin\varphi$	Q is the reactive power	var
$S = \sqrt{P^2 + P_q^2}$	S is the total power	VA
$\frac{U_2}{U_1} = \frac{N_2}{N_1} = k$	k is the transformation ratio	
	N_1 and N_2 are the numbers of turns in the primary and secondary	
	U_1 and U_2 are voltages across the primary and secondary of the transformer	V
	Electromagnetic oscillations and waves	
$T = 2\pi\sqrt{LC}$	T is the period of natural oscillations of an oscillatory circuit	s
$\lambda = cT$	λ is the length of the electromagnetic waves	m
	c is the velocity of light in vacuum	m/s
	Geometrical optics	
$n = c/v$	n is the absolute refractive index	
	v is the velocity of light in a medium	m/s
$n_{2,1} = \frac{\sin\varepsilon}{\sin\varepsilon'}$	$n_{2,1}$ is the relative refractive index	
	ε is the angle of incidence	deg
	ε' is the angle of refraction	deg

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$n_{2,1} = \frac{v_1}{v_2}$	v_1 is the velocity of light in the first medium	m/s
	v_2 is the velocity of light in the second medium	m/s
$\sin i_{cr} = 1/n$	i_{cr} is the critical angle of total internal reflection	deg
$\frac{1}{f} = \frac{1}{a} + \frac{1}{a'}$	f is the focal length of a lens or mirror	m
	a is the distance from the object to the lens (mirror)	m
	a' is the distance from the image to the lens (mirror)	m
$\Phi = 1/f$	Φ is the optical power of a lens	D
$f = \frac{R}{2}$	f is the focal length of a mirror	m
	R is the radius of curvature of the mirror	m
$\beta = \frac{H}{h}$	β is the linear magnification	
	H is the height of the image	m
	h is the height of the object	m
	Photometry	
$\Phi = Q/t$	Φ is the luminous flux	lm
	Q is the luminous energy	J
$\Phi_{tot} = 4\pi I$	Φ_{tot} is the total luminous flux	lm
$I = \Phi/\omega$	I is the luminous intensity	cd
	ω is the solid angle	sr
$E = \Phi/S$	E is the illuminance	lx
Laws of illumination		
$E = I/r^2$	r is the distance from a source of light to the surface being illuminated	m
$E = E_0 \cos\alpha$	E_0 is the illuminance due to the normal rays	lx
or		
$E = \frac{I}{r^2}\cos\alpha$	α is the angle of incidence	deg
	Wave and quantum properties of radiation	
$k\lambda = d\sin\varphi$	d is the grating constant	m
	φ is the angle to the diffraction maximum	deg
	k is the spectral order	

28. Basic Formulas in Physics (*cont.*)

Formula	Quantities appearing in formulas	Units
$\varepsilon = h\nu$	ε is the energy of a photon	J
	h is the Planck constant	J·s
$m = \frac{\varepsilon}{c^2} = \frac{h}{c\lambda}$	m is the mass of a photon	kg
$mc = \frac{h}{\lambda}$	mc is the momentum of a photon	kg·m/s
$p = \frac{I}{c} = N\frac{h\nu}{c}$	p is the pressure of light	Pa
	N is the number of photons	
	I is the radiation intensity	J/(m^2·s)
Einstein's equation for photoelectric effect		
$h\nu = A + \frac{m_e v^2}{2}$	A is the electron work function	J

29. Some Mathematical Formulas

Algebra

1. $$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2,\\ (a-b)^2 &= a^2 - 2ab + b^2,\\ (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3,\\ (a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3,\\ a^2 - b^2 &= (a-b)(a+b),\\ a^3 - b^3 &= (a-b)(a^2+ab+b^2),\\ a^3 + b^3 &= (a+b)(a^2-ab+b^2).\end{aligned}$$

2. The following inequality means that the arithmetic mean is larger than or equal to the geometric mean

$$\frac{a+b}{2} \geqslant \sqrt{ab}, \quad a \geqslant 0, \quad b \geqslant 0.$$

The equality arises when $a = b$.

3. For a given quadratic equation

$$ax^2 + bx + c = 0,$$

its two solutions can be found from the formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

or

$$x_{1,2}=\frac{-\frac{b}{2}\pm\sqrt{\left(\frac{b}{2}\right)^2-ac}}{a}.$$

The latter is more convenient when b is even.

For a reduced quadratic equation

$$x^2+px+q=0,$$

its two solutions are given by

$$x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}.$$

4. Suppose we are given a system of two equations in two unknowns:

$$\begin{cases} a_1x+b_1y=c_1, \\ a_2x+b_2y=c_2. \end{cases}$$

In order to find its solution, one unknown from one of the equations (say, the first) must be expressed in terms of the other unknown (for instance, y in terms of x) and substituted into the other equation:

$$y=\frac{c_1-a_1x}{b_1},$$

$$a_2x+b_2\,\frac{c_1-a_1x}{b_1}=c_2.$$

Solving this equation, we obtain

$$x=\frac{c_1b_2-c_2b_1}{a_1b_2-a_2b_1}.$$

Substituting x into the expression for y, we find that

$$y=\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}.$$

The same method can be used to solve a system with a larger number of equations: we express one unknown from one of the equations of the system and substitute it into the remaining equations. This reduces the number of equations and unknowns by one. Similarly, we eliminate in turn the other unknowns until we are left with one equation in one unknown, which can be solved. The remaining unknowns are determined in reverse order.

5. Some formulas for approximate calculations.

If $\varepsilon \ll 1$ (at least by a factor of 10), we have

$$\frac{1}{1+\varepsilon}=1-\varepsilon,$$

$$\frac{1}{1-\varepsilon}=1+\varepsilon,$$

$$(1+\varepsilon)^2=1+2\varepsilon,$$

$$(1-\varepsilon)^2=1-2\varepsilon,$$

$$(1+\varepsilon)^3=1+3\varepsilon,$$

$$(1-\varepsilon)^3=1-3\varepsilon,$$

$$\sqrt{1+\varepsilon}=1+\frac{\varepsilon}{2},$$

$$\sqrt{1-\varepsilon}=1-\frac{\varepsilon}{2}.$$

These formulas yield quite accurate results. For example, let us calculate $\sqrt{1/3.96}$:

$$\sqrt{\frac{1}{3.96}}=\frac{1}{\sqrt{4-0.04}}=\frac{1}{\sqrt{4\left(1-\frac{0.04}{4}\right)}}=\frac{1}{2\sqrt{1-0.01}}$$

$$=\frac{1}{2\left(1-\frac{0.01}{2}\right)}=\frac{1}{2}\,\frac{1}{1-0.005}$$

$$=\frac{1}{2}(1+0.005)=0.5025.$$

Geometry

1. Given a right triangle with sides a and b and hypothenuse c. Then

$$a^2+b^2=c^2.$$

2. The area of a triangle is

$$S=\frac{1}{2}ah,$$

where h is the height dropped on side a from the opposite vertex.

The area of a rectangle with sides a and b is

$$S=ab.$$

The area of a trapezium with bases a and b and height h is

$$S=\frac{1}{2}(a+b)h.$$

The area of a circle of radius r is

$$S = \pi r^2.$$

3. The area of the surface and the volume of a sphere of radius r are

$$S = 4\pi r^2, \quad V = \frac{4}{3}\pi r^3.$$

The area of the lateral surface of a right cylinder with radius r and length h is

$$S = 2\pi r h.$$

The total surface area and the volume of the right cylinder are

$$S = 2\pi r (r + h), \quad V = \pi r^2 h.$$

Trigonometry

1. The trigonometric functions of the sums and differences of angles are

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta,$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

2. The functions of double the angle are

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha.$$

3. The functions of half the angle are

$$2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha,$$

$$2\cos^2\frac{\alpha}{2} = 1 + \cos\alpha.$$

4. The sum and the difference of the functions are

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2},$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2},$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2},$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$

5. The trigonometric functions in terms of the tangent are

$$\sin\alpha = \frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}, \quad \cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha}}.$$

6. Cosine theorem: for an arbitrary triangle with sides a, b, and c, we have

$$a^2 = b^2 + c^2 - 2bc\cos A,$$

where A is the angle opposing side a.

The sine theorem:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

where A, B, and C are the angles opposing sides a, b, and c respectively.

7. For $\varepsilon \ll 1$, we have

$$\sin\varepsilon = \varepsilon,$$
$$\cos\varepsilon = 1,$$
$$\tan\varepsilon = \varepsilon.$$

8. Some derivatives and integrals:

$$(x^n)' = nx^{n-1},$$
$$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}},$$
$$(\sqrt{x})' = \frac{1}{2\sqrt{x}},$$
$$(\sin x)' = \cos x,$$
$$(\cos x)' = -\sin x,$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1},$$
$$\int \frac{dx}{x} = \ln|x|,$$
$$\int \sin x\,dx = -\cos x,$$
$$\int \cos x\,dx = \sin x.$$

PERIODIC TABLE

Periods	Rows	Groups of				
		I	II	III	IV	V
1	I	(H)				
2	II	Li 3 6.94$_1$ $2s^1$ 1 2 Lithium	Be 4 9.01218 $2s^2$ 2 2 Beryllium	5 B 10.81 3 2 $2p^1$ Boron	6 C 12.011 4 2 $2p^2$ Carbon	7 N 14.0067 5 2 $2p^3$ Nitrogen
3	III	Na 11 22.98977 $3s^1$ 1 8 2 Sodium	Mg 12 24.305 $3s^2$ 2 8 2 Magnesium	13 Al 26.98154 3 8 2 $3p^1$ Aluminium	14 Si 28.085$_5$ 4 8 2 $3p^2$ Silicon	15 P 30.97376 5 8 2 $3p^3$ Phosphorus
4	IV	K 19 39.098$_3$ $4s^1$ 1 8 8 2 Potassium	Ca 20 40.08 $4s^2$ 2 8 8 2 Calcium	Sc 21 44.9559 $3d^1 4s^2$ 2 9 8 2 Scandium	Ti 22 47.90 $3d^2 4s^2$ 2 10 8 2 Titanium	V 23 50.9415 $3d^3 4s^2$ 2 11 8 2 Vanadium
	V	29 Cu 63.54$_6$ 1 18 8 2 $3d^{10} 4s^1$ Copper	30 Zn 65.38 2 18 8 2 $4s^2$ Zinc	31 Ga 69.72 3 18 8 2 $4p^1$ Gallium	32 Ge 72.5$_9$ 4 18 8 2 $4p^2$ Germanium	33 As 74.9216 5 18 8 2 $4p^3$ Arsenic
5	VI	Rb 37 85.467$_8$ $5s^1$ 1 8 18 8 2 Rubidium	Sr 38 87.62 $5s^2$ 2 8 18 8 2 Strontium	Y 39 88.9059 $4d^1 5s^2$ 2 9 18 8 2 Yttrium	Zr 40 91.22 $4d^2 5s^2$ 2 10 18 8 2 Zirconium	Nb 41 92.9064 $4d^4 5s^1$ 1 12 18 8 2 Niobium
	VII	47 Ag 107.868 1 18 18 8 2 $5s^1$ Silver	48 Cd 112.41 2 18 18 8 2 $5s^2$ Cadmium	49 In 114.82 3 18 18 8 2 $5p^1$ Indium	50 Sn 118.6$_9$ 4 18 18 8 2 $5p^2$ Tin	51 Sb 121.7$_5$ 5 18 18 8 2 $5p^3$ Antimony
6	VIII	Cs 55 132.9054 $6s^1$ 1 8 18 18 8 2 Cesium	Ba 56 137.33 $6s^2$ 2 8 18 18 8 2 Barium	La* 57 138.905$_5$ $5d^1 6s^2$ 2 9 18 18 8 2 Lanthanum	Hf 72 178.4$_9$ $5d^2 6s^2$ 2 10 32 18 8 2 Hafnium	Ta 73 180.947$_9$ $5d^3 6s^2$ 2 11 32 18 8 2 Tantalum
	IX	79 Au 196.9665 1 18 32 18 8 2 $5d^{10} 6s^1$ Gold	80 Hg 200.5$_9$ 2 18 32 18 8 2 $6s^2$ Mercury	81 Tl 204.3$_7$ 3 18 32 18 8 2 $6p^1$ Thallium	82 Pb 207.2 4 18 32 18 8 2 $6p^2$ Lead	83 Bi 208.9804 5 18 32 18 8 2 $6p^3$ Bismuth
7	X	Fr 87 [223] $7s^1$ 1 8 18 32 18 8 2 Francium	Ra 88 226.0254 $7s^2$ 2 8 18 32 18 8 2 Radium	Ac** 89 [227] $6d^1 7s^2$ 2 9 18 32 18 8 2 Actinium	Ku 104 [261] $6d^2 7s^2$ 2 10 32 32 18 8 2 Kurchatovium	105 $6d^3 7s^2$ 2 11 32 32 18 8 2

* LANTHANIDES

Ce 58 140.12 $4f^2 6s^2$ 2 8 20 18 8 2 Cerium	Pr 59 140.9077 $4f^3 6s^2$ 2 8 21 18 8 2 Praseodymium	Nd 60 144.2$_4$ $4f^4 6s^2$ 2 8 22 18 8 2 Neodymium	Pm 61 (145] $4f^5 6s^2$ 2 8 23 18 8 2 Promethium	Sm 62 150.4 $4f^6 6s^2$ 2 8 24 18 8 2 Samarium	Eu 63 151.96$_5$ $4f^7 6s^2$ 2 8 25 18 8 2 Europium	Gd 64 157.2 $4f^7 5d^1 6s^2$ 2 9 25 18 8 2 Gadolinium

** ACTINIDES

Th 90 232.0381 $6d^2 7s^2$ 2 10 18 32 18 8 2 Thorium	Pa 91 231.0359 $5f^2 6d^1 7s^2$ 2 9 20 32 18 8 2 Protactinium	U 92 238.02$_9$ $5f^3 6d^1 7s^2$ 2 9 21 32 18 8 2 Uranium	Np 93 237.0482 $5f^4 6d^1 7s^2$ 2 8 23 32 18 8 2 Neptunium	Pu 94 [244] $5f^6 7s^2$ 2 8 24 32 18 8 2 Plutonium	Am 95 [243] $5f^7 7s^2$ 2 8 25 32 18 8 2 Americium	Cm 96 [247] $5f^7 6d^1 7s^2$ 2 9 25 32 18 8 2 Curium

OF THE ELEMENTS

elements

VI	VII	VIII		
	1 H 1.0079 $1s^1$ Hydrogen (1)			2 He 4.00260 $1s^2$ Helium (2)
8 O 15.999_4 $2p^4$ Oxygen (6 2)	9 F 18.998403 $2p^5$ Fluorine (7 2)			10 Ne 20.17_9 $2p^6$ Neon (8 2)
16 S 32.06 $3p^4$ Sulphur (6 8 2)	17 Cl 35.453 $3p^5$ Chlorine (7 8 2)			18 Ar 39.94_8 $3p^6$ Argon (8 8 2)
Cr 24 51.996 $3d^5 4s^1$ Chromium (1 13 8 2)	Mn 25 54.9380 $3d^5 4s^2$ Manganese (2 13 8 2)	Fe 26 55.84_7 $3d^6 4s^2$ Iron (2 14 8 2)	Co 27 58.9332 $3d^7 4s^2$ Cobalt (2 15 8 2)	Ni 28 58.70 $3d^8 4s^2$ Nickel (2 16 8 2)
34 Se 78.9_6 $4p^4$ Selenium (6 18 8 2)	35 Br 79.904 $4p^5$ Bromine (7 18 8 2)			36 Kr 83.80 $4p^6$ Krypton (8 18 8 2)
Mo 42 95.94 $4d^5 5s^1$ Molybdenum (1 13 18 8 2)	Tc 43 98.9062 $4d^5 5s^2$ Technetium (2 13 18 8 2)	Ru 44 101.0_7 $4d^7 5s^1$ Ruthenium (1 15 18 8 2)	Rh 45 102.9055 $4d^8 5s^1$ Rhodium (1 16 18 8 2)	Pd 46 106.4 $4d^{10} 5s^0$ Palladium (0 18 18 8 2)
52 Te 127.6_0 $5p^4$ Tellurium (6 18 18 8 2)	53 I 126.9045 $5p^5$ Iodine (7 18 18 8 2)			54 Xe 131.30 $5p^6$ Xenon (8 18 18 8 2)
W 74 183.8_5 $5d^4 6s^2$ Tungsten (2 12 32 18 8 2)	Re 75 186.207 $5d^5 6s^2$ Rhenium (2 13 32 18 8 2)	Os 76 190.2 $5d^6 6s^2$ Osmium (2 14 32 18 8 2)	Ir 77 192.2_2 $5d^7 6s^2$ Iridium (2 15 32 18 8 2)	Pt 78 195.0_9 $5d^8 6s^2$ Platinum (1 17 32 18 8 2)
84 Po 209 $6p^4$ Polonium (6 18 32 18 8 2)	85 At [210] $6p^5$ Astatine (7 18 32 18 8 2)			86 Rn [222] $6p^6$ Radon (8 18 32 18 8 2)

Atomic mass — Atomic number

U 92 238.02_9 $5f^3 6d^1 7s^2$ Uranium (2 9 21 32 18 8 2)

Distribution of electrons by unfilled and following completed sublevels — Distribution of electrons by levels

Atomic masses conform with the International Table of 1977. The accuracy of the last significant digit is ±1 or ±3 if it is set in small type. The numbers in brackets are the mass numbers of the most stable isotopes. The names and symbols of elements in parentheses are not generally adopted.

Tb 65 158.9254 $4f^9 6s^2$ Terbium (2 8 27 18 8 2)	Dy 66 162.5_0 $4f^{10} 6s^2$ Dysprosium (2 8 28 18 8 2)	Ho 67 164.930_4 $4f^{11} 6s^2$ Holmium (2 8 29 18 8 2)	Er 68 167.2_6 $4f^{12} 6s^2$ Erbium (2 8 30 18 8 2)	Tm 69 168.9343 $4f^{13} 6s^2$ Thulium (2 8 31 18 8 2)	Yb 70 173.0_4 $4f^{14} 6s^2$ Ytterbium (2 8 32 18 8 2)	Lu 71 174.96_7 $5d^1 6s^2$ Lutetium (2 9 32 18 8 2)

Bk 97 [247] $5f^8 6d^1 7s^2$ Berkelium (2 8 27 32 18 8 2)	Cf 98 [251] $5f^{10} 7s^2$ Californium (2 8 28 32 18 8 2)	Es 99 [254] $5f^{11} 7s^2$ Einsteinium (2 8 29 32 18 8 2)	Fm 100 [257] $5f^{12} 7s^2$ Fermium (2 8 30 32 18 8 2)	Md 101 [258] $5f^{13} 7s^2$ Mendelevium (2 8 31 32 18 8 2)	(No) 102 [255] $5f^{14} 7s^2$ (Nobelium) (2 8 32 32 18 8 2)	(Lr) 103 [256] $6d^1 7s^2$ (Lawrencium) (2 9 32 32 18 8 2)

Answers

1.1. Brownian movement and diffusion. 1.2. Molecular adhesive forces in liquids are much stronger than in gases. 1.3. During painting, the particles of a paint diffuse into pores (intermolecular spaces) of the surface being painted. 1.4. In the region where the salt dissolves a high salt concentration is created. As a result, diffusion takes place to regions with a lower concentration. 1.5. The end faces of Johansson blocks are highly polished; for this reason, molecular adhesive forces come into play. 1.6. Gluing is based on molecular adhesive forces and wetting. 1.7. About 316 mol. 1.8. 24 kg, 28 kg, and 4 kg. 1.9. 1.6 kg. 1.10. 6.02×10^{26} and 6.02×10^{23}. 1.11. About 2600 m^3. 1.12. About 0.14 mol, 8.2×10^{22}. 1.13. About 5.7×10^{-7} m^3. 1.14. The mass of 1 m^3 of brass is 8500 kg. 1.15. 10^5 mol. 1.16. 5.32×10^{-26} and 2.66×10^{-26} kg, 4.65×10^{-26} and 2.33×10^{-26} kg, 6.64×10^{-27} kg. 1.17. 16×10^{-3} kg/mol, 2.66×10^{-26} kg. 1.18. 1.46×10^{20}. 1.19. About 1.54×10^{16}. 1.20. The ratio is approximately 2×10^{14}. 1.21. The ratio is 14.3 (considering that equal volumes of gases under normal conditions contain the same number of molecules). 1.22. 7.3×10^{-26} kg, 1.97 kg/m^3. 1.23. 9.9×10^{-26} kg. 1.24. 2×10^{-3} kg/mol. 1.25. 4.2×10^{-9} m. 1.26. 1.26×10^{-25} kg, 1.01×10^{-28} m^3, 5.8×10^{-10} m. 1.27. 9×10^9 m, 225 times. 1.28. 16×10^{-3} kg/mol. 1.29. $1.4 \times 10^{-5}/V_0$, or 6.3%, where V_0 is the molar volume of the gas under normal conditions. 1.30. 4.0×10^{-8} m. 1.31. 1.12×10^{-4} s^{-1}. 1.32. 550 m/s. 1.33. 6.5×10^{-8} m. 1.34. 9.3×10^{-8} m. 1.35. About 2×10^{-6} m. 1.36. 2.1×10^{-23} kg·m/s. 1.37. About 3.9×10^{-8} m.

2.1. 470 m/s and 510 m/s for air and 450 m/s and 480 m/s for oxygen. 2.2. The mean kinetic energies of translational motion of helium and neon are the same. 2.3. At 322 K. 2.4. 2.25. 2.5. About 631 and 8640 m/s. 2.6. 6.6×10^{-22} J, 1.2×10^{-19} J. 2.7. About 3860 K. 2.8. 1.25×10^4 J, 6.2×10^5 J. 2.9. 290 K. 2.10. 9.9×10^{-11} Pa, high vacuum. 2.11. 1.45×10^{25}. 2.12. About 6.5×10^{-21} J, 9.3×10^{25}. 2.13. 1.3×10^{-6} Pa. 2.14. 1350 m/s, 6.07×10^{-21} J. 2.15. 3.6×10^{12} molecules/cm^3. 2.16. 10^5 Pa. 2.17. 1.3×10^{27}. 2.18. 2.1×10^{-21} J. 2.19. 3.6×10^{21}. 2.20. By 1.7%. 2.21. 4.46×10^{25}, 148 g. 2.22. The thermodynamic temperature and the kinetic energy have increased fourfold.

3.1. About 1.2 l. 3.2. About 64.4 kPa. 3.3. 323 K. 3.4. 614 l. 3.5. 1.82 m^3. 3.6. 44 $\times$ 10^{-3} kg/mol, carbon dioxide. 3.7. 1.2 mol. 3.8. About 2 kg. 3.9. 1.4 mol. 3.10. 4.7 $\times$ 10^{-3} m^3. 3.11. 2.6 times.

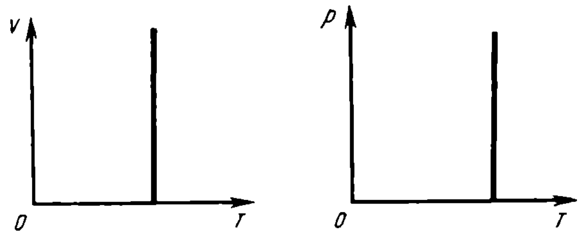


Fig. 153

3.12. About 3.5 kPa. 3.13. 513 K. 3.14. 1.62 mol, 195 kPa. 3.15. 3 MPa. 3.16. 58 $\times$ 10^{-3} kg/mol, 2.2 $\times$ 10^{25}. 3.17. 932 K. 3.18. Hydrogen. 3.19. 111 kg. 3.20. 606 K. 3.21. 73.3 MPa. 3.22. About 10^5 Pa. 3.23. 1.2 kg/m^3. 3.24. 0.46 kg/m^3, 9.6 $\times$ 10^{24} m^{-3}, 444 m/s. 3.25. 28 $\times$ 10^{-3} kg/mol, nitrogen. 3.26. 1.66 kg. 3.27. 0.7 MPa. 3.28. 2 $\times$ 10^5 Pa. 3.29. See Fig. 153. 3.30. 53. 3.31. See Fig. 154; (1) in joules, (2) isotherm *2* for the second equation will lie further from the V-axis since it corresponds to a higher temperature. 3.32. 296, 963, and 17.6 g. 3.33. See Fig. 155. 3.34. Isobar *1* corresponds to the process occurring at a higher pressure. An arbitrarily chosen temperature T_1 corresponds to two values V_1 and V_2 of the volume, and hence to two values p_1 and p_2 of pressure. But the product of pressure and volume at a constant temperature is constant, i.e. $p_1V_1 = p_2V_2$. Therefore, volume V_1 on isobar *1* corresponds to a higher pressure. 3.35. The gas leaks. 3.36. 8.5 l. 3.37. To 225 K. 3.38. By 82 K. 3.39. See Fig. 156. 3.40. 38.3 kN.

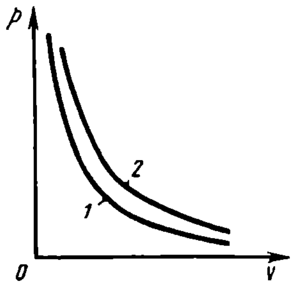


Fig. 154

4.1. 250 J. 4.2. 380 J. 4.3. Aluminium, 1.9. 4.4. 4187 J, 460 J. 4.5. The specific heat of copper is larger than that of aluminium, the specific heat of lead is the smallest of the three. 4.6. Copper weight will be heated to a higher temperature. No, it does not. 4.7. The energy required to heat a gas at constant pressure is spent both to increase the internal energy of the gas and to do work of expansion. 4.8. 25.5 MJ. 4.9. 1 kg. 4.10. 1.73 $\times$ 10^8 J. 4.11. 1026 J. 4.12. 86 l. 4.13. 293 K. 4.14. 68 kJ. 4.15. 2.79 MJ. 4.16. 124 J/(kg·K). 4.17. 169 g. 4.18. 380 J/(kg·K). 4.19. 125 J/(kg·K). 4.20. 781°C. 4.21. 1216 K. 4.22. 120 J/(kg·K), lead. 4.23. 47%. 4.24. 57 kg. 4.25. 4 g.

4.26. 24 K. 4.27. 0.7 K. 4.28. 1.9 × 10^6 m^3. 4.29. 225 MJ. 4.30. 187 K. 4.31. To about 78 m. 4.32. About 0.2 K. 4.33. 0.11 K. 4.34. 25. 4.35. 19 K. 4.36. 174 m/s. 4.37. 250 K. 4.38. 29%. 4.39. 30%. 4.40. 1029 kg. 4.41. About 24%. 4.42. About 17 kW. 4.43. About 550 km. 4.44. 4.28 kJ. 4.45. 26.4 kJ. 4.46. 831 J. 4.47. 7462 J, 1662 J. 4.48. 48.1 kJ. 4.49. 32 × 10^{-3} kg/mol, oxygen. 4.50. 416 J. 4.51.

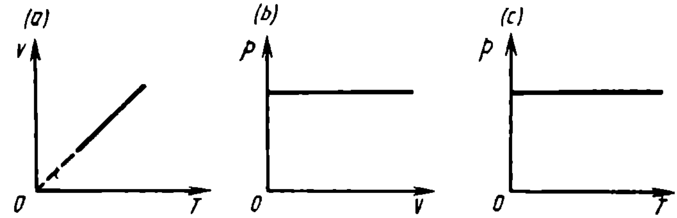


Fig. 155

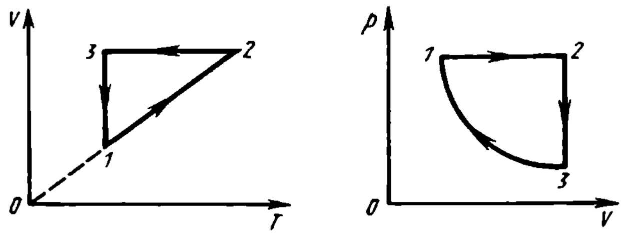


Fig. 156

106 J. 4.52. 1.8 kJ. 4.53. 8.31 J; no, it does not. 4.54. 0.31 kg. 4.55. Yes, it can, for example, during adiabatic expansion or compression. 4.56. The pressure will change more during the adiabatic compression.

5.1. This can be done if the pressure is reduced somehow. 5.2. No, they are not; the internal energy of steam is higher. 5.3. In such vessels, evaporation takes place both from the water surface and through the vessel walls, which contain pores to facilitate cooling. 5.4. The pressure gauge indicates the excess pressure over the atmospheric pressure. 5.5. The number density of molecules increases. 5.6. At night, when the air temperature falls considerably, the water vapour in the air saturates it and partially condenses to form water drops, i.e. mist. 5.7. These substances have different boiling points. 5.8. Boiling water quenches fire more quickly. 5.9. No, it is not. 5.10. The work of expansion of the gas is done at the expense of a decrease in the internal energy of the gas, as a result of which the temperature of the released gas drops, and the valve is covered by frost. 5.11. The difference between the humidities of air in the two climates determines different evaporation rates from the skin. 5.12. No, it is not. In the

critical state, the specific latent heat of vaporization is zero for all liquids. 5.13. 300 J. 5.14. The energy liberated during the condensation of water vapour is about eight times more than that liberated during the condensation of mercury vapour. 5.15. 11.3 MJ, 2.1 MJ. 5.16. 4.0 kJ. 5.17. 2.6×10^5 J. 5.18. 100 g. 5.19. 36.6°C. 5.20. 53 kg. 5.21. About 10°C. 5.22. 71.6°C. 5.23. About 3.3 kg, see Fig. 157. 5.24. 84 kJ. 5.25. 2.25×10^6 J/kg, 0.01×10^6 J/kg, 0.44%. 5.26. 334 g. 5.27. 2.32 kJ. 5.28. 0.04 m³. 5.29. 17 min. 5.30. 51.8 kg. 5.31. 6.5 kg. 5.32. 35%. 5.33. 10^{-2} kg/m³, 78%. 5.34. 19.4×10^{-3} kg/m³, 22°C. 5.35. 17.3×10^{-3} kg. 5.36. 1°C. 5.37. At about 7°C. 5.38. 9.4 mg, 65%. 5.39. To 284.5 K. 5.40. 9.4×10^{-3} kg/m³, 48%. 5.41. At 4°C. 5.42. 18°C. 5.43. 60%, about 11 g. 5.44. 17°C, 13°C. 5.45. 19°C, the relative humidity increases. 5.46. 0.4 m³. 5.47. It decreases to 54%. 5.48. 1.21 kg, 271 g. 5.49. 2.33 kPa. *Hint.* $p_s = p_a/B$, p_a can be determined from the equation of state: $p_a = mRT/MV$, $p_s = mRT/MVB$. 5.50. 12.3×10^{-3} kg/m³, about 48%. *Hint.* Solve the problem by using the equation of state.

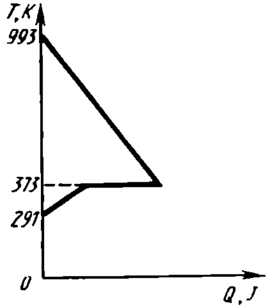


Fig. 157

6.1. The surface tension changes: it decreases when soap is placed in water and increases when sugar is placed. 6.2. Under the action of the forces of surface tension, a pellet acquires the shape for which the surface area is at a minimum. 6.3. Yes, it will. 6.4. Water is a nonwetting liquid for a surface covered with grease. 6.5. See answer to Problem 6.4. 6.6. This is done to create conditions in which the solder is wetting. 6.7. A nonwetting liquid. 6.8. This is done in order to prevent moisture from rising in the capillaries of the building walls 6.9. The presence of capillaries in caked soil makes it possible for water to rise closer to the surface. 6.10. See answer to Problem 6.9. 6.11. 9.6×10^{-3} N. 6.12. 8.4×10^{-3} N. 6.13. 1.37×10^{-3} and 0.12 N .6.14. 1.56×10^{-3} N, towards the water. 6.15. 0.105 N. 6.16. 1.44×10^{-4} J. 6.17. 7.2×10^{-4} J. 6.18. 3.15×10^{-2} N/m. 6.19. 0.022 N/m, ethyl alcohol. 6.20. 2.9 cm. 6.21. 2.56 mg. 6.22. 15 mg. 6.23. In the test tube of pure water. 6.24. 1.14×10^3 kg/m³. 6.25. 0.7 cm.

7.1. Strong heating may recrystallize steel, which makes its mechanical properties worse. 7.2. The shape of the single crystal will change. 7.3. Glass is brittle. 7.4. The melting point of lead is considerably lower than the melting points of other metals. 7.5. Lead can be melted by heating it in a hermetically sealed vessel with water. The temperature of the water can then be raised considerably above the melting point of lead. 7.6. In summer, the temperature of the layers of air close to the surface of the Earth is above 0°C, and small ice crystals formed in the upper cold layers of the atmosphere melt on their way

to the ground. Larger crystals have no time to melt and reach the ground in the form of hail. 7.7. The melting point of the mixture of snow and common salt is lower than 0°C, and hence snow melts even below 0°C. 7.8. 0.4 cm. 7.9. 1.5 mm, 7.5×10^{-4}. 7.10. 4 mm, about 1.4 J. 7.11. It will decrease by a factor of four. 7.12. 9 mm. 7.13. 3×10^7 Pa. 7.14. 1.07×10^{11} Pa. 7.15. 5×10^{-3} m. 7.16. 9.4 kN. 7.17. 25. 7.18. 7.5 cm^2. 7.19. About 4.2 km. 7.20. 3 J. 7.21. 0.3 mm.

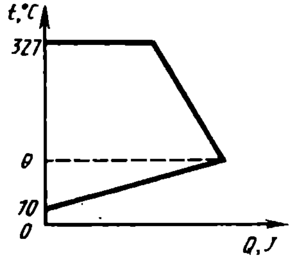


Fig. 158

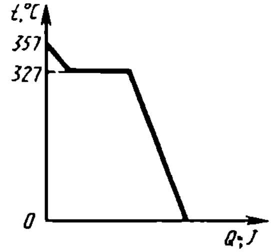


Fig. 159

7.22. No, it will not. 7.23. Alcohol has a lower freezing point. 7.24. About 159 kJ. 7.25. About 380 kJ. 7.26. 25.1°C. 7.27. $Q_l : Q_t = \rho_l\lambda_l : \rho_t\lambda_t = 0.67$. 7.28. 320 K, see Fig. 158. 7.29. About 12 g, see Fig. 159. 7.30. 3.5×10^5 J/kg. 7.31. 19 kg. 7.32. 227 kg. 7.33. 17%. 7.34. About 80 kg. 7.35. 4.5 W. 7.36. 20%. 7.37. 20 g. 7.38. 0.9 kg. 7.39. 0.8. 7.40. 3.35×10^5 J/kg. 7.41. 97 g.

8.1. The alloy has a very small coefficient of linear expansion and it is used to ensure that the mechanism is accurate independently of the thermal conditions. 8.2. The oval hole allows for linear expansion during which the gap between the rails is reduced. 8.3. Different coefficients of linear expansion would weaken the tube during its operation (when it is heated); no, it cannot. 8.4. It will increase. 8.5. The sleeve should be heated. 8.6. The relation is approximate. *Hint.* We write the expression for a unit volume after heating a body through 1 K: $1 + \beta = (1 + \alpha)^3$. After raising the right-hand side to the third power, we can neglect the terms $3\alpha^2$ and α^3 in view of their smallness, which gives $\beta = 3\alpha$. 8.7. This is due to the relatively small thermal expansion coefficient. 8.8. Thermal expansivity should be taken into account. Its value for liquids is much larger than that for the material of which the tanks are made. 8.9. During heating, stresses may emerge in teeth, which causes cracks in the enamel. 8.10. The strip will bend towards the metal with the smaller coefficient of linear expansion; such strips are used in thermal relays used to control temperatures automatically. 8.11. By 12 mm. 8.12. 10 K. 8.13. Tungsten. 8.14. 533.128 and 532.872 m. 8.15. About 903°C. 8.16. By 417 K. 8.17. To 391 K. 8.18. The aluminium component is 330 m longer than the steel component. 8.19. 1.99 m. 8.20. 739.2 kN. 8.21. 15 K. 8.22. 79.2 kN. 8.23. 500°C. 8.24. About 4 kJ. 8.25. 37 kN. 8.26. 16.2 cm^3. 8.27. This relation is approximate. 8.28.

0.306 m^2. **8.29.** 46 K. **8.30.** 0.09 l, 0.087 l. **8.31.** 0.08 l. **8.32.** 1.85×10^3 kg/m^3. **8.33.** 400 K. **8.34.** About 3.6 kg. **8.35.** 299 K. **8.36.** 9.48 l, 7.49 kg. **8.37.** 9.49 l, 7.5 kg. **8.38.** 57.8 m^3, about 40.5 t. **8.39.** 2.14 m^3. **8.40.** About 3.6 m^3. **8.41.** 1.8×10^{-4} K^{-1}. **8.42.** 42.5 kJ.

K

9.1. Two. **9.2.** $+4.8 \times 10^{-19}$ C. **9.3.** No, it cannot. **9.4.** This can be done if the end of the rod held in the hand is coated with an insulating material. **9.5.** The sphere should be brought in contact with two similar neutral spheres. **9.6.** The static charge that may be formed as the car moves is conducted to the earth through the chain. **9.7.** Yes, it can. **9.8.** The surface charge density will increase. **9.9.** At first the ball will touch the electrically charged rod and then will be repelled from it. **9.10.** No, it will not. **9.11.** 0, 2. **9.12.** 1.44 mN, by a factor of 81. **9.13.** 2.1×10^{-8} C. **9.14.** 9 GN, 4.5 GN. **9.15.** 95 km.

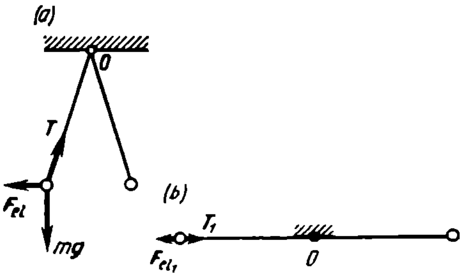


Fig. 160

9.16. (1) 6-9, (2) in kerosene and wax paper, (3) to water. **9.17.** Yes; it will decrease by a factor of 3.2. **9.18.** The drops will approach each other with an initial acceleration of 3.4 m/s^2, which will increase further as the drops approach each other. Yes, it will. **9.19.** 1.84×10^{-7} N. **9.20.** 3.45×10^{11}, about 0.01 N, yes, it will; see Figs. 160*a* and *b*. **9.21.** 2.25×10^{-4} N. **9.22.** 5×10^{-7} C/m^2. **9.23.** 4×10^{-6} C. **9.24.** 4×10^{-7} and 2×10^{-7} C, 0.42 m. **9.25.** 4.2×10^{42}. **9.26.** 1.77×10^{-11} F/m, 5.2×10^{-7} C. **9.27.** No, they do not. **9.28.** This is done for electrostatic shielding. **9.29.** (a) yes, they can, (b) no, they cannot. **9.30.** The field strength will be greatest where the curvature of the surface is the largest, namely, at a point. For this reason, charge may leak from the point, forming an "electric wind". **9.31.** See the answer to Problem 9.30. **9.32.** 0. **9.33.** It means that the electric field acts on a charge of one coulomb placed at this point with a force of 300 N. **9.34.** 7×10^3 V/m. **9.35.** 1.6×10^{-5} N. **9.36.** 3.2×10^{-14} N. **9.37.** 3×10^4 V/m. **9.38.** 2.2. **9.39.** 1.44×10^5 V/m; the field strength will be doubled. **9.40.** 3.75×10^4 V/m, 42 nC. **9.41.** 10^4 electrons. **9.42.** Concentric spherical surfaces. **9.43.** No, it will not. **9.44.** 9×10^3 V/m, 2.7×10^4 V/m. **9.45.** 1.71×10^4 V/m. **9.46.** About 3.6×10^5 V/m. **9.47.** 1.13 kV/m. **9.48.** (a) 0, (b) 0, (c) $\sigma/\varepsilon_0\varepsilon$. **9.49.** 0.1 mg. **9.50.** 9.93 m/s^2. **9.51.** 0, $\sqrt{2}Q/(16\pi\varepsilon_0\varepsilon R^2)$. **9.52.** Due to electrostatic charging by induction. **9.53.** About 5.9×10^5 C, 8.3×10^8 V. **9.54.** 5.6 kV.

9.55. 38 V. 9.56. 3×10^{-4} J. 9.57. 6 kV, 6 kJ. 9.58. 1.44×10^{6} V. 9.59. 2.4×10^{-5} J, 150 V. 9.60. 2.56 kV, 4.1×10^{-16} J. 9.61. 3.5×10^{4} V/m, 2.4 kV. 9.62. 36 V. 9.63. 1.8 kV. 9.64. 8 cm, 10^{-7} s. 9.65. Yes, it can be changed by bringing another conductor close to the charged one. 9.66. $2C$. 9.67. 9×10^{6} km, $R_{\text{sph}}/R_{\text{Earth}} \simeq 1400$. 9.68. The capacitance, charge, and energy of the capacitor will increase, while its voltage will remain unchanged. 9.69. 1.1×10^{-11} F, 1.1×10^{-5} μF, 11 pF. 9.70. About 711 μF. 9.71. 4.5 cm, 3×10^{-7} C/m^2. 9.72. The oxide film used as the dielectric in these capacitors is very thin. 9.73. About 455 and 1786 V; the charge from the smaller sphere will be transferred to the larger sphere until the potentials become equal. 9.74. About 100 pF. 9.75. 1.8 mm. 9.76. 6.6×10^{-10} C. 9.77. 1.1×10^{-7} C; it will decrease by a factor of six. 9.78. 6 mm. 9.79. 5×10^{-9} F. 9.80. 3.8×10^{-5} J. 9.81. (1) 1.09 μF, 2.2×10^{-2} J, (2) 12 μF, 0.24 J. 9.82. 0.6 μF. 9.83. 0.24 μF, 2.4×10^{-5} C. 9.84. 300 V. 9.85. 100 V, 10^{-2} C, 0.5×10^{-2} C. 9.86. 1 μF. 9.87. The capacitor with the higher capacitance. 9.88. 36 J, 13 kW. 9.89. 4.8 μF, 9.6×10^{-4} C. 9.90. In the second case. 9.91. $C = \dfrac{4\pi\varepsilon_0 \varepsilon r^2}{\Delta r}$; for vacuum, 555 pF. 9.92. 0.215 J/m^3.

10.1. 7.5×10^{16}. 10.2. 6.3×10^{21}. 10.3. 10^{3} A. 10.4. 3×10^{23} cm^{-3}. 10.5. 1.1×10^{-4} m/s. 10.6. 3.6×10^{-2} V/m, 518 m. 10.7. 5.4×10^{-4} m^2/(V·s). 10.8. 2.5×10^{4} A/m^2; yes, it is. 10.9. 2 A/m^2. 10.10. About 23 mm. 10.11. 6.2 A/mm^2. 10.12. 0.054 Ω; it will be half. 10.13. Nichrome wire, 2.5 times. 10.14. The current in the tungsten wire is 1.87 times larger than that in the steel wire provided that the internal resistance of the accumulator can be neglected in view of it being small. 10.15. The resistance decreased by a factor of four. 10.16. 12 μF. 10.17. 1.2 A, about 42 Ω. 10.18. 272 m. 10.19. 60 Ω. 10.20. 736 m. 10.21. 2.7 mA. 10.22. 3 V. 10.23. 2.7 V, 2.67 A/mm^2. 10.24. 15 mm^2. 10.25. 7.5 kN. 10.26. 0.017 Ω. 10.27. The resistance of the nichrome wire is larger by a factor of 3.125. 10.28. Steel. 10.29. Will increase by 3.24 Ω for cast iron and by 0.34 Ω for the ferro-aluminium high-resistance alloy. 10.30. The temperature resistance coefficient must be as large as possible. 10.31. 230 Ω. 10.32. 25 K. 10.33. 6 Ω. 10.34. 250 K. 10.35. 558 V. 10.36. 146 mm^2. 10.37. About 1260 kV. 10.38. $I_2 > I_1$, the resistance will increase; the current in the circuit will decrease. The readings on ammeters A_1 and A_2 will be the same. 10.39. 4 Ω. 10.40. 0.033 Ω. 10.41. 22 Ω, 10 A, $U_1 = 50$ V, $U_2 = 70$ V, $U_3 = 100$ V. 10.42. 20Ω. 10.43. 24Ω, 3Ω. 10.44. 3.24, 2.12, 1.2, and 0.37 A. 10.45. 0.03Ω. 10.46. Into two parts; in parallel. 10.47. 25 Ω. 10.48. 44 Ω, 5 A and 0.5 A. 10.49. 0.64 Ω, 3.2, 1.6, and 0.2 A. 10.50. 1.8 Ω and 1.2 Ω. 10.51. 0.55 A, 0.275 A, 1.65 A, and 0.275 A. 10.52. 6 Ω, $I_1 = 2$ A, $I_3 = I_4 = 1$ A. 10.53. 2.1 Ω, 21 V, $U_1 = 12$ V, $I_3 = 7.5$ A, $U_3 = 9$ V, $I_2 = I_4 = I_5 = 2.5$ A, $U_2 = U_4 = U_5 = 3$ V. 10.54. The resistance of the cold filament is lower than that of the hot filament. Consequently, the switch-on current is larger than the switch-off current. 10.55. 4 Ω, 3 A. 10.56. 508 V, 488 V, 475 V. 10.57. 27 V. 10.58. 4.5 Ω, 2 Ω, 1.5 Ω, 0.67 Ω, 0.46 Ω, 0. 10.59. 0.78 Ω. 10.60. 2.6 Ω. 10.61. 1 A, 0.5 and 0.5 A. 10.62. 440 Ω, 110 V, 0.25 A. 10.63. 146 V, 73 V, 0.33 A, 0.17 A. 10.64. 0.014 Ω. 10.65.

About 2.3 Ω, 52.5 A, $I_1 = 22.5$ A, $I_2 = 30$ A, $I_3 = I_4 = I_5 = 7.5$ A. **10.66.** 20 A, $I_1 = 5$ A, $I_2 = 15$ A, $I_3 = I_4 = 2.5$ A, $I_6 = I_7 = 10$ A, $I_8 = 7.5$ A. **10.67.** 2.5×10^{-3} Ω. **10.68.** 60 A. **10.69.** 400 Ω. **10.70.** 10 kΩ. **10.71.** 250 Ω, 225 V. **10.72.** 17 m. **10.73.** 5. **10.74.** It measures the voltage of the external circuit. **10.75.** 1.5 V, 0.24 V. **10.76.** 1.93 V, 0.25 Ω. **10.77.** 0.29 A, 1.02 V, 70%. **10.78.** 230 V, 229 V. **10.79.** 10 A. **10.80.** 0.1 Ω. **10.81.** 10^{-5} C. **10.82.** 1.5×10^{-5} C. **10.83.** 12 V, 2 Ω. **10.84.** 147 V, 120 V. **10.85.** 0.6 A, 3 A. **10.86.** 225 V. **10.87.** 236 V, 220 V. **10.88.** 222 V, 21 mm². **10.89.** 535 V. **10.90.** 230 V, 195 V. **10.91.** 2 A, 540 V. **10.92.** (a) 1.2 A, (b) 17.5 A, (c) 4.7 A. **10.93.** 0.29 A, 0.8 A. **10.94.** About 3.6 A. **10.95.** 0.6 Ω. **10.96.** The internal resistance of acid accumulators is very small, and therefore a large current emerging in a circuit may cause the plate to break down. **10.97.** 2.5 A. **10.98.** 1.25 A, 0.75 A. **10.99.** $I_{par}/I_{ser} = 3$. **10.100.** They should be connected in two parallel groups of three series-connected cells. **10.101.** They should be connected in three parallel groups of four series-connected cells. **10.102.** 1.3 V, the emf will be reduced by a factor of seven. **10.103.** Series connection. **10.104.** It is more advantageous when the external resistance is higher than the internal one. **10.105.** 52 V, 50 V. **10.106.** 3.42 V, 3.36 V. **10.107.** 0.4 Ω. **10.108.** 0.77 A, 1.26 V. **10.109.** $I = \dfrac{\mathscr{E}_1 r_2 + \mathscr{E}_2 r_1}{r_1 r_2 + R\,(r_1 + r_2)} = 1.24$ A.

11.1. 18 C, 216 J. **11.2.** 317 MJ, 88 kWh. **11.3.** 1125 kWh, 5 rubles 76 kopecks. **11.4.** 91%, 1 ruble 6 kopecks. **11.5.** 800 MW, 7×10^9 kWh. **11.6.** 176 kWh. **11.7.** 234 MJ, 65 kWh. **11.8.** 48 W. **11.9.** 4 Ω, $U_{CD} = 0$, $I_5 = 0$, 144 W. **11.10.** 54.5 A. **11.11.** 8.4 kW. **11.12.** 1.35 MW. **11.13.** 2.7 MW. **11.14.** (a) 13.8 MJ, (b) 8.64×10^5 J, (c) 3.46 MJ. **11.15.** 82.5 kW, 74.25 kW. **11.16.** 102 kW, 34 kW. **11.17.** 125 kW, 110 kW, 0.88. **11.18.** 550 W, 5.5 kWh. **11.19.** 2.8 mm². **11.20.** The power is higher for the parallel connection. **11.21.** 16. **11.22.** 0.16 m/s, 12.5 s. **11.23.** 63.4 km/h. **11.24.** 462 A. **11.25.** The melting point of lead is relatively small (327°C). **11.26.** The fuses have different cross-sectional areas. **11.27.** The coil of the 127-V hot plate is thicker. **11.28.** 570 MJ. **11.29.** (a) 110 V, 60 W, 0.22 MJ, (b) 88 V, 132 V; the 40-W bulb is under a voltage higher than the nominal value; 38.4 W, 57.5 W, 0.14 MJ, (c) 138 V, 82 V; the 60-W bulb is under a voltage higher than the nominal value; 94 W, 56 W, 3.38×10^5 J, 0.2 MJ. **11.30.** 26 g. **11.31.** 79%, 0.8 kopecks. **11.32.** 2 Ω. **11.33.** 24 V, 0.1 Ω. **11.34.** 2.64 kW, 90%. **11.35.** The wire should be shortened to 3 m. **11.36.** 20 or 80 Ω. **11.37.** 15 kg.

12.1. No, we cannot. **12.2.** No, it cannot. Otherwise, the oxide film would be destroyed by electrolysis. **12.3.** Current reversal means that its direction changes. When the article becomes an anode during the current reversal, the uneven regions (protrusions) are dissolved more vigorously and the edges or surfaces are smoothed over; yes, it can. **12.4.** 9.32×10^{-8} kg, 1.118×10^{-6} kg, 0.33×10^{-6} kg, 6.25×10^{18}. **12.5.** 283 mg. **12.6.** 362 g. **12.7.** 3.3×10^{-7} kg, 6.6×10^{-7} kg. **12.8.** 2.96×10^{-4} kg. **12.9.** 3.33×10^{-7} kg/C, 0.004×10^{-6} kg/C, 1.2%. **12.10.** Copper. **12.11.** Silver, 1. **12.12.** 1.6×10^{-19} C, 3.2×10^{-19} C, 4.8×10^{-19} C. **12.13.** 1.118×10^{-6} kg/C, 6.8×10^{-7} kg/C. **12.14.**

2.24×10^{22}. **12.15.** 4.13×10^{-3} kg, 1.88×10^{22}, 1.26×10^{22}. **12.16.** 11.8 kg. **12.17.** 64.5 GJ, 0.12 Ω. **12.18.** 278 A/m². **12.19.** 9.65, 18.4 g. Iron is liberated at the cathode and chlorine at the anode. Positive metal ions move to the cathode, while negative chlorine ions move to the anode. **12.20.** 9.4 h. **12.21.** 52 min. **12.22.** 2.6×10^4 kWh, 520 rubles. **12.23.** 4.5 kg. **12.24.** 12.5 h. **12.25.** 2.7×10^{-9} m/s. **12.26.** 312 K. **12.27.** No, it was not. A correction of 0.1 A is required. **12.28.** 1.045×10^{-8} kg/C, 2.38×10^{-7} kg/C, 1.26×10^{-7} kg/C. **12.29.** 50 C, 56 mg. **12.30.** About 29 W. **12.31.** 369 mg. *Hint.* See Fig. 161. The area of the trapezium is equal to the amount of electricity passing through the electrolyte from the moment when the current starts changing. **12.32.** About 28.2 A/m². **12.33.** 1.9×10^{22}.

12.34. $\mathscr{E}_{\text{pol}} = U - \frac{m}{kT} R = 0.8$ V.

13.1. As a result of recombination, charged particles are transformed into neutral atoms, and the gas becomes an insulator. **13.2.** *OA* is the region of dynamic equilibrium between ionization and recombination (the region where Ohm's law is approximately valid), *AB* is the region of the saturation current (the current depends only on the intensity of the ionizer), and *BC* is the region of impact ionization (self-sustained discharge). **13.3.** (1) Charge carriers in a gas are formed by an ionizer, while in solutions they are formed due to the effect of a solvent with thermal motion taken into account. (2) Gases exhibit electron and ion conductivities, while solutions only have ion conductivity. (3) Conductivity in a gas does not obey Ohm's law. **13.4.** The temperature should be increased. **13.5.** No, it is not. For a small mean free path, the electric field strength should be high. **13.6.** The large current in the spark reduces the voltage across the electrodes, which interrupts the discharge. **13.7.** (a) Coronas formed around high-voltage power lines; (b) the corona discharge can be used as a filtre for the purification of flue gas. **13.8.** Glow discharge; electrons, gas ions, and mercury vapour. **13.9.** When the gas is being rarefied, the mean free path increases, and hence the energy $W_i = eE\lambda$ required for ionization can be acquired at a lower electric field strength. **13.10.** The activation of the cathode (coating it with a layer of a more active metal) reduces the work function of cathode electrons and saves energy. **13.11.** The horizontal lines of the graph correspond to saturation currents for different temperatures of the filament. As the temperature of the filament increases, the emission of electrons is increased. **13.12.** The electron beam can be controlled by using electric or magnetic fields. In CRT tubes, this is done using parallel-plate capacitors. Current-carrying coils on the neck of the tube are used to produce magnetic fields. **13.13.** In a gas under normal conditions, charge carriers are absent, or their number is negligibly small. Plasma is a mixture of electrically charged particles, such that the total negative

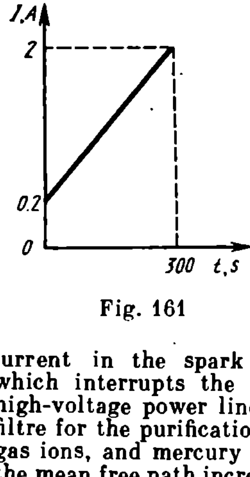


Fig. 161

charge of the particles is equal in magnitude to the total positive charge. The presence of charged particles in the plasma ensures its electrical conductivity. This cannot be said of a gas. **13.14.** 4.3 V. **13.15.** It will increase by a factor of 1.2, 1.6×10^7 m/s, 1.9×10^7 m/s. **13.16.** 1.03×10^6 m/s. **13.17.** 1.03×10^7 m/s, 9.7×10^{-10} s. **13.18.** 7.95×10^5 m/s. **13.19.** 2.18×10^6 m/s. **13.20.** 6.4×10^{-14} A/m^2. **13.21.** 3 μm. **13.22.** 1.26×10^{13} m^{-3}. **13.23.** 1.46×10^{-4} m^2/(V·s).

14.1. Curve *1*. **14.2.** Electrons and holes. **14.3.** The number of holes is equal to the number of electrons; no, it is not. **14.4.** *n*-type (phosphorus is pentavalent); *p*-type (aluminium is trivalent). **14.5.** It will decrease. **14.6.** It will increase by a factor of 2.4×10^4. **14.7.** $b = 2.5 \times 10^{-3}$ m^2/(V·s), $n = 10^{18}$ m^{-3}. **14.8.** It is a semiconductor with terminals to be connected to a circuit. For these thermistors, the current does not depend linearly on the applied voltage. **14.9.** Thermistors are based on the temperature dependence of resistance, while photoresistors make use of the dependence of resistance on illuminance. **14.10.** A transistor is a semiconductor device in the form of a crystal containing two *p-n* junctions. These regions are the base, the collector, and the emitter. **14.11.** If the thickness of the base is much smaller than the mean free path, minority charge carriers getting into it have no time to recombine. **14.12.** $I_{em} = I_b + I_{col}$. **14.13.** The application of semiconductor devices in radio engineering saves energy and makes appliances more compact and durable.

15.1. If we mentally divide the current-carrying conductor into a large number of elements, for each element there exists an element in which the current has the opposite direction, and hence these two elements repel each other. With such interaction between the elements, the conductor will assume the circular shape. **15.2.** Connecting the

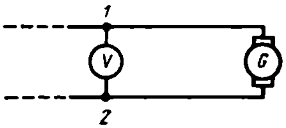


Fig. 162

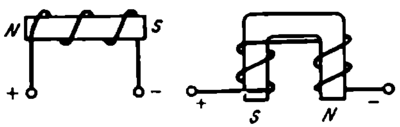


Fig. 163

voltmeter to points *1* and *2* (Fig. 162), we can determine the point with a higher potential. Then we can bring a magnetic needle to one of the conductors and determine the direction of current from the deflection of the north pole. **15.3.** See Fig. 163. **15.4.** See Fig. 164. **15.5.** See Fig. 165. **15.6.** See Fig. 166. **15.7.** The magnet will be repelled by the solenoid and will rise. **15.8.** The coil will be deflected to the right. **15.9.** For the indicated direction of the current, the loop will be rotated clockwise if the north pole is on the right; through 90°. **15.10.** 0.46 N. **15.11.** 0.8 m. **15.12.** 50 A. **15.13.** 2.8 m. **15.14.** 0.4 N, 0.2 N, 0. **15.15.** 1.2×10^{-2} T. **15.16.** We must bring the magnet to the lamp. The filament in the a.c. circuit will oscillate and will

appear as blurred. 15.17. 0.42 N. 15.18. $I = \rho Sg/B$; from N to M. 15.19. 15.7 N. 15.20. 275 A. 15.21. 26 A/m, 3.25×10^{-5} T. 15.22. 5×10^{-5} T, 4 cm. 15.23. 7×10^{-5} and 1×10^{-5} T. 15.24. 3.14×10^{-5} T, 6.9 cm. 15.25. 125 A/m, 10 A. 15.26. 16 A, 16 A/m. 15.27. 1.1×10^{-1} A·m². 15.28. About 2.4 A, 16 A/m. 15.29. 5.7×10^{-6} N·m. 15.30. 3×10^{-4} T. 15.31. 7.5×10^{3} A/m, 9.4×10^{-3} T. 15.32.

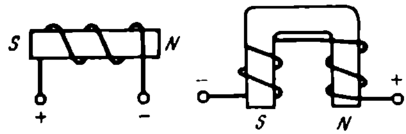


Fig. 164

1.5 mm. 15.33. 800, 280. The permeability of steel decreases. When the magnetization of a ferromagnetic attains saturation, the magnetic induction increases only due to increasing magnetic field strength. 15.34. 0.5 T, 1.5 mWb. 15.35. 7.9×10^{-7} Wb. 15.36. 2×10^{-4} Wb, 2.4×10^{-4} J. 15.37. 144 mWb. 15.38. 7.2×10^{-4} Wb. 15.39. 0.2 T,

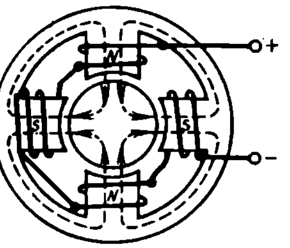


Fig. 165

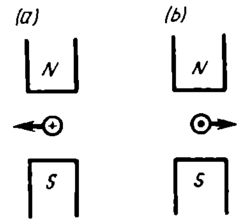


Fig. 166

100. 15.40. At the initial moment, the force is directed vertically downwards; a circle. 15.41. 3.0×10^{-16} N. 15.42. 1.2 m, 4.0×10^{-7} s, 2.5 MHz. 15.43. The velocity vector must be perpendicular to the plane containing vectors **E** and **B**. By hypothesis, the electron moves uniformly in a straight line, and hence $F_L = 0$, $v = E/B$. 15.44. 1.4×10^{-14} J. 15.45. 3.9 mm, 4.4 cm. 15.46. $T_{1_1} = T_{1_2}$. 15.47. $T_1 > T_e$.

16.1. An emf will be induced in the case (a). 16.2. When the poles of the magnet are connected (disconnected), the magnetic field induction changes, which leads to the emergence of an induced current. 16.3. From N to M; from M to N. 16.4. An emf is induced when the frame enters the magnetic field or leaves it since in these cases the magnetic flux piercing the frame changes. 16.5. No. 16.6. The current induced in the upper part of the frame is directed away from us in the cases (a) and (d) and towards us in the cases (b) and (c). 16.7. The

second rod is nonmagnetized. 16.8. The current through the galvanometer flows in the upward direction. 16.9. The induced current has the counterclockwise direction. 16.10. When the magnet enters the cylinder and emerges out of it, the magnetic field produced by the current induced in the cylinder decelerates the motion. If the cylinder is long, $a = g$ in it. 16.11. Yes, it is. The galvanometer reading is 0 since the emf's induced in each half of the conductor are equal in magnitude and have opposite directions. 16.12. 3.4×10^{-2} V. 16.13. 6 A. 16.14. 1.4 T. 16.15. 0.46 V. 16.16. About 20 m/s. 16.17. 2 V, 0.5 A. 16.18. 30°. 16.19. 64 V. 16.20. 6 mWb. 16.21. 100 V. 16.22. 1.5 s. 16.23. 0.8 V, 8 mWb. 16.24. 0.4 Wb. 16.25. 0.1 H. 16.26. A core made of a ferromagnetic must be inserted into the solenoid. 16.27. A self-inductance emf emerging during the disconnection of the circuit has the same direction as the emf of the source, and their joint action causes sparkling. A capacitor connected to the switch eliminates this effect. 16.28. 7 A. 16.29. About 2.7 A. 16.30. About 2.7 J. 16.31. 0.216 J, 0.036 Wb, 0.72 V. 16.32. 1.5×10^{-5} C. 16.33. 4 mA. *Hint.* $I = \mathscr{E}/R$, $\mathscr{E} = -\Delta\Phi/\Delta t$, $\Delta\Phi = \Delta\,(Bl^2)$. Since l^2 does not change, $\Delta\Phi = l^2\Delta B$, $I = l^2\Delta B/(R\,\Delta t)$.

17.1. 5 s. 17.2. 2. 17.3. 2 s, 0.5 Hz. 17.4. 2.51 rad/s, 2.5 s. 17.5. 1 s, 1 Hz. *Hint.* The time elapsed between the impact against the floor and the moment the bounced ball attains the upper point is $T/2$. 17.6. The frequency of vibrations will not change, but the vibrations will attenuate sooner. 17.7. 2 Hz, 0.5 s. 17.8. (a) In phase; (b) and (c) in antiphase. 17.9. $\omega = 2\pi n/t = 3.46$ rad/s. 17.10. 4.77 Hz, 0.21 s. 17.11. 0.5 Hz, 2 s, 3.14 rad/s. 17.12. 4.25 m, 0.3 rad/s, 0.75 rad, 20.9 s, 0.9 rad. 17.13. 0, 2 cm, 0, —1 cm. 17.14. 2π rad, π rad, $\pi/2$ rad. 17.15. 3.54 cm. 17.16. 0.85 m, 0.6 m, —0.6 m. 17.17. —1.04 m, 0.85 m, 0; see Fig. 167. 17.18. 4 m. 17.19. 14.2 cm. 17.20. 6 m, 2 Hz, 0.5 s, $-\pi/2$ rad, $x = 6 \sin \pi\,(4t - 0.5)$. 17.21. 5.2 m. 17.22. π rad, 0. 17.23. (a) $\Delta\varphi = 3\pi/2$ rad or $\Delta\varphi = -\pi/2$ rad; (b) $\Delta\varphi = \pi/4$ rad. *Hint.* (a) $\varphi_1 = 3\pi t/2 + 5\pi/2$, $\varphi_2 = 3\pi t/2 + \pi/2 + \pi/2$; $\Delta\varphi = \varphi_1 - \varphi_2$, (b) $\varphi_1 = \omega t/4 + 7\pi/4$, $\varphi_2 = \omega t/4 - \pi + \pi/2$. 17.24. (a) $x = 2 \sin \pi\,(t + 1/3)$, (b) $x = \sqrt{3} \sin (5\pi/6)\,(t - 1)$. *Hint.* (a) $x = A \times \sin(\omega t + \varphi_0)$, $\sqrt{3} = A \sin \varphi_0$, $1 = A \sin [(2\pi/T)\,(T/4) + \varphi_0] = A \times \sin(\pi/2 + \varphi_0) = A \cos \varphi_0$. Solving these equations together, we obtain $A = 2$, $\varphi_0 = \pi/3$. (b) $0 = A \sin(\omega + \varphi_0)$, consequently, $\omega = -\varphi_0$, $\sqrt{3}/2 = A \sin \omega$, $-3/2 = A \sin 2\omega = 2A \sin \omega \cos \omega$, $-3/2 = \sqrt{3} \cos \omega$, $\cos \omega = -\sqrt{3}/2$, $\omega = 5\pi/6$, $A = \sqrt{3}$. 17.25. (a) $x = 0.09 \sin 40\pi t$, (b) $x = 5 \sin \pi\,(t + 1/4)$, (c) $x = \sin \pi\,(t/3 - 1/4)$. 17.26. 4.17×10^{-2} s, 8.33×10^{-2} s, 8 m. *Hint.* The displacement equal to half the amplitude corresponds to the phase $\pi/6$, or $t_1 = T/12$. Knowing the period, we can determine the time $t_1 = 0.0417$ s. The displacement equal to half the amplitude (from the point of maximum deviation) corresponds to the change in phase by $\pi/3$, or $t_2 = T/6$. Hence $t_2 = 0.0833$ s. During 10 s, the body completes 20 vibrations, and hence $s = n \times 4$ A $= 20 \times 0.4$ m $= 8$ m. 17.27. The force of gravity does. The equilibrium position is. The force is at a maximum in the extreme positions and at a minimum in the equilibrium position. 17.28. $v = A\omega \cos(\omega t + \varphi_0)$. The motion of the ball is not uniformly

variable since the restoring force varies all the time. **17.29.** Yes, they will. **17.30.** The aluminium ball will come to a halt first. **17.31.** The acceleration for the amplitude value of the displacement is maximal and in the equilibrium position is minimal. The velocity has the maximum value in the equilibrium position. **17.32.** $F = mg \sin \alpha = 6.93 \times 10^{-2}$ N. **17.33.** 204 g. **17.34.** 30°. **17.35.** The period of oscillations of a pendulum depends on the free fall acceleration which is different at different latitudes. **17.36.** The length of the pendulum becomes smaller with decreasing temperature and hence the period changes (the clock will be fast). The correct pace can be restored by changing the length of the pendulum by displacing its load. **17.37.** The period will double. **17.38.** The length of the pendulum should be reduced by a factor of 6.05. *Hint.* By hypothesis, the periods of the pendulum on the Earth and on the Moon must be the same. Expressing the periods by the formula $T = 2\pi \sqrt{l/g}$ for the conditions on the

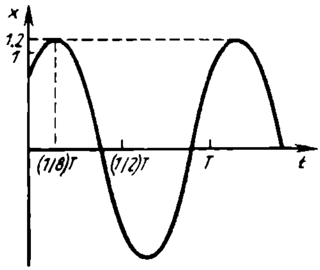


Fig. 167

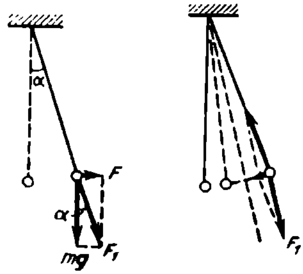


Fig. 168

Moon and on the Earth, we obtain $(l_{\text{Earth}}/l_{\text{Moon}})(g_{\text{Moon}}/g_{\text{Earth}}) = 1$, $l_{\text{Earth}}/l_{\text{Moon}} = g_{\text{Earth}}/g_{\text{Moon}} = 6.05$, $l_{\text{Moon}} = l_{\text{Earth}}/6.05$. **17.39.** 2.004 s, 2.003 s, 2.008 s, 4.92 s. **17.40.** 9.82 m/s². **17.41.** (a) The clock will keep correct time. (b) In the freely falling lift, weightlessness sets in, and the restoring force vanishes. If the pendulum is in an extreme position, no oscillation will take place, while in the equilibrium position the pendulum will uniformly rotate in the vertical plane. **17.42.** (a) The force must be directed upwards and equal to $0.75mg$; (b) the force must be directed downwards and equal to $0.5625mg$. *Hint.* If the period increases under the action of force F, the acceleration imparted by this force has the minus sign. $T_0 = 2\pi \sqrt{l/g}$, $2T_0 = 2\pi \sqrt{l/(g - g_1)}$, $1/2 = \sqrt{(g - g_1)/g}$, $g = 4(g - g_1)$, $g_1 = 0.75g$. **17.43.** $G = mg$, it is directed upwards. **17.44.** (a) $0.5mg$, $\alpha = 0.464$ rad or $\alpha = 26°30'$, (b) $1.2mg$, $\alpha = 0.876$ rad or $\alpha = 50°12'$. *Solution.* The resultant force acting on the ball can be found from the right-angled triangle (Fig. 168): $F_1 = ma_1 \sqrt{m^2g^2 + F^2}$, whence $a = \sqrt{g^2 + F^2/m^2}$. Solving this equation together with the two equations for the periods of oscillations $T_0 = 2\pi \sqrt{l/g}$ and $kT_0 = 2\pi \sqrt{l/g_1}$, where $k = T/T_0$, we obtain $F = mg \sqrt{1/(k^4 - 1)}$. The equilibrium

position corresponds to $\tan\alpha = F/(mg)$. **17.45.** $T = 0.99T_0$ (see solution of Problem 17.44). **17.46.** 1.9 s (see solution to Problem 17.44). **17.47.** 0.9 Hz (see solution to Problem 17.44). **17.48.** Yes, they will be harmonic; the force of gravity changes the position of equilibrium. **17.49.** 1.13 Hz. **17.50.** 126 N/m. **17.51.** 0.03 m, 31.3 rad/s, 0.2 s, $-\pi/2$ rad, $x = 0.03 \sin(31.3t - \pi/2)$. **17.52.** 8×10^{-3} J. 0.14 m/s, 1 m/s^2. **17.53.** 6.32 J, the energy does not depend on the initial phase. **17.54.** 0.192 J, 0.6 m/s, 0.1795 J, 0.0125 J, $x = 1.96 \times 10^{-2} \sin(31.6t - 0.5\pi)$. *Hint.* When the engine is switched off, the spring is stretched to the amplitude of the displacement. In this case, $m \cdot 2g = kA$. **17.55.** $x = 3\cos 0.2 \times \sin(\pi t - 0.2\pi)$, $3\cos 0.2$ m, 0.5 Hz, 2 s, -0.2π rad/s, π rad/s. *Hint.* By using the formula for the sine of double angle $\sin 2\alpha = 2\sin\alpha\cos\alpha$, the equation can be transformed to the conventional form. **17.56.** The pendulum clock is 47.6 s slow. *Solution.* The motion of the rocket can be divided into three parts (Fig. 169). On the first segment, the acceleration is directed upwards, and $a_1 = g$; on the second segment, which lasts until the engine is switched off, the rocket falls freely, and on the third segment of deceleration, the acceleration is directed upwards. Let us first determine t_2 and t_3. In order to find t_2, we shall use the formula for a uniformly accelerated motion: $h = h_0 + v_0 t + gt^2/2$. In the case under consideration, $h = H_2$, $h_0 = H_1$, and $v_0 = v_1$. Taking into account the sign of the acceleration and the quantities $v_1 = gt_1$ and $H_1 = gt_1^2/2$, we can write the equation $H_2 = gt_1^2/2 + gt_1t_2 - gt_2^2/2$, whence $t_2 = t_1 + \sqrt{2(t_1^2 - H_2/g)} = 70$ s. Let us now consider the third segment of the path. Here $v_2 = v_1 - gt_2 = g(t_1 - t_2) = -392$ m/s, $a_3 = -v_2^2/(2H_2) = -8g$, $t_3 = v_2/a_3 = 5$ s. On the first segment, the period of the pendulum clock is $T = 2\pi\sqrt{l/2g} = T_0/\sqrt{2}$. The clock is $\Delta t_1 = t_1(\sqrt{2} - 1) = 12.4$ s fast. On the second segment, the pendulum clock has zero speed, and on the third segment it is fast: $T = T_0/3$, $\Delta t_3 = t_3(3 - 1) = 10$ s. In total, $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3 = 12.4 \text{ s} - 70 \text{ s} + 10 \text{ s} = -47.6$ s. **17.57.** See Fig. 170. **17.58.** 2 m, 3 m, 5 m (Fig. 171). **17.59.** $x = 0$, $x = -\sin\omega t$, 0, -1 m (Fig. 172). **17.60.** (a) $2\sqrt{2}$ m, (b) 5.4 m, (c) 5.4 m (Fig. 173). **17.61.** $\varphi = \arctan 0.4 = 0.38$ rad $= 21°48'$ (Fig. 173c). **17.62.** 3.6 m, $30°27'$ (Fig. 174). **17.63.** The particle vibrates about its equilibrium position. **17.64.** See Fig. 175. **17.65.** (a) To the right, (b) to the left. **17.66.** The propagation velocity depends on surface tension. **17.67.** 0. **17.68.** 16.7 m/s. **17.69.** The velocity of sound is higher in iron; no, it cannot. **17.70.** The sound emerges as a result of friction between metal surfaces in the hinge. Having a larger surface, the door augments the

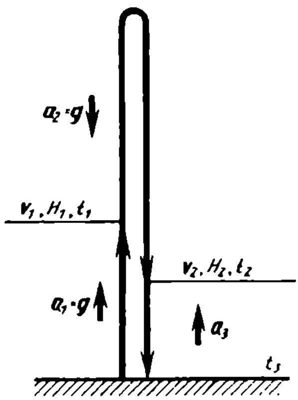


Fig. 169

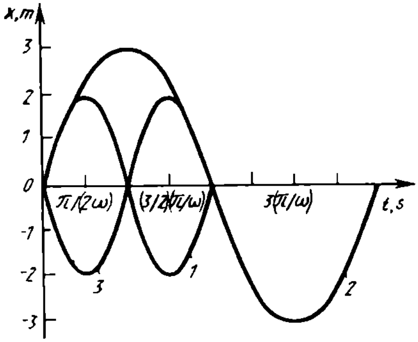


Fig. 170

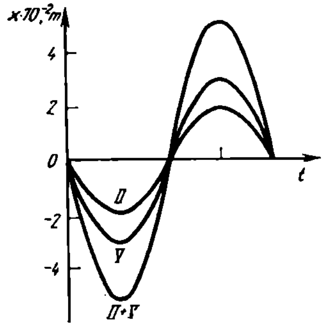


Fig. 171

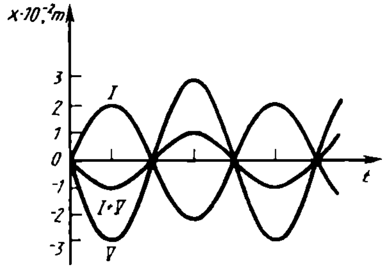


Fig. 172

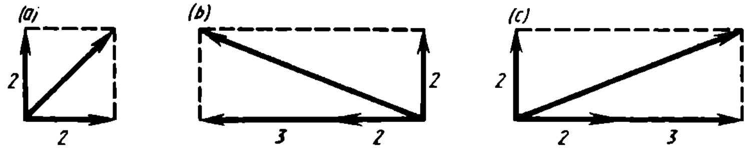


Fig. 173

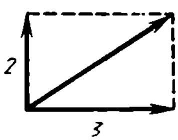


Fig. 174

Fig. 175

vibrations. 17.71. Elastic forces. 17.72. The velocity of propagation of sound depends on temperature and pressure. 17.73. An acoustic wave propagating in the rail is partially reflected from the surface of the rail inwards, and hence the intensity of the sound does not attenuate as rapidly as in the spherical wave propagating in air. 17.74. The sound will not be detected. 17.75. The vibrations of the tuning fork are transmitted through the surface of the table which is much larger than the surface of the tuning fork. 17.76. The sound from the tuning fork in contact with the table will die away sooner since the energy of the tuning fork is spent on the vibration of the table. 17.77. 22.7 cm-11.3 cm. 17.78. 0.66 m. 17.79. 0.5π rad. 17.80. The air column in the cap will start vibrating. 17.81. 1545 Hz. 17.82. 6 cm. 17.83. The frequency of the sound will increase. 17.84. 1445 m/s, 3638 m/s. 17.85. The values of v and T will not change, while λ will increase by a factor of 14.7. 17.86. 1200 m. 17.87. 2.04 mm, 5.1 mm. 17.88. 3 km. 17.89. 13.2 s. 17.90. 2×10^{-3} %, 6 cm.

18.1. 0.02 s, 100 times. 18.2. 4.24 A. 18.3. No, it cannot. 18.4. 177 V, 314 rad/s. 18.5. 63.8 A, $(314t + \pi/4)$ rad, $\pi/4$ rad, 50 Hz. 18.6. 0, 126.6 V, 179 V, 89.5 V, −155 V, −179 V, −126.6 V, 0. 18.7. −3 A, 1.5 A. 18.8. (a) If the plane of the frame is perpendicular to the magnetic field lines, (b) if the plane of the frame is parallel to the magnetic field lines. 18.9. 5×10^{-4} V. 18.10. The induced emf will not change. 18.11. (a) The induced emf decreases by a factor of 1.3125; (b) increases by a factor of 1.19; (c) increases by a factor of 1.143. 18.12. 120. 18.13. 0.046 T. *Hint.* The beginning of oscillations of the emf corresponds to the position of the frame in which its plane is perpendicular to the magnetic field lines. 18.14. 0.15π V, 20π rad/s, 0.1 s, $-\pi/4$ rad, $e = 0.15\pi \sin \pi (20t - 0.25)$. 18.15. The impedance of the conductor increases sixfold; the impedance is equal to the resistance of the conductor. 18.16. 4 Ω, 3 Ω. 18.17. 152.6 Ω. 18.18. The resistance will be infinitely large. 18.19. 31.8 Ω, 0.53 Ω. 18.20. 531 Ω. 18.21. 18.5 Ω. 18.22. 28.2 A. 18.23. 50 μF. 18.24. 127 V, 179 V. 18.25. $U_C = 16$ V, $U_L = 96$ V, 200 Hz. *Hint.* The answer is obtained by solving the two equations: $X_L - X_C = U/I_1$ and $X_L/2 - 2X_C = U/I_2$. 18.26. See Fig. 176. 18.27. (a) See Figs. 177*a* and *b*, (b) see Figs. 177*c* and *d*. 18.28. (a) See Fig. 178*a*, (b) see Fig. 178*b*. 18.29. (a) See Figs. 179*a* and *b*, (b) see Fig. 179*c*. 18.30. (a) See Fig. 180*a*, (b) see Fig. 180*b*. 18.31. (a) See Fig. 181*a*, (b) see Figs. 181*b* and *c*. 18.32. See Fig. 180*a* to Problem 18.30. 18.33. See Fig. 180*b* to Problem 18.30. 18.34. (a) 100 Ω, 1.2 A, 72 V, 96 V, see Fig. 182*a*, (b) 4 Ω, 30 A, 114 V, 42 V, see Fig. 182*b*, (c) 16 Ω, 7.5 A, 180 V, 60 V, see Fig. 182*c*. 18.35. (a) 5 Ω, 44 A, 220 V, 1866 V, 1866 V, see Fig. 183*a*, (b) 42.4 Ω, 5.2 A, 156 V, 327 V, 171 V, see Fig. 183*b*. 18.36. (a) 12 A, 9 A, 15 A, 0.8, see Fig. 184*a*, (b) 36 A, 18 A, 40.25 A, 0.894, see Fig. 184*b*, (c) 18 A, 9 A, 20 A, 0, see Fig. 184*c*. 18.37. (a) 6 A, 12 A, 12 A, 6 A, 1, see Fig. 185*a*, (b) 4 A, 3.6 A, 7.5 A, 6 A, 0.67, see Fig. 185*b*, (c) 9 A, 18 A, 6 A, 15 A, 0.6, see Fig. 185*c*. 18.38. The inductance should be increased by 0.184 H; 120 V. *Hint.* From the condition $\dfrac{R}{\sqrt{R^2 + \omega^2 L^2}} = \dfrac{\sqrt{3}}{2}$ we obtain $\dfrac{L}{R} = \dfrac{1}{\omega\sqrt{3}}$. The equality of the phase differences leads to

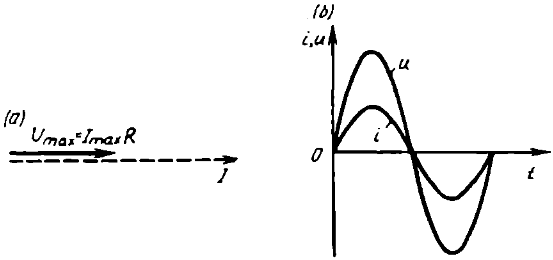


Fig. 176

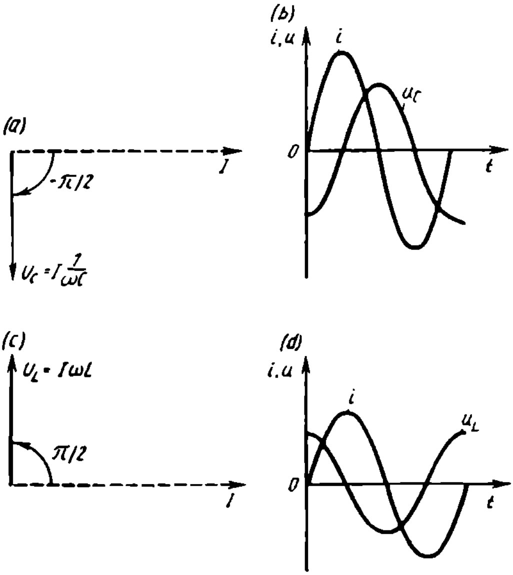


Fig. 177

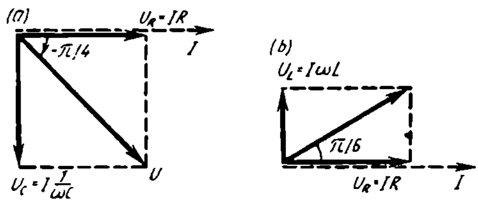


Fig. 178

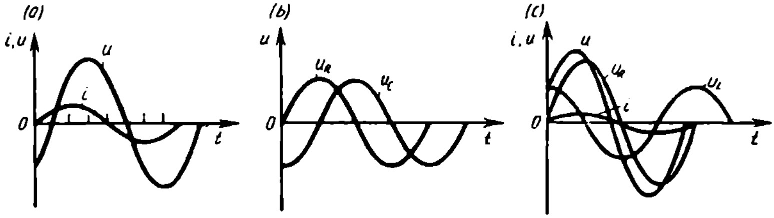


Fig. 179

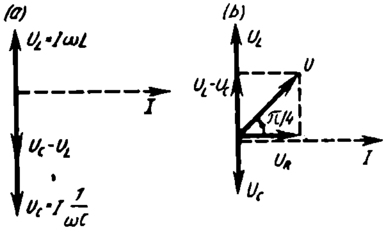


Fig. 180

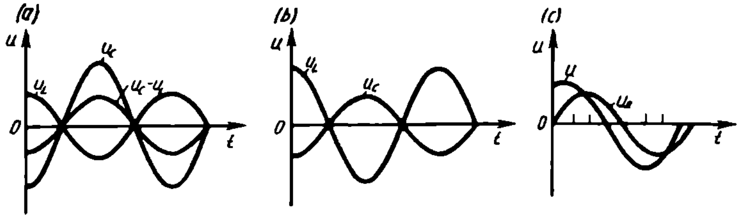


Fig. 181

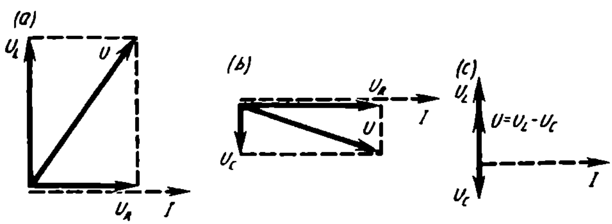


Fig. 182

the equation $\dfrac{R+\Delta R}{\sqrt{(R+\Delta R)^2+\omega^2(L+\Delta L)^2}} = \dfrac{R}{\sqrt{R^2+\omega^2L^2}}$. Hence $\Delta L = \Delta R\,\dfrac{L}{R} = \Delta R\,\dfrac{1}{\omega\sqrt{3}}$. **18.39.** 17.3 Ω, 0.04 H. **18.40.** $U_R =$ 32 V, $U_L = 48$ V, $U_C = 32$ V, $U_{tot} = 36$ V, 26°34′, see Fig. 186. **18.41.** 1594 μF. **18.42.** Hydraulic turbines cannot ensure the required

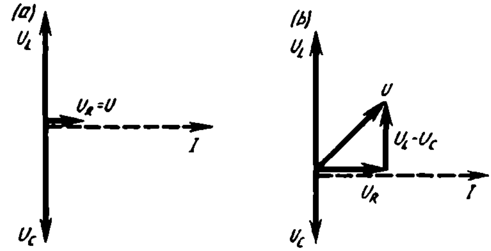


Fig. 183

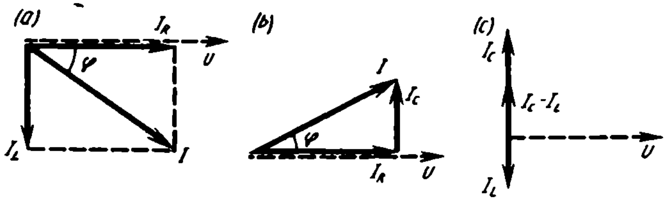


Fig. 184

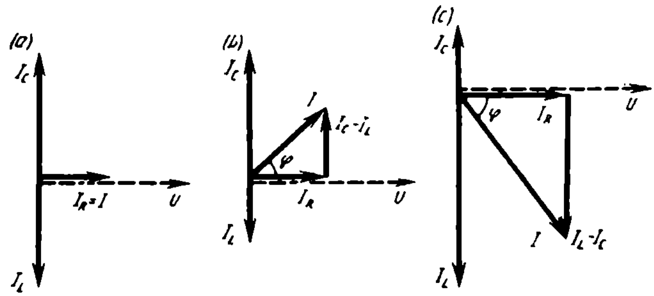


Fig. 185

speed of rotation of the generator's rotor. In order to obtain the standard frequency (50 Hz), multipole generators are used. An increase in the number of pole pairs is equivalent to an increase in the speed of rotation of the rotor. **18.43.** 20. *Hint.* The frequency ν of alternat-

ing current and the speed of rotation n of the rotor are connected through the relation $\nu = pn$, where p is the number of pole pairs. **18.44.** 100 Hz. **18.45.** No, it will not. **18.46.** The transformer consumes an insignificant amount of energy.
18.47. The huge current passing through a short-circuited turn leads to heating of the transformer and may damage it. **18.48.** 2. **18.49.** 10. **18.50.** 696. **18.51.** 16. **18.52.** 0, 28.2 var, 28.2 VA. **18.53.** 72 W, 96 var, 120 VA, 0.6, 20 V. **18.54.** 108 W, 144 var, $R = 27\ \Omega$, $X_L = 36\ \Omega$, $X_C = 72\ \Omega$. **18.55.** 0.995, 380 W, 36 var. **18.56.** 270 VA, 200 W, 182 var, 0.74. *Hint.* For the parallel connection, the power factor can be determined by two methods: $\cos\varphi = P/S$ or $\cos\varphi = \frac{1/R}{\sqrt{1/R^2 + 1/X_C^2}}$ which follows from the vector diagram for currents. **18.57.** 0.019 H, 240 W, 602 var. **18.58.** 60 Ω, 41.8 Ω, 240 W, 344.7 var, 420 VA, 0.57. **18.59.** (a) 1, 160 W, 0; (b) 0.6, 120 W, 160 var. **18.60.** 0.97, 8067 W, 2020 var.

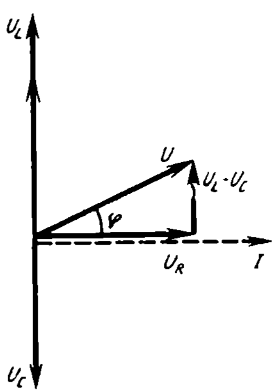


Fig. 186

18.61. $X = 12.5\ \Omega$, $R = 7.2\ \Omega$, 0.865, 400 VA, 6.25 Ω.

19.1. The frequency increases. **19.2.** Yes, it will; the period will increase. **19.3.** 500 m. **19.4.** 7.94×10^4 Hz, 1.26×10^{-5} s. **19.5.** The reflection of electromagnetic waves takes place. **19.6.** Using ultrashort waves penetrating through the ionosphere. **19.7.** 53 and 2640 m. **19.8.** About 200 m, 1.6 MHz. **19.9.** 70 pF. **19.10.** 45 km. **19.11.** The power is proportional to the fourth power of frequency. **19.12.** C_1. **19.13.** The signal is amplified at the expense of the source of electric energy. **19.14.** 2.25×10^8 m/s, 0.13 m. **19.15.** 7.5 mH. **19.16.** 0.5 MHz, 0.1 H. **19.17.** 1.6 mm. **19.18.** 10^{-5} s, 300 m. **19.19.** 4.65 A. **19.20.** 4×10^{-3} s, 4 μF, $E_{mag} = E_{el} = 2 \times 10^{-6}$ J. **19.21.** $L = \frac{\nu_2^2 - \nu_1^2}{4\pi^2 \Delta C \nu_1^2 \nu_2^2} = 2.3$ mH.

20.1. By 40°. **20.2.** By 30°. **20.3.** The mirror should be placed on the path of the rays at an angle of 78 or 12° to the horizontal. **20.4.** (a) 68°, 22°; (b) 22°, 68°; see Fig. 187. **20.5.** The mirror should be placed at an angle of 45° to the horizontal with the mirror surface facing upwards. **20.6.** 160 cm. **20.7.** In the vertical plane. **20.8.** The image of the lamp will be virtual and symmetric about the mirror, see Fig. 188. **20.9.** 11.5 cm, see Fig. 189. **20.10.** The mirrors should be placed at right angles to the lateral sides of the triangle S_1SS_2 at equal distances from the vertices, see Fig. 190. **20.11.** (a) 3, (b) 5. **20.12.** As a result of multiple reflections by the mirrors, a large number of images will be formed. **20.13.** By making the distance to the building large. **20.14.** The rays emitted by the Sun can be treated as parallel, therefore, the point of their convergence will be the focus. **20.15.** 39 cm,

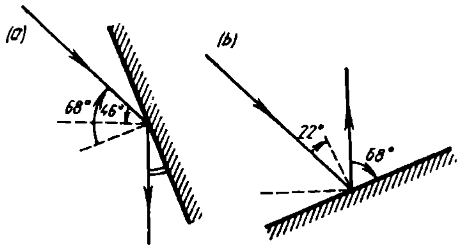


Fig. 187

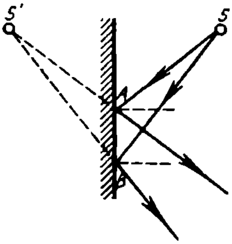


Fig. 188

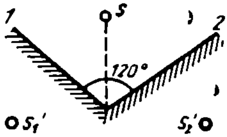


Fig. 189

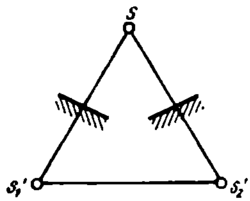


Fig. 190

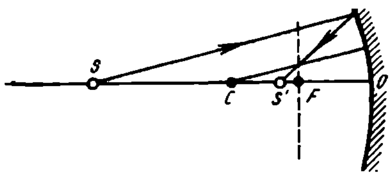


Fig. 191

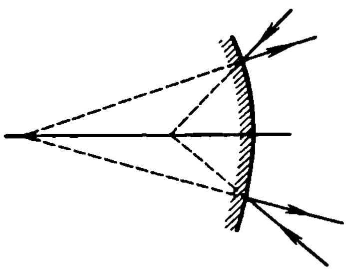


Fig. 192

19.5 cm. **20.16.** See Fig. 191. **20.17.** 1.2 m. **20.18.** 10.5 cm, 21 cm. **20.19.** 20 cm from the mirror. **20.20.** 20 cm. **20.21.** 24 cm, —1.7. **20.22.** 60 cm, 40 cm. **20.23.** 0.2 m from the mirror. **20.24.** 14 cm. **20.25.** The field of vision can be increased by using convex mirrors.

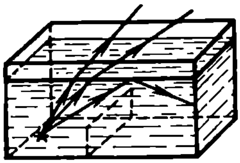


Fig. 193

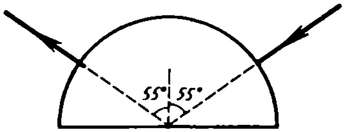


Fig. 194

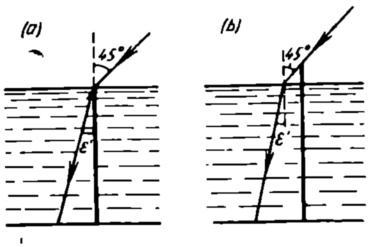


Fig. 195

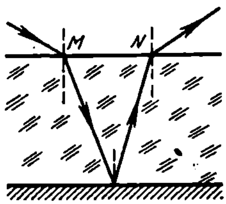


Fig. 196

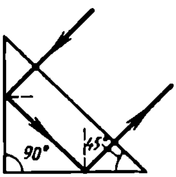


Fig. 197

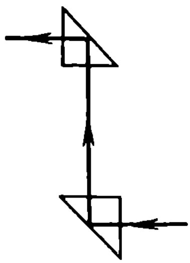


Fig. 198

20.26. 3 m. **20.27.** 1.5 m. **20.28.** —0.2 m, the image is virtual and reduced. **20.29.** 0.24 m, see Fig. 192. **20.30.** 86 cm. **20.31.** 1.8. **20.32.** 35°. **20.33.** 43°. **20.34.** (a) About 8°, (b) 12°20′. **20.35.** 50°. **20.36.** The image of the fish in water is virtual and displaced. **20.37.** 1.41, 54°40′. **20.38.** (1) For $n_1 = n_2$, (2) when $\varepsilon = 0$. **20.39.** About 38°, 225000 km/s. **20.40.** 2.3×10^5 km/s. **20.41.** 2×10^5 km/s, 1.5. **20.42.** 35°. **20.43.** 57°. **20.44.** See Fig. 193. **20.45.** About 24°. **20.46.** 1.5. **20.47.** 33°20′. **20.48.** Air bubbles on the surface of the blackened ball create the

conditions of the total reflection at the water-air interface. **20.49.** The effect is explained by the total reflection. **20.50.** See Fig. 194. **20.51.** 26 cm. **20.52.** 0.75 m, see Fig. 195*a*. **20.53.** 82.5 cm, see Fig. 195*b*. **20.54.** 4.5 cm. **20.55.** 0.4 cm. **20.56.** $MN = 6.6$ cm, see Fig. 196. **20.57.** All points are displaced similarly; the displacement is notice-

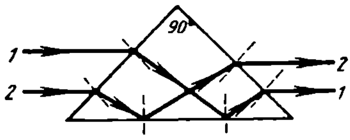


Fig. 199

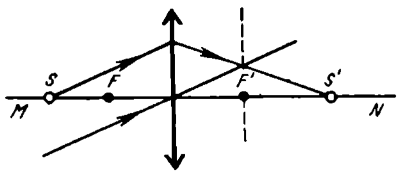


Fig. 200

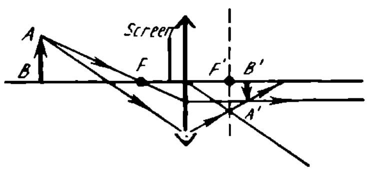


Fig. 201

able if the thickness of the glass is nonuniform. **20.58.** 2.65 cm, 5 cm. **20.59.** See Fig. 197. **20.60.** See Fig. 198. **20.61.** See Fig. 199. **20.62.** 34°. **20.63.** 17°. **20.64.** 35°30′. **20.65.** 38°. **20.66.** $n = \sin\frac{A+\delta}{2} : \sin\frac{A}{2}$. **20.67.** 8 D, 2 D. **20.68.** —4 D, —2.5 D. **20.69.** 25 cm, —20 cm, —50 cm. **20.70.** It will be diverging if the transparent medium in which the lens is placed is optically denser than the material of the lens. **20.71.** See Fig. 200. **20.72.** The brightness of the image will be reduced, see Fig. 201. **20.73.** See Fig. 202. **20.74.** They should be arranged so that the foci of the lenses coincide. **20.75.** 30 cm from the lens, the image will be real and reduced by a factor of two. **20.76.** —10 cm, 30 cm, 20 cm, and about 17 cm. **20.77.** (1) The height of the image decreases, (2) the image is real, inverted, and full-sized, (3) the object should be between the focus and the lens. **20.78.** 10 cm, 10 D. **20.79.** See Fig. 203. In order to determine the optical centre O, we

draw a ray from point A to A'. It will intersect the optical axis at the optical centre. Then we draw another ray from point A parallel to the optical axis, which after the refraction by the lens will pass through the focus F' and will arrive at point A'. In this problem, the converging lens produces a real image. **20.80.** 24 cm. **20.81.** About

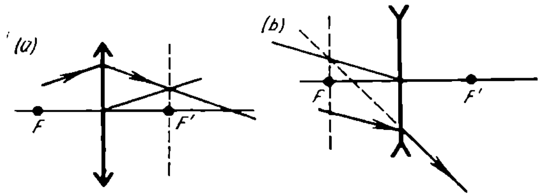


Fig. 202

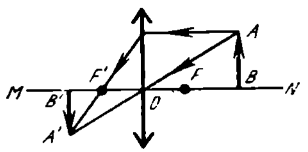


Fig. 203

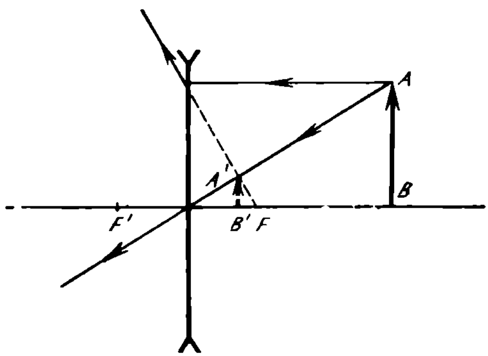


Fig. 204

3.1 D, 4. **20.82.** 0.3 m. **20.83.** 4 cm. **20.84.** 1.75 m from the lens, the image is magnified ×2.5. **20.85.** —2 m, the image is virtual, erect, and magnified ×5. **20.86.** 24 cm. **20.87.** —12 cm, 3.6 cm. **20.88.** 16 cm. **20.89.** Yes, it will, $f = \infty$. **20.90.** 2.5. **20.91.** —7.5 cm, about —13 D; see Fig. 204. **20.92.** 0.5. **20.93.** 4. **20.94.** 0.15 m, about 7 D. **20.95.** 10 D, 5 D. *Hint.* The optical power Φ of the system is the sum of the optical powers of the lenses that are in contact and form the

system. 20.96. About -6.7 D, $\Phi_{\text{concave}} = \dfrac{l}{d\,(l-d)} - \Phi_{\text{convex}}$. 20.97. 30. 20.98. 5 D. 20.99. 2.63 m. 20.100. 4 D. 20.101. For improving the sharpness of the picture. 20.102. 19 m. 20.103. About 8.9 D. 20.104. 7.25. 20.105. 2.5 cm, 16 cm.

21.1. 2.51×10^3 lm. 21.2. 2.4 cd. 21.3. 3.167 lx, 188.5 lm. 21.4. 8×10^{-3} lm. 21.5. 10^5 lm, 10^4 lm. 21.6. 39 lx. 21.7. 25 lx. 21.8. 2. 21.9. $E_1 = 13.3$ lx, $E_2 = 17.4$ lx, in the latter case the illuminance is higher. 21.10. At about 41°. 21.11. The illuminance of the vertical wall is 6.33 times higher. 21.12. 224×10^6 km. 21.13. Solar rays are incident on the slopes at a smaller angle to their surface, consequently, the same surface area absorbs a higher energy per unit time. 21.14. 35 lx. 21.15. 67 lx, about 38 lx. 21.16. 32.3 lx. 21.17. 200 cd. 21.18. 1 m, 0.71 m. 21.19. The illuminance under each bulb is the same and equal to 60 lx. 21.20. 55 lx. 21.21. About 700 cd. 21.22. 19 lx. 21.23. Over the area smaller than 942 m^2. 21.24. About 31 lx. 21.25. 2 m. 21.26. About 9 and 6.7 m. 21.27. 225 cd. 21.28. 0.8 m from the lamp with a lower luminous intensity. 21.29. Yes, it decreases by a factor of n. 21.30. 123 lx. *Hint*. The illuminance on the screen is determined as the sum of the illuminances produced by the lamp and its image formed by the plane mirror. 21.31. 2.6 s. *Hint*. In order to obtain photographs of the same quality, the following condition must be fulfilled during printing: the same amount of energy must be supplied to the photographic paper: $W_1 = W_2$. Since $W = \Phi t = ESt$, we obtain $E_1St_1 = E_2St_2$. Using this equality and having determined the illuminances, we calculate the time t_2.

22.1. A bright spot (enhancement of light) is obtained when the optical path difference of the waves is equal to an even number of half-waves. A dark spot (attenuation of light) is obtained when the path difference of the waves is equal to an odd number of half-waves. 22.2. Constructive interference will occur at point *1* and destructive interference at point *2*. 22.3. The effect is explained by interference. 22.4. The phenomenon appears due to interference of light. Yes, it will; alternating dark and bright fringes are formed, whose colour is determined by the colour of rays incident on the surface of water. 22.5. The interference will be constructive (bright fringe). 22.6. 1.52×10^{-6} m, red. 22.7. When the soap film is illuminated by white light, interference occurs: in various regions, depending on the film thickness, path difference is created such that it causes enhancement of some wavelengths and attenuation of others. 22.8. 3.2 mm. 22.9. 652 nm. 22.10. The radii will decrease. 22.11. It is based on interference: the film coating the objective is chosen so that the waves in the medium part of the spectrum are mainly attenuated. Red and violet rays are attenuated insignificantly and produce a bluish-violet tinge. 22.12. $\lambda = dh/L = 750$ nm, where d is the separation between the light sources, h is the distance between two adjacent interference fringes and L is the distance from the screen to the line connecting the light's sources. 22.13. 4.6 mm. 22.14. About 3.6 m. 22.15. This is explained by diffraction of light. Eyelashes play the role of a diffraction grating. 22.16. Unlike the spectrum obtained with a prism, a diffraction spectrum is uniform-

ly extended in all regions. Besides, spectra of several orders are obtained to the right and to the left of the central bright line. 22.17. 0.4 μm. *Hint*. For small angles, sines can be replaced by tangents; $k\lambda = d \sin \varphi$. 22.18. 018 nm, 484 nm. 22.19. 0.76 μm. 22.20. 2×10^{-6} m, 5×10^3 cm^{-1}. 22.21. 4.34 cm. 22.22. 500. 22.23. 4. *Hint*. $k\lambda = d \sin \varphi$, $k = d \sin \varphi/\lambda$, the spectrum order will be maximum for $\sin \varphi = 1$.

23.1. The velocity of light in water is higher. 23.2. 1.33, 10^{-3} s. 23.3. 5×10^{14} Hz. 23.4. 759 nm, 0.4 μm. 23.5. About 1.23×10^8 m/s. 23.6. 380 nm, 6×10^{14} Hz. 23.7. Green, the colour of the light is determined by the frequency of oscillations which does not change as light propagates from one medium to another. 23.8. The wavelength will increase by 19 nm. 23.9. 1.85×10^8 m/s, 1.8×10^8 m/s, 3.75×10^{14} and 7.5×10^{14} Hz, 494 and 240 nm. 23.10. About 605 nm and 485 nm. 23.11. Ice. 23.12. The word would not be seen through a red medium since it does not transmit green light. 23.13. The ultraviolet radiation with a wavelength of about 290 nm corresponds to the maximum energy. 23.14. Eyes begin to react to complementary colours due to eye strain. 23.15. By investigating solar spectrumby absorption lines (spectral analysis). 23.16. 6000 K. *Hint*. For determining the radiant exitance of the Sun, use the tabulated value of the radius, $e = E/(4\pi R^2)$. The temperature of the Sun's surface is determined from Stefan-Boltzmann's law $e = \sigma T^4$. 23.17. The nebula consists of stars. If it contained a gaseous substance, it would have a line spectrum. 23.18. 4.4×10^9 kg, 4.4×10^{26} kg. 23.19. 3683 K. *Hint*. Make use of Wien's law.

24.1. About 1.67 Pa. 24.2. 141 kJ. 24.3. 3.86×10^{-7} Pa. 24.4. 3.75×10^{-19} J, 2.34 eV. 24.5. 2000. 24.6. 2×10^{-18} J, 12.5 eV. 24.7. 10^{11}. 24.8. 5.52×10^{-19} J, 0.6×10^{-35} kg, 1.84×10^{-27} kg·m/s. 24.9. 5×10^{-16} J, 5.5×10^{-32} kg, 1.66×10^{-23} kg·m/s; 4×10^{-18} J, 4.4×10^{-35} kg, 1.33×10^{-26} kg·m/s. 24.10. Approximately to the east. 24.11. 4.9×10^{-19} J. 24.12. 653 nm. 24.13. 273 nm. 24.14. Yes, it will since $\lambda < \lambda_{th}$. 24.15. Silver will not be charged since $\lambda > \lambda_{th}$. 24.16. 1.44×10^{-19} J. 24.17. About 3.93×10^{-17} J. 24.18. 2.4×10^5 m/s. 24.19. 2.9×10^{-19} J, 1.3×10^6 m/s. 24.20. 2.85×10^8 m/s. *Hint*. The electron velocity obtained indicates that the relativistic formula has to be used. Since the electron work function for molybdenum is negligible in comparison with the energy of a photon, it can be neglected in calculations, and then the formula $h\nu = A + E_0\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$ becomes $h\nu = E_0\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$, where $E_0 = 0.51$ MeV is the rest energy of the electron. 24.21. 2.84×10^{-19} J. *Solution*. $\frac{hc}{\lambda_1} = A + \frac{mv_1^2}{2}$, $\frac{hc}{\lambda_2} = A + \frac{mv_2^2}{2}$, $\frac{hc}{\lambda_1} - \frac{mv_1^2}{2} = A$, $\frac{hc}{\lambda_2} - \frac{mv_2^2}{2} = A$, $\frac{hc}{\lambda_1} - \frac{mv_1^2}{2} = \frac{hc}{\lambda_2} - \frac{mv_2^2}{2}$, $hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{3mv_2^2}{2}$, $\frac{mv_2^2}{2} = \frac{hc}{3}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$, $A = \frac{hc}{\lambda_2} - \frac{hc}{3}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$, $A = \frac{hc}{3}\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$.

25.1. $\Delta l = 0.87$ m — 0.66 m $= 0.21$ m. **25.2.** $v = 0.6c$. **25.3.** 2.61×10^8 m/s; no, it will not. **25.4.** In the form of a square. **25.5.** 1.2×10^{-13} cm, by a factor of 1.67. **25.6.** About 3.57 years. **25.7.** 10 years. **25.8.** About 40.2×10^{12} km; about 2.68×10^5 AU; about 1.2 years. **25.9.** 44.4 years for the observer and 19.4 years for the cosmonaut. **25.10.** 3.73×10^{-13} kg. **25.11.** $0.866c$. **25.12.** $0.968c$. **25.13.** 1.05×10^{-30} kg, 2.09×10^{-30} kg. **25.14.** 9.2×10^7 m/s. **25.15.** 8.33 kg, 2.2×10^4 kg/m^3. **25.16.** The mass and density will not change for the observer in the rocket. **25.17.** $0.89c$. **25.18.** 0.51 MeV, 1.02 MeV. **25.19.** 2.05×10^{-22} kg$\cdot$m/s. **25.20.** 900 m/s. **25.21.** $1.25c$, which contradicts the postulate of the theory of relativity on the impossibility to exceed the velocity of light; $\simeq 0.9c$.

26.1. It is the flow of doubly ionized helium atoms. Their mass is almost 8000 times as large as the electron mass. **26.2.** The Lorentz force does. **26.3.** γ-radiation. **26.4.** γ-radiation is electromagnetic waves following X-rays in the electromagnetic spectrum and differing from them in a higher frequency (of the order of 10^{20} Hz) and hence in a higher energy. **26.5.** 6. **26.6.** An electron is born as a result of conversion of a neutron into a proton. **26.7.** About 4 days, 2×10^{-6} s^{-1}. **26.8.** $T_{Bi} = 5$ days, $T_{Po} = 138$ days. **26.9.** About 1.2%. **26.10.** 7.9 h, 1.3×10^{-5} s^{-1}. **26.11.** 4.5×10^9 years, about 5×10^{-18} s^{-1}. **26.12.** The atom of ${}^3_1H$ contains one proton and two neutrons, helium atom has two protons and two neutrons, aluminium atom has 13 protons and 14 neutrons, uranium atom has 92 protons and 146 neutrons, neptunium atom has 93 protons and 144 neutrons. The number of neutrons is increasing. **26.13.** They differ in the number of neutrons, tabulated chlorine consists of 75% chlorine with the mass number of 35 and 25% chlorine with the mass number 37. **26.14.** The nucleus of silicon atom; the nuclei of helium, carbon, nitrogen, oxygen, neon, magnesium, sulphur, and calcium. **26.15.** 4.8×10^{-19} C, 4.64×10^{-18} C, 1.47×10^{-17} C. **26.16.** 1.2×10^{-8} C. **26.17.** $R_{He} = 2.2 \times 10^{-15}$ m, $R_U = 8.7 \times 10^{-15}$ m, $\rho_{He} = 1.44 \times 10^{17}$ kg/m^3, $\rho_U = 1.4 \times 10^{17}$ kg/m^3. **26.18.** ${}^{227}_{89}Ac \rightarrow {}^{223}_{87}Fr + {}^4_2He$, α-decay. **26.19.** It will be converted into neptunium: ${}^{237}_{92}U \rightarrow {}^{237}_{93}Np + {}^{0}_{-1}e$. The mass number does not change. The nucleus will be shifted to the right. **26.20.** A neutron. ${}^7_3Li + {}^2_1H \rightarrow {}^8_4Be + {}^1_0n$. **26.21.** $Z = 6$, $A = 14$, carbon isotope: ${}^{14}_7N + {}^1_0n \rightarrow {}^{14}_6C + {}^1_1p$. **26.22.** It is an iodine atom ${}^{127}_{53}I$. **26.23.** 1.02 MeV, 2.5×10^{20} Hz. **26.24.** ${}^{14}_7N + {}^{15}_7N + 2{}^1_0n \rightarrow {}^{15}_7N + {}^{16}_7N + \gamma$. **26.25.** The energy must be higher than the sum of the rest energies of an electron and a positron, i.e. larger than 1.02 MeV. **26.26.** ${}^{27}_{13}Al + {}^4_2He \rightarrow {}^{30}_{14}Si + {}^1_1p$, the nucleus of silicon isotope. **26.27.** Identical tracks are left by identical particles, viz. nuclei of helium atoms formed in the reaction ${}^{11}_5B + {}^1_1p \rightarrow 3{}^4_2He$. **26.28.** The missing particle is an α-particle. **26.29.** Curium ${}^{242}_{96}Cm$, neutron. **26.30.** A proton is converted into a neutron. **26.31.** 0.12 amu or 1.99×10^{-28} kg. **26.32.** 0.07 amu, about 65.2 MeV. **26.33.** 1.97 amu or 3.27×10^{-27} kg, 1833 MeV or about 2.9×10^{-10} J. **26.34.** (1) Energy is absorbed, (2) liberated, (3) liberated. **26.35.** 4.0 MeV or 6.4×10^{-13} J. **26.36.** 770 kg. **26.37.** 1.67 t. **26.38.** In all cases, the energy is approximately equal to 2×10^{13} J. **26.39.** About 64 g. **26.40.** 4.8×10^{12} J. **26.41.** In fast neutron reactors, natural uranium is utilized with a higher

efficiency, and besides the nuclear fuel (plutonium) is reproduced. **26.42.** Gamma-quantum, electron, neutrino, and proton. **26.43.** 3×10^{-11} J. **26.44.** About 1.13×10^{10} J. **26.45.** About 3.2×10^{19} J.

27.1. 8 min 19 s. **27.2.** 4.02×10^{13} km, 1.31 pc. **27.3.** 5.54 h. **27.4.** 6.2×10^{5} pc. **27.5.** Sirius, Canopus. **27.6.** Canis Major; this constellation is in the southern part of the celestial sphere. **27.7.** 8.9 years. **27.8.** 3.04×10^{4} pc. **27.9.** Vega; Lyra. **27.10.** The north and south points on the celestial meridian and points of east and west on the celestial equator. **27.11.** Serpens constellation. **27.12.** The list of constellations may change depending on geographical latitude. **27.13.** See the answer to Problem 27.12. **27.14.** The Sun rises exactly at the east and sets exactly at the west only during the vernal and autumnal equinoxes. **27.15.** Aquarius, Capricorn, Sagittarius, Scorpius, and Libra. **27.16.** At the poles. **27.17.** About 55°. **27.19.** 30°. **27.21.** By 1°. **27.22.** 0, 0. **27.23.** 0, 12 h. **27.24.** 57°42′. **27.25.** 1.9°. **27.26.** 1.392×10^{6} km. **27.27.** 5.71×10^{7} km. **27.28.** 8.2 pc. **27.29.** 84.01 years. **27.30.** 9.539 AU. **27.31.** 5.203 AU, 778×10^{6} km. **27.32.** In the form of a narrow crescent. **27.33.** This is valid for any latitude. **27.34.** In the first quarter. **27.35.** 7.5×10^{4} years. **27.36.** 0.545″. **27.37.** The inclination of the Venus axis to the plane of its orbit is close to 90°. **27.38.** 1400 kg/m^3. **27.39.** 7.7×10^{3} m/s.

TO THE READER

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(MOSTLY PHYSICISTS AND ENGINEERS)

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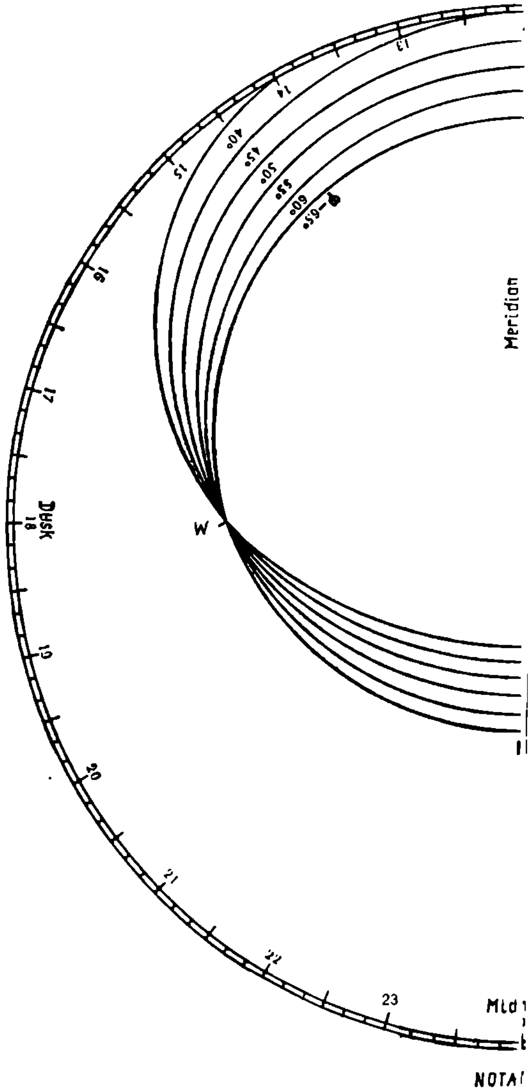

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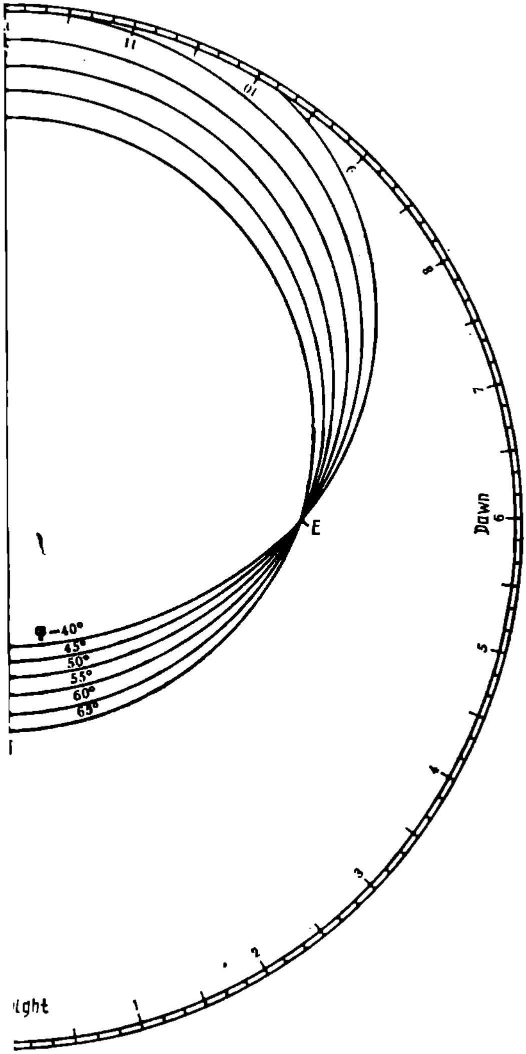

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